Equation of a Circle with given centre and radius

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 11
CHAPTER NAME : CONIC SECTIONS

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Introduction to Conic Sections

Circle, parabola, ellipse, hyperbola are called conic sections. Conic Sections were studied extensively by ancient Greeks, who discovered properties that enable us to state their definitions in terms of a fixed point and a fixed line in the plane.

<u>Definition</u>

The conic section is the locus of a point which moves in a plane such that its distance from a fixed point always bears a constant ratio to its perpendicular distance from a fixed straight line in the plane.

The locus of a second-degree equation in x and y is called a conic.

Introduction to Conic Sections

Then

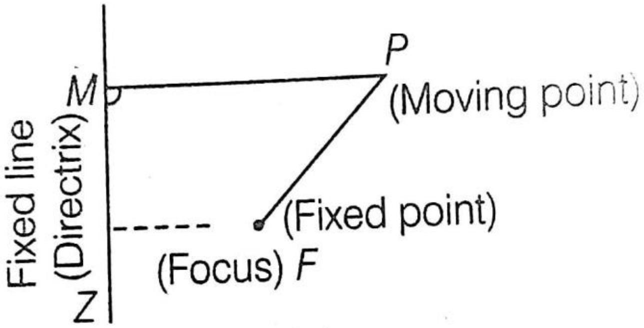
(i) the fixed point is called **focus** and is denoted by F.

(ii) the constant line ZM is called **directrix.**

(iii) the constant ratio is called **eccentricity**

and is denoted by e.

In the given figure, $\frac{PM}{PM} = e = constant$



Introduction to Conic Sections

If $e = 0$, then the conic is called a circle.

If $e = 1$, then the conic is called a parabola.

If $e < 1$, then conic is called an ellipse.

If $e > 1$, then conic is called a hyperbola.

(iv) The straight line passing through the focus and perpendicular to the directrix is called the **axis** of the conic.

(v) The point of intersection of the cone and its axis is called **the vertex** of the conic.

(vi) The point which bisects every chord of the conic passing through it is called the **centre** of the conic.

Introduction to Conic Sections

The name conic section is derived from the fact that these are the curves from a cone by taking its cross-section in various ways.

