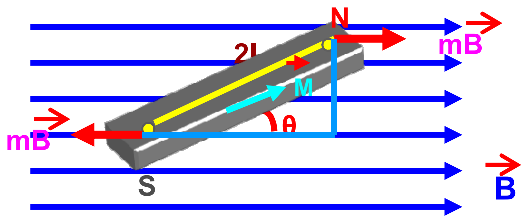
Torque on a magnetic dipole (bar magnet) in a uniform magnetic field

CLASS-XII

SUBJECT : PHYSICS
CHAPTER NUMBER: 05
CHAPTER NAME : MAGNETISM AND MATTER

Torque on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:

The forces of magnitude mB act opposite to each other and hence net force acting on the bar magnet due to external uniform magnetic field is zero. So, there is no translational motion of the magnet.



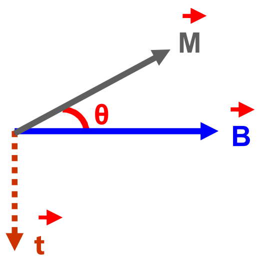
However, the forces are along different lines of action and constitute a couple. Hence the magnet will rotate and experience torque.

Torque = Magnetic Force x ⊥ distance

t = mB (2l sin θ)

= M B sin θ

$$\vec{t} = \vec{M} \times \vec{B}$$



Direction of Torque is perpendicular and into the plane containing $\vec{M}$ and $\vec{B}$.

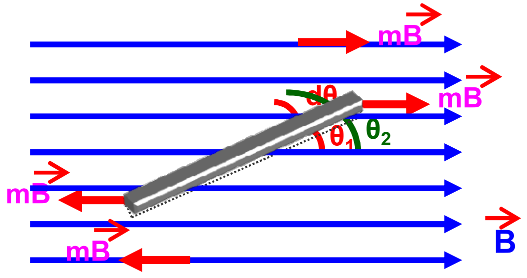
Work done on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:

$dW = \tau d\theta$

$= M B \sin\theta \, d\theta$

$W = \int_{\theta_1}^{\theta_2} M B \sin\theta \, d\theta$

$W = M B (\cos\theta_1 - \cos\theta_2)$



If Potential Energy is arbitrarily taken zero when the dipole is at 90°, then P.E in rotating the dipole and inclining it at an angle θ is

Potential Energy = $- M B \cos\theta$

Potential energy is minimum (= –MB) at θ = 0° (most stable position) and maximum (= +MB) at θ = 180°

(most unstable position).

Note:

Potential Energy can be taken zero arbitrarily at any position of the dipole.

Torque on a Magnetic Dipole (Bar Magnet) in Uniform Magnetic Field:

- When $\vec{M}$ is aligned with $\vec{B}$, the torque is zero.
- In the non uniform field, the net force will be non-zero. In addition, there will, in general, be a torque on the system as before.
- In the non uniform field, when $\vec{M}$ is parallel to $\vec{B}$ or antiparallel to $\vec{B}$. In either case, the net torque is zero, but there is a net force on the dipole if $\vec{B}$ is not uniform

Determination of magnitude of magnetic field using a compass needle

- We may at times be required to determine the magnitude of $\vec{B}$ accurately. This is done by placing a small compass needle of known magnetic moment $\vec{M}$ and moment of inertia I and allowing it to oscillate in the magnetic field. The equation of motion of the dipole is

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$$I \frac{d^2\theta}{dt^2} = -MB \sin\theta$$

$$\frac{d^2\theta}{dt^2} + \frac{MB}{I} \sin\theta = 0$$

For small values of θ in radians, the above equation becomes

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \text{ (angular SHM)}$$

The angular frequency and time period are respectively

$$\omega = \sqrt{\frac{MB}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$\Rightarrow \boxed{B = \frac{4\pi^2 I}{MT^2}}$$

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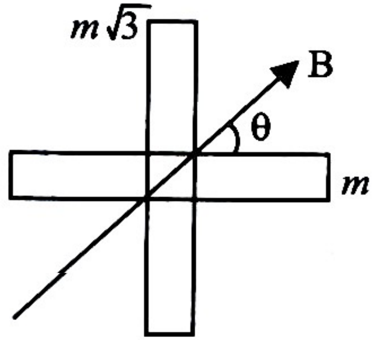
NUMERICAL

The magnetic needle has magnetic moment $6.7 \times 10^{-2} A \cdot m^2$ and moment of inertia $I = 7.5 \times 10^{-6} \text{kg} \cdot m^2$. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field? (NCERT)

$$B = \frac{4\pi^2 I}{MT^2}$$

Numerical

Question: Two magnets of magnetic moments m and $m\sqrt{3}$ are joined to form a cross. The combination is suspended in a uniform magnetic field B. The magnetic moment m now makes an angle θ with the field direction find the value of angle θ.

