Combination of cells in series and parallel

CLASS-XII

SUBJECT : PHYSICS
CHAPTER NUMBER: 03
CHAPTER NAME : CURRENT ELECTRICITY

CHANGING YOUR TOMORROW

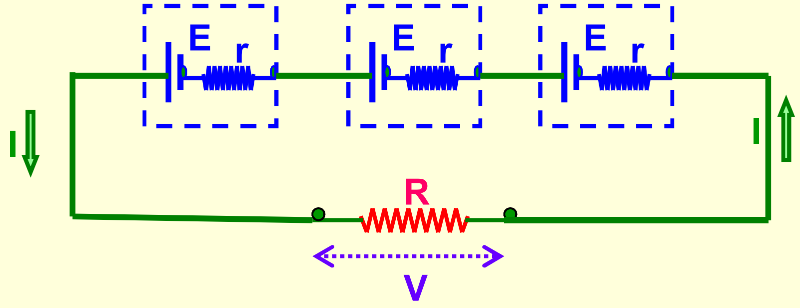
Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: **1800 120 2316**
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Cells in Series combination:

Cells are connected in series when they are joined end to end so that the same quantity of electricity must flow through each cell.

NOTE:

1. The emf of the battery is the sum of the individual emfs
2. The current in each cell is the same and is identical with the current in the entire arrangement.
3. The total internal resistance of the battery is the sum of the individual internal resistances.



Total emf of the battery = nE (for n no. of identical cells)

Total Internal resistance of the battery = nr

Total resistance of the circuit = nr + R

Current $I = \frac{nE}{nr + R}$

(i) If R << nr, then I = E / r (ii) If nr << R, then I = n (E / R)

Conclusion: When internal resistance is negligible in comparison to the external resistance, then the cells are connected in series to get maximum current.

NOTE

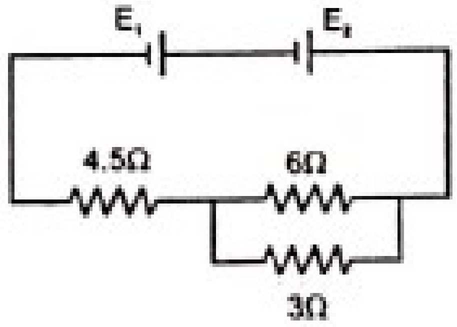
For cells being wrongly connected, their EMFs are taken to be negative but internal resistances are positive. E.g.

In the given figure net emf = (10-4)V= 6V ,

but net internal resistance = $r_1 + r_2$

NUMERICAL

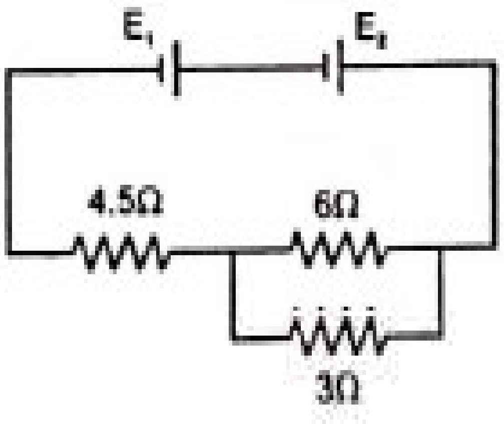
In the given circuit cells E_1 and E_2 have EMFs 3V and 5V respectively and internal resistances 0.3Ω and 1.2Ω respectively. Find the current through the circuit.



NUMERICAL

In the given circuit cells E_1 and E_2 have EMFs 3V and 5V respectively and internal resistances 0.3Ω and 1.2Ω respectively. Find the current through the circuit.

SOLUTION



For the given circuit ;

$$E_{eq} = E_1 + E_2 = 3V + 5V = 8V$$

$$r_{eq} = r_1 + r_2 = 0.3\Omega + 1.2\Omega = 1.5\Omega$$

$$R_{eq} = 4.5\Omega + \frac{6\times3}{6+3}\Omega = 6.5\Omega$$

So current is ;

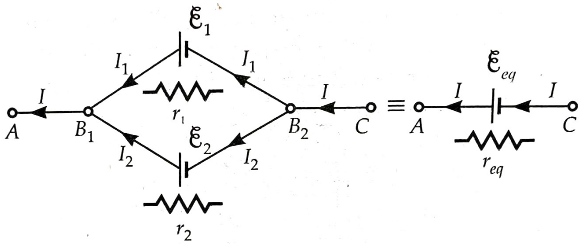
$$I = \frac{E_{eq}}{r_{eq} + R_{eq}} = \frac{8V}{1.5\Omega + 6.5\Omega} = 1A$$

ODM PUBLIC SCHOOL

Cells in Parallel combination

In a parallel combination of cells, the potential difference of each cell is the same, and current through the combination is equal to the sum of the current through individual cells.