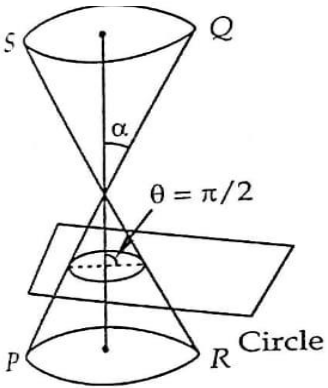


Fig. 25.2

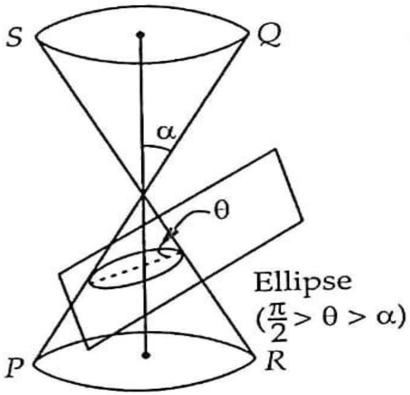


Fig. 25.3

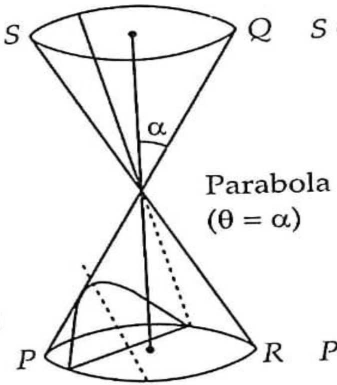


Fig. 25.4

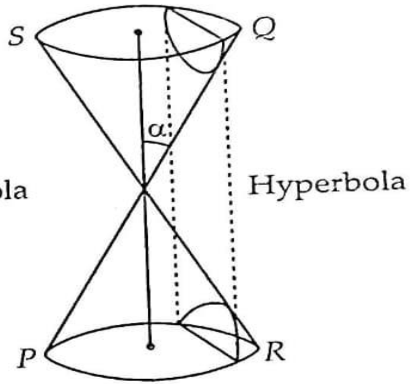
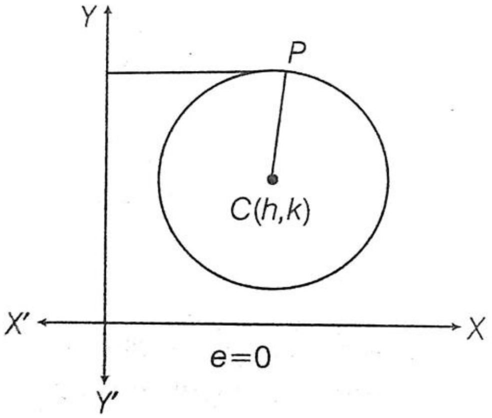


Fig. 25.5

Circle

A circle is the set of all points in a plane that are equidistant from a fixed point in that plane. Or, The locus of a point which moves on the plane such that it remains at a constant distance from a fixed point of the plane, is called a circle. The fixed point is called the center and the constant distance is called the radius of the circle.



Standard Equation of a Circle

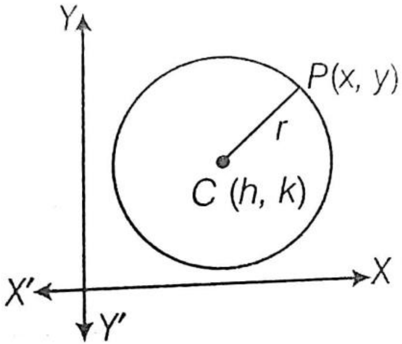
Let $C(h,k)$ be the center of the circle, $P(x,y)$ be any point on the circumference of the circle and r be the radius of the circle.

Then $|CP| = r$

$\Rightarrow \sqrt{(x-h)^2+(y-k)^2} = r$

$\Rightarrow (x-h)^2+(y-k)^2 = r^2 \ldots (1)$

which is the required equation of the circle.



This equation is also known as central form of the equation of a circle.

If the centre is at origin $i.e. h = 0, k = 0$, then the equation of the circle is $x^2 + y^2 = r^2$.

Example

Find the equation of a circle with centre $(2, 3)$ and radius is 5.

Sol: The required equation of the circle is $(x-2)^2+(y-3)^2=5^2=25$

Equation of Circle in Special Cases

1) When the circle passes through the origin

If the circle passes through the origin $(0,0)$, then radius $r = |OC| = \sqrt{h^2 + k^2}$

Hence the equation of the circle is $(x-h)^2 + (y-k)^2 = h^2 + k^2$

2) When the center lies on $x - axis$ or $y - axis$

If the center lies on $x - axis$, then $k = 0$

Then, the equation of the circle is $(x-h)^2 + y^2 = r^2$

If the center lies on the $y - axis$, then $h = o$

Then, the equation of the circle is $x^2 + (y-k)^2 = r^2$.

Equation of Circle in Special Cases

3) When the circle touches the $x - axis$

Since the circle touches $the\ x - axis$, so $r = |k|$. In this case, circles may lie on the upper of $the\ x - axis$ or the lower of the $x - axis$. Therefore, the equations of such circles are $(x - h)^2 + (y \mp r)^2 = r^2$

4) When the circle touches the $y - axis$.

Since, the circle touches $y - axis$, so $|h| = r$

In this case, circles may lie on the right of $the\ y - axis$ or the left of the $y - axis$. Therefore, the equation of such circles are $(x \mp r)^2 + (y - k)^2 = r^2$

5) When circles touch both the coordinate axes

Since the circle touches both the axes, so $|h| = |k| = r$

In this case, circles may lie in any of the four quadrants.

Then, the equation of such circles are $(x \pm r)^2 + (y \pm r)^2 = r^2$

Examples

1: Find the equation of a circle whose centre is $(1, 2)$ and touches $the\ x - axis$.

2: Find the equation of the circle which touches both the axes and whose radius is 5.