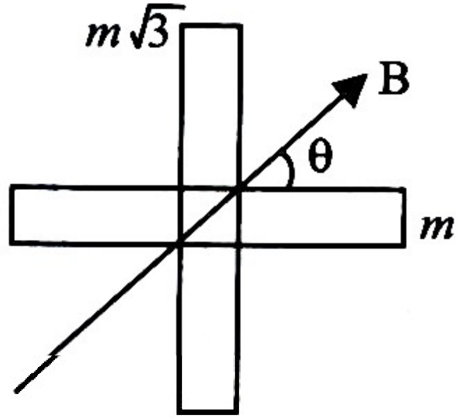


Numerical

In the position of equilibrium

$$mB\sin\theta = \sqrt{3}\, mB\sin(90^\circ - \theta) = \sqrt{3}\, mB\cos\theta$$

$$\tan\theta = \sqrt{3} \quad \text{or } \theta = 60^\circ$$



Numerical

Question: Answer the following:

a) A magnetic needle is placed in a uniform magnetic field with its axis tilted w.r.t its position of stable equilibrium. Deduce an expression for the time period of (small amplitude) oscillation of this magnetic dipole about an axis, passing through its centre and perpendicular to its plane.

b) If this magnet is replaced by a combination of two similar bar magnets, placed over each other, how will the time period vary?

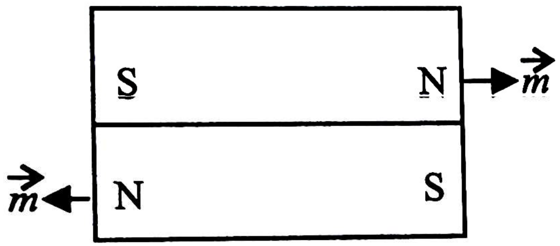
Numerical

(*b*) (*i*) If two magnets are placed as shown below

$m_{net} = 0$

Net moment of inertia = 2I

So $T = \infty$

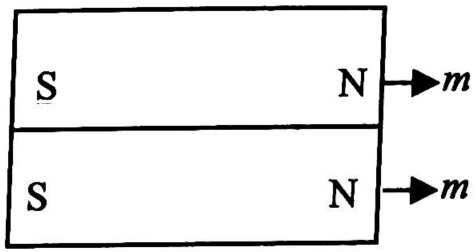


(*ii*) $m_{net} = 2m$

$I_{net} = 2I$

$$T = 2\pi\sqrt{\frac{2I}{2mB}} = 2\pi\sqrt{\frac{I}{mB}}$$

T will remain same.



NUMERICAL

A short bar magnet placed with its axis at 30° with an external field of 800 G experiences a torque of $0.016 Nm$.

(a) What is the magnetic moment of the magnet?

(b) What is the work done in moving it from its most stable to most unstable position? (NCERT)

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SOLUTION

$$\tau = MB\,sin\,\theta$$

$$\mathrm{M} = 0.4A \cdot m^2$$

$$W = -MB\,cos\,180^\circ + MB\,cos\,0^\circ = 2mB$$

$$\Rightarrow W = 0.064J$$

NUMERICAL

A bar magnet of magnetic moment 6J/T is aligned at 60^0 with a uniform external magnetic field of 0.44T. Calculate

(a)The work done in turning the magnet to align its magnetic moment:

(i) Normal to the magnetic field

(ii) Opposite to the magnetic field

(b)Torque on the magnet in the final orientation in case (ii).

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(a) Given, magnetic moment, $M = 6$ J/T

Aligned angle, $\theta_1 = 60°$

External magnetic field,

$B = 0.44$ T

(i) When the bar magnet is align normal to the magnetic field, i.e. $\theta_2 = 90°$

∴ Amount of work done in turning the magnet,

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$
$$= -6 \times 0.44 (\cos 90° - \cos 60°)$$
$$= +6 \times 0.44 \times \frac{1}{2} \quad \begin{pmatrix} \because \cos 90° = 0 \\ \text{and } \cos 60° = 1/2 \end{pmatrix}$$
$$= 1.32 \text{ J}$$

(1)

(ii) When the bar magnet align opposite to the magnetic field, i.e. $\theta_2 = 180°$

$$\therefore W = -MB(\cos 180° - \cos 60°)$$
$$= -6 \times 0.44 \left(-1 - \frac{1}{2}\right) \quad (\because \cos 180° = -1)$$
$$= 6 \times 0.44 \times \frac{3}{2}$$
$$= 3.96 \text{ J}$$

(1)

(b) We know that, torque,

$$\tau = \mathbf{M} \times \mathbf{B} = MB\sin\theta$$

For case (ii), $\theta = 180°$

$$\therefore \quad \tau = MB\sin 180° \quad (\because \sin 180° = 0)$$
$$= 0$$

∴ Amount of torque is zero for case (ii). **(1)**

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MCQ Ques

1. A bar magnet is held perpendicular to a uniform field. If the couple acting on the magnet is to be halved, by rotating it, the angle by which it is to be rotated is
(a) 30°
(b) 60°
(c) 45°
(d) 90°

2. A magnetic needle is kept in a non-uniform magnetic field. It experiences
(a) a torque but not a force.
(b) neither a force nor a torque.
(c) a force and a torque.
(d) a force but not a torque.

Assertion: When the magnet lies along the direction of the magnetic field then torque is minimum.

Reason: The direction of the torque $\vec{\tau}$ is given by the right-hand screw rule.

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Ans: (b)

THANKING YOU

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