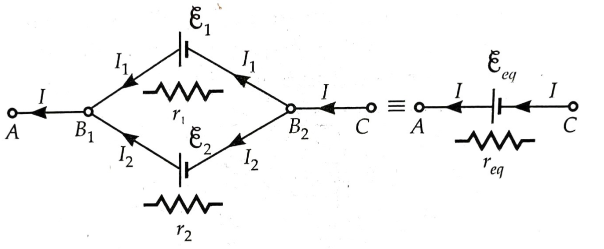
Let two cells of EMFs ξ_1 and ξ_2 with internal resistances, r_1 and r_2 respectively are in parallel across a resistance R.



I = current through the combination.

Cells in Parallel combination

In a parallel combination of cells, the potential difference of each cell is the same, and current through the combination is equal to the sum of the current through individual cells.



Let two cells of EMFs ξ_1 and ξ_2 with internal resistances, r_1 and r_2 respectively are in parallel across a resistance R.

I = current through the combination.

For the first cell $V = \xi_1 - I_1 r_1$

$$\Rightarrow I_1 = \frac{\xi_1 - V}{r_1} = \frac{\xi_1}{r_1} - \frac{V}{r_1}$$

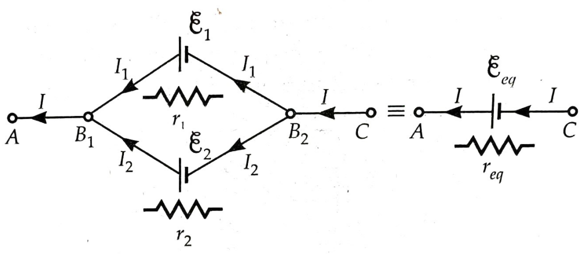
Similarly for the second cell $I_2 = \frac{\xi_2}{r_2} - \frac{V}{r_2}$

Now for equivalent circuit $I = \frac{\xi_{eq}}{r_{eq}} - \frac{V}{r_{eq}}$

Cells in Parallel combination

As in parallel combination $I = I_1 + I_2$

$$\Rightarrow \frac{\xi_{eq}}{r_{eq}} - \frac{V}{r_{eq}} = \frac{\xi_1}{r_1} - \frac{V}{r_1} + \frac{\xi_2}{r_2} - \frac{V}{r_2} = \left(\frac{\xi_1}{r_1} + \frac{\xi_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$



$$\Rightarrow \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \Rightarrow r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

And $\frac{\xi_{eq}}{r_{eq}} = \frac{\xi_1}{r_1} + \frac{\xi_2}{r_2} \Rightarrow \frac{\xi_{eq}}{r_{eq}} = \frac{\xi_1 r_2 + \xi_2 r_1}{r_1 r_2} \Rightarrow \frac{\xi_{eq}}{r_1 r_2 / (r_1 + r_2)} = \frac{\xi_1 r_2 + \xi_2 r_1}{r_1 r_2} \Rightarrow \xi_{eq} = \frac{\xi_1 r_2 + \xi_2 r_1}{r_1 + r_2}$

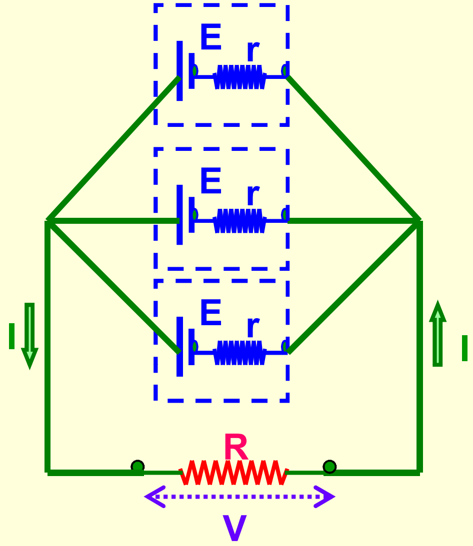
Current in the circuit is $I = \frac{\xi_{eq}}{r_{eq} + R}$

For large no. of cells in parallel, $\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots\ldots\ldots$ $\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + \ldots\ldots\ldots$ And $I = \frac{\xi_{eq}}{r_{eq} + R}$

ODM PUBLIC SCHOOL

Cells in Parallel combination:

Cells are said to be connected in parallel when they are joined positive to positive and negative to negative such that current is divided between the cells.



NOTE:

1. The emf of the battery is the same as that of a single cell.
2. The current in the external circuit is divided equally among the cells.
3. The reciprocal of the total internal resistance is the sum of the reciprocals of the individual internal resistances.