Diameter Form of the Equation of the Circle

Let (x_1, y_1) and (x_2, y_2) be the endpoints of the diameter of a circle. Then, equation of circle drawn on the diameter is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Example: Find the equation of the circle, whose endpoints of a diameter are $A(1, 5)$ and $B(-1, 3)$.

General Equation of a Circle

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General Equation of a Circle

Equation (1) $(i.e. (x-h)^2 + (y-k)^2 = r^2)$
can be written as $x^2 - 2hx + h^2 + y^2 - 2ky + k^2 = r^2$
$\Rightarrow \; x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$
which is of the form $x^2 + y^2 + 2gx + 2fy + c = 0$
The above equation is called the general equation of a circle.
Conversely, consider an equation of the form $x^2 + y^2 + 2gx + 2fy + c = 0 \ldots (2)$
$\Rightarrow \; x^2 + y^2 + 2gx + 2fy = -c$
$\Rightarrow \; x^2 + y^2 + 2gx + 2fy + c = 0$
$\Rightarrow \; x^2 + 2gx + g^2 + y^2 + 2fy + f^2 = g^2 + f^2 - c$
$\Rightarrow \; \{x - (-g)\}^2 + \{y - (-f)\}^2 = \left(\sqrt{g^2 + f^2 - c}\right)^2 \ldots (3)$
Comparing equations (1) and (3), we get eq. (2) always represents a circle with centre at $(-g, -f)$ and radius $r = \sqrt{g^2 + f^2 - c}$

Notes:

- Eq. (2) represents a real circle if $g^2 + f^2 > c$
- If $g^2 + f^2 = c$, then $r = 0$, then the circle is called a point circle.
- If $g^2 + f^2 < c$, then we get an imaginary circle.
- The general second degree equation in x and y in which the coefficients of x^2 and y^2 are equal and there is no term containing the product xy always represent a circle.
- If $ax^2 + ay^2 + 2gx + 2fy + c = 0$, represents a circle, then the centre is at $\left(-\frac{g}{a}, -\frac{f}{a}\right)$ and radius $= \sqrt{\frac{g^2}{a^2} + \frac{f^2}{a^2} - \frac{c}{a}}$

Examples

1. Find the centre and radius of each of the following circle.

$(i)\ x^2 + (y + 2)^2 = 9$

$(ii)\ x^2 + y^2 + 8x + 10y - 8 = 0$

Examples

2. Find the equation of the circle which passes through the points $(3, 7)$ and $(5, 5)$ and has its centre on the line $x - 4y = 1$.

Position of a point with respect to a Circle

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be a given circle and $P(\alpha, \beta)$ be a given point. Then the centre is at $(-g, -f)$ and radius $r = \sqrt{g^2 + f^2 - c}$.

The point P will lie inside or on or outside the given circle according as

$$|CP| \; < \; = \; > \; r$$

$$\Rightarrow \sqrt{(\alpha + g)^2 + (\beta + f)^2} < = > \sqrt{g^2 + f^2 - c}$$

$$\Rightarrow \alpha^2 + 2g\alpha + g^2 + \beta^2 + 2f\beta + f^2 < = > g^2 + f^2 - c$$

$$\Rightarrow \alpha^2 + \beta^2 + 2g\alpha + 2f\beta + c < = > 0.$$

Example 3: Does the point $(-3, 2)$ lies inside or outside or on the circle $x^2 + y^2 = 9$?

Example

4. Find the equation of the circle of radius 5 whose canter lies on the $x - axis$ and passes through the point $(2, 3)$.

Equation of a Circle passing through three non – collinear points

Example 5: Find the equation of the circle which passes through points $(1, 2), (3, -4)$ and $(5, -6)$.

Concentric Circles

Two circles having the same center $C(h, k)$ but different radii r_1 and r_2 are called concentric circles.
Thus the circles $(x-h)^2+(y-k)^2= r_1^2$ and $(x-h)^2+(y-k)^2= r_2^2,\ r_1 \neq r_2$ are concentric circles.

Example: Find the equation of the circle which passes through the center of the circle $x^2+y^2+8x+10y-7=0$ and is concentric with the circle $2x^2+2y^2-8x-12y-9=0$.

Example: Find the equation of the circle concentric with the circle $2x^2+2y^2+8x+10y-39=0$ and having its area equal to 16π sq. units.