Total emf of the battery = E

Total Internal resistance of the battery = r / n

Total resistance of the circuit = (r / n) + R

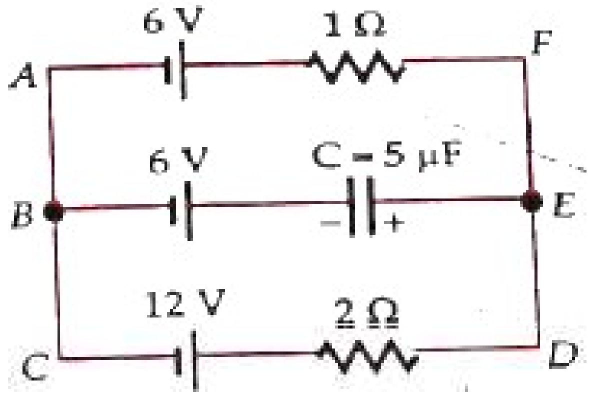
$$\text{Current } I = \frac{nE}{nR + r}$$

(i) If R << r/n, then I = n(E / r) (ii) If r/n << R, then I = E / R

Conclusion: When external resistance is negligible in comparison to the internal resistance, then the cells are connected in parallel to get maximum current.

Numerical

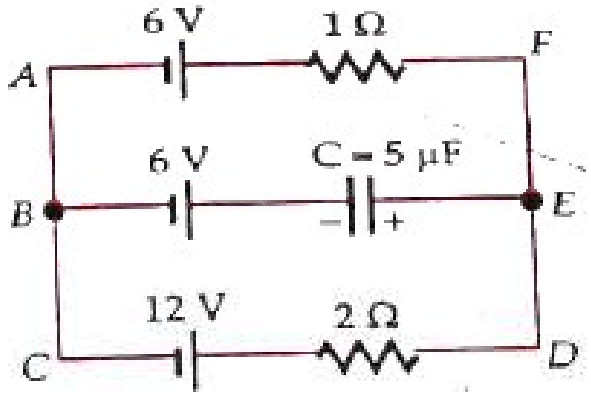
Question: In the given circuit find the charge and energy stored in the capacitor at a steady state



Numerical

Question: In the given circuit find the charge and energy stored in the capacitor at a steady state.

Solution :



In the figure, there is no current through arm BE.

So the total current is, $I = \frac{12V - 6V}{2\Omega + 1\Omega} = 2A$

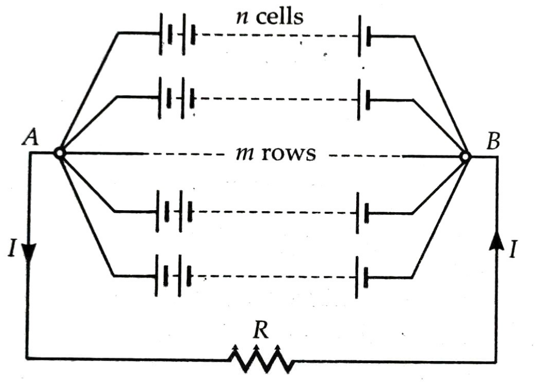
So, $V_{AF} = V_{BE} = V_{CD} = 12V - 2A \times 2\Omega = 8V$

Voltage across capacitor = 8V-6V=2V

$\therefore q = CV = 5\mu F . 2V = 10\mu C$ and $U = \frac{1}{2}CV^2 = \frac{1}{2}5\mu F(2V)^2 = 10\mu J$

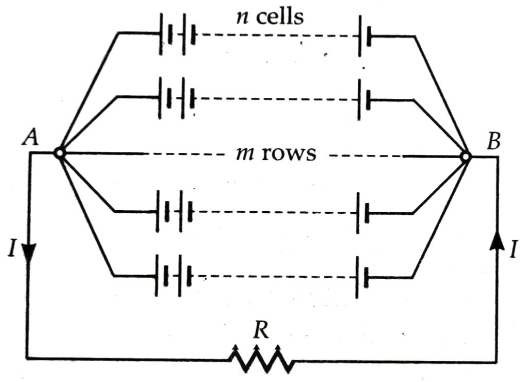
Mixed grouping

Mixed grouping of m x n identical cells each of emf ξ and internal resistance r in m rows with each row carrying n cells :



Mixed grouping

Mixed grouping of m x n identical cells each of emf ξ and internal resistance r in m rows with each row carrying n cells :



In the combination, each row is equivalent to a series combination of n identical cells each of emf ξ and internal resistance r.

So equivalent emf of each row is $\xi_s = n\xi$

the internal resistance of each row is $r_s = nr$

Now the combination is equivalent to a parallel grouping of identical cells each of emf ξ_s and internal resistance r_s .