Equation of a Parabola in different Forms

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Parabola

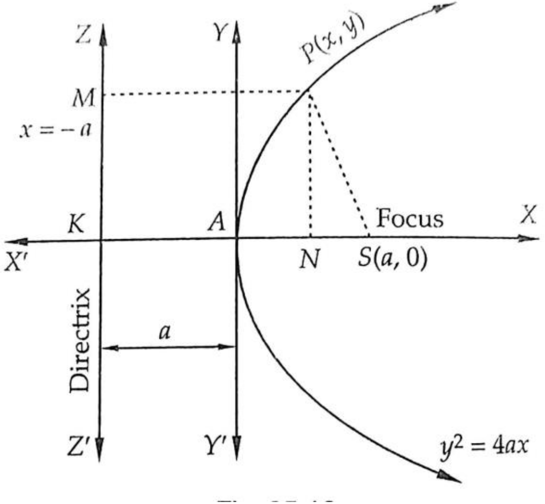
Consider a parabola whose vertex is at origin and focus along the x-axis at $F(a, 0)$. If $P(x, y)$ be any point on the parabola, then $|\mathrm{PF}| = |\mathrm{PM}|$

$\Rightarrow \sqrt{(x-a)^2 + y^2} = x + a$

$\Rightarrow \ (x-a)^2 + y^2 = \ (x+a)^2$

$\Rightarrow \ \ y^2 = (x+a)^2 - (x-a)^2$

$\Rightarrow \ \ y^2 \ = 4ax$, which is the equation of the parabola



Properties

1. The parabola $y^2 = 4ax$ opens to right.
2. The equation of the directrix is $x = -a\ i.e\ x + a = 0$.
3. The equation of the axis is $y = 0$.
4. The parabola $y^2 = 4ax$ is symmetrical about the x-axis. Since x cannot be negative ,so no portion of the curve lies in the 2nd and 3rd quadrant.
5. The distance of any point on the parabola from the focus is called the focal distance. For the parabola $y^2 = 4ax$, the focal distance is $x + a$.
6. The line segment joining any two points of the parabola is called a chord. If a chord is passing through the focus, it is called a focal chord.

   If a focal chord is perpendicular to the axis, it is called the latus rectum.

   The end points of the latus rectum are at $L(a, 2a), R(a. -2a)$. The length of the latus rectum is 4a.

Corollary:

1. Let the vertex of the parabola is at $(0,0)$.If the focus of is at$(-a, 0), i.e.$
   the parabola opens to the left, then the equation of the parabola is $y^2 = 4ax$.
   Then the equation of the directrix is $x = a$ $i.e$ $x - a = 0$.
   Equation of the axis is $y = 0$.
   The end points of the latus rectum are $L(-a, 2a), R(-a, -2a)$.
2. Let the vertex of the parabola is at origin .If the focus is $F(0, a), i.e$
   the parabola open upwards, then the equation of the parabola
   is $x^2 = 4ay$.
   The equation of the axis is $x = 0$.
   The equation of the directrix is $y = -a$.
   The end points of the latus rectum are $L(-2a, a), R(2a, a)$.

Corollary

3. Let the vertex of the parabola is at the origin .If the focus is at $F(0.-a), i.e.$ the parabola opens downwards, then the equation of the parabola is $x^2 = -4ay$.
   The equation of the axis is $x = 0$.
   The equation of the directrix is $y = a, i.e\ y - a = 0$.
   The end points of the latus rectum are at $L(-2a, -a), R(2a, -a)$.

Different parabolas

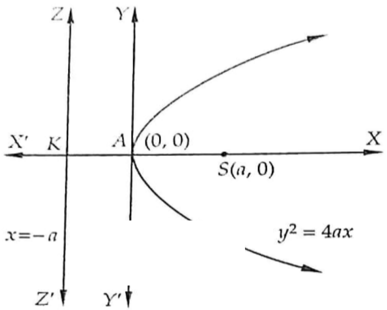


Fig. 25.12

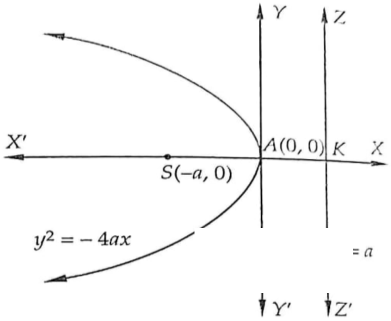
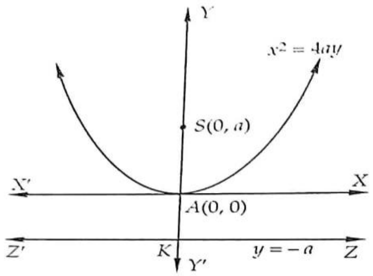
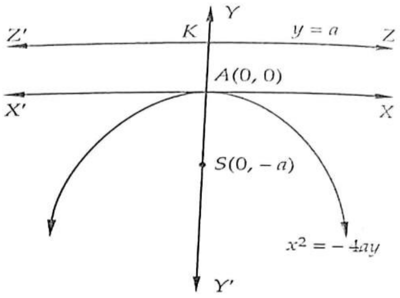


Fig. 25.13





General Equation of a Parabola

1. If the vertex is at (h, k) and axis is parallel to the x-axis ,then the equation of the parabola is $(y-k)^2 = \pm 4a(x-h)$.
2. If the vertex is at (h, k) and axis is parallel to y-axis ,then the equation of the parabola is $(x-h)^2 = \pm 4a(y-k)$.

   The general equation of a parabola can be written either of the following forms:

   $x^2 + ax + by + c = 0$ (axis parallel to y-axis) or,

   $y^2 + dy + ex + f = 0$ (axis parallel to x-axis)

Thus ,the general equation of a parabola is either quadratic in x and linear in y or quadratic in y and linear in x.

Parametric Representation:

The parametric representation of the equation of the parabola : $y^2 = 4ax$ are $x = at^2$,$y = 2at$, where t is a parameter.

Formation of Equation of a Parabola from given conditions

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Example

1: Find the vertex, focus, equation of the axis, equation of the directrix, and length of the latus rectum, endpoints of the latus rectum of the parabola $y^2 = -8x$.

2. Find the equation of parabola which is symmetric about the x-axis and passing through points $(2, -3)$.

3. A double ordinate of the parabola $y^2 = 4ax$ is of length $8a$. Prove that the lines from the vertex to its ends are at right angles.

Example

4. Find the equation of the parabola with focus $(2, 0)$ and directrix $x = -2$.
5. Find the coordinates of the focus, axis of the parabola, the equation of the directrix, and length of the latus rectum of the parabola $x^2 = -9y$.
6. Find the equation of a parabola when the vertex is at $(0, 0)$ and focus is at $(0, -4)$.

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Equation of an Ellipse in different forms

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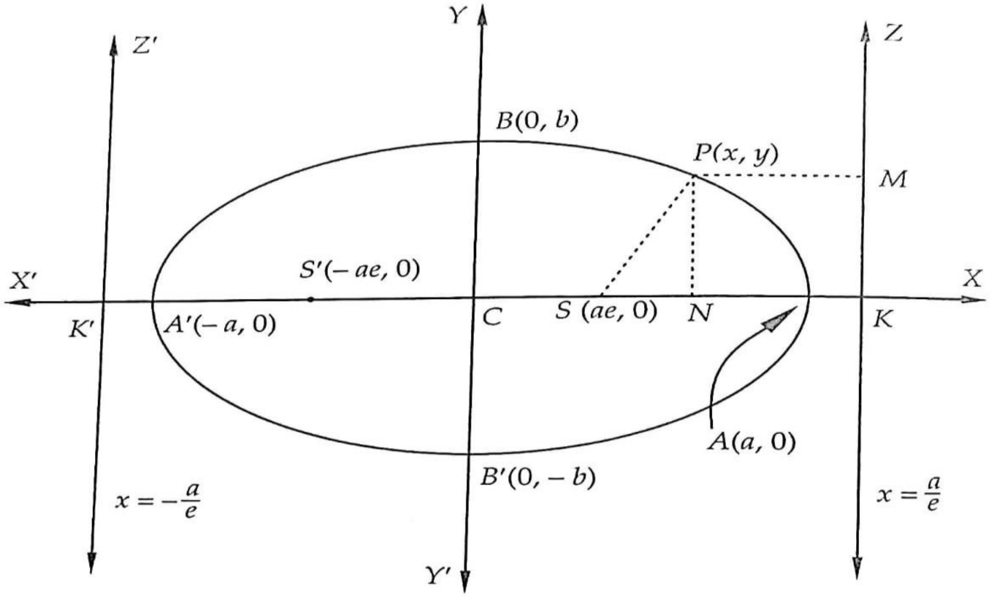
The Ellipse