So equivalent emf is $\xi_{eq} = \xi_s = n\xi$

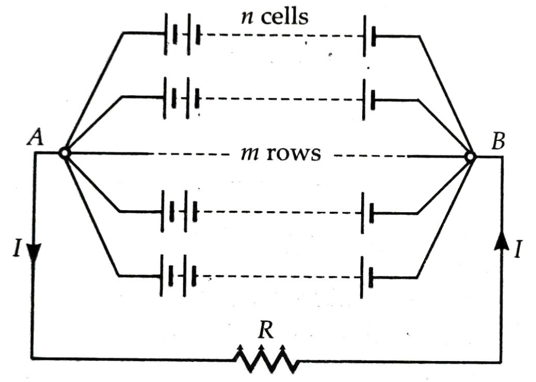
equivalent internal resistance is $r_{eq} = \frac{r_s}{m} = \frac{nr}{m}$

So current in the circuit is

$$I = \frac{\xi_{eq}}{r_{eq} + R} = \frac{n\xi}{\frac{nr}{m} + R} = \frac{mn\xi}{nr + mR}$$

Mixed grouping

Condition for maximum current :



Mixed grouping

Condition for maximum current :

Now $nr + mR = (\sqrt{nr})^2 + \left(\sqrt{mR}\right)^2 = \left(\sqrt{nr} - \sqrt{mR}\right)^2 + 2\sqrt{nrmR}$ ....(i)

For current to be maximum, (nr + mR) is to be minimum

From equation (i) we get that (nr+mR) will be minimum if

$$\left(\sqrt{nr} - \sqrt{mR}\right)^2 = 0 \Rightarrow nr = mR \Rightarrow R = \frac{nr}{m}$$

This is the condition for the maximum current.

Using the condition we get the maximum current to be

$$I_{max} = \frac{mn\xi}{nr + nr} = \frac{m\xi}{2r} \quad \text{or} \quad I_{max} = \frac{mn\xi}{mR + mR} = \frac{n\xi}{2R}$$

HOME ASSIGNMENT

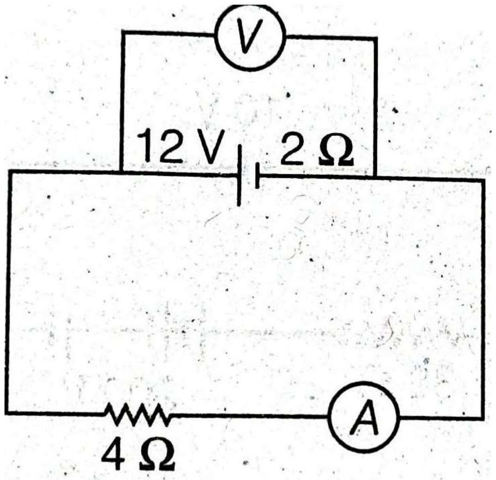
Question: Two cells of EMFs 1V and 2V with internal resistance 2Ω and 1Ω respectively are connected in series and parallel across the same resistor separately. Find the external resistance so that current through in both cases is the same .

Question: Twelve identical cells each of emf 1.5 V and internal resistance 1Ω are connected in m rows with each row containing n cells across an external resistance of 3Ω . Find the values of m and n to get maximum current through external resistance. What is the maximum current?

ODM
PUBLIC SCHOOL

Home Assignment

1. A battery of emf $12V$ and internal resistance 2Ω is connected to a 4Ω resistor as shown in the figure.
   a) Show that a voltmeter when placed across the cell and across the resistor, in turn, gives the same reading.
   b) To record the voltage and the current in the circuit, why is voltmeter placed in parallel and ammeter in series in the circuit?



HOME ASSIGNMENT

2. A cell of emf E and internal resistance r is connected to two external resistances R_1 and R_2 and a perfect ammeter. The current in the circuit is measured in four different situations:

   a) Without any external resistance in the circuit.

   b) With resistance R_1 only.

   c) With R_1 and R_2 in series combination

   d) With R_1 and R_2 in parallel combination

   The current measured in the four cases are $0.42\ \mathrm{A}$, $1.05\ \mathrm{A}$, $1.4\ \mathrm{A}$ and $4.2\ \mathrm{A}$, but not necessarily in that order. Identify the currents corresponding to the four cases mentioned above.

3. A battery of emf $10\mathrm{V}$ and internal resistance 3Ω is connected to a resister. If the current in the circuit is $0.5\mathrm{A}$, find

   a) the resistance of the resister

   b) the terminal voltage of the battery

THANKING YOU

ODM EDUCATIONAL GROUP

ODM
PUBLIC SCHOOL
SCHOOL OF HUMAN EXCELLENCE