The locus of a point which moves on the plane in such a way that the sum of its distances from two fixed points remains constant is called an ellipse.

Each of the fixed point is called a focus.

Equation of an Ellipse



Equation of an Ellipse

Consider an ellipse whose foci are along the x-axis and the mid-point of the line segment joining the foci is at the origin. If $|\mathrm{FF}'| = 2c$,then the foci are at $F(c,0)$ and $F'(-c,0)$.

If $P(x,y)$ be any point on the ellipse, then $|PF'|+|PF|= constant$

$$= 2a \ \ (say)$$

$$\Rightarrow \sqrt{(x+c)^2+y^2} + \sqrt{(x-c)^2+y^2} = 2a$$

$$\Rightarrow \sqrt{(x+c)^2+y^2} = 2a - \sqrt{(x-c)^2+y^2}$$

$$\Rightarrow (x+c)^2+y^2 = 4a^2 + (x-c)^2+y^2 - 4a\sqrt{(x-c)^2+y^2}$$

$$\Rightarrow (x+c)^2-(x-c)^2 - 4a^2 = -4a\sqrt{(x-c)^2+y^2}$$

Equation of an Ellipse

$$\Rightarrow\ 4cx - 4a^2 = -4a\sqrt{(x-c)^2+y^2}$$

$$\Rightarrow\ cx - a^2 = -a\sqrt{(x-c)^2+y^2} \qquad ....(1)$$

$$\Rightarrow\ c^2x^2 + a^4 - 2a^2cx = a^2\{(x-c)^2+y^2\}$$

$$\Rightarrow\ c^2x^2 + a^4 - 2a^2cx = a^2x^2 + a^2c^2 - 2a^2cx + a^2y^2$$

$$\Rightarrow\ a^2x^2 - c^2x^2 + a^2y^2 = a^4 - a^2c^2$$

$$\Rightarrow\ (a^2+c^2)x^2 + a^2y^2 = a^2(a^2-c^2)$$

$$\Rightarrow\ b^2x^2 + a^2y^2 = a^2b^2$$

$\Rightarrow\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which is the required equation of the ellipse. .... (2)

Properties

1. The ellipse(2) meets the x-axis at $A'(-a,0)$ and $A(a,0)$. These two points are called vertices.
   The line segment $A'A$ is called major axis and its length is 2a.
   The equation of the major axis is y=0.
2. The ellipse(2) meets the y-axis at $B(0,b)$ and $B'(0,-b)$.
   The line segment BB' is called the minor axis and its length is 2b.
   The equation of minor axis is x=0.
3. The length of the major axis is greater than minor axis and foci lie on the major axis.
4. The point of intersection of the major and minor axis is called centre. Here the centre is at $(0,0)$.
5. The chords through foci and perpendicular to the major axis are called latera recta.
6. The end-points of the latera recta are at $L\left(c\,,\frac{b^2}{a}\right), R(c,-\frac{b^2}{a})$

$$L'\left(-c\,,\frac{b^2}{a}\right), R'(-c,-\frac{b^2}{a}).$$

Properties

The length of each latus rectum is $\frac{2b^2}{a}$.

The equation of the latera recta are $x = \pm c$.

7. From equation(1), we get

$$cx - a^2 = -a\sqrt{(x-c)^2 + y^2}$$

$$\Rightarrow \sqrt{(x-c)^2 + y^2} = a - \frac{cx}{a}$$

$$= \frac{c}{a}\left(\frac{a^2}{c} - x\right)$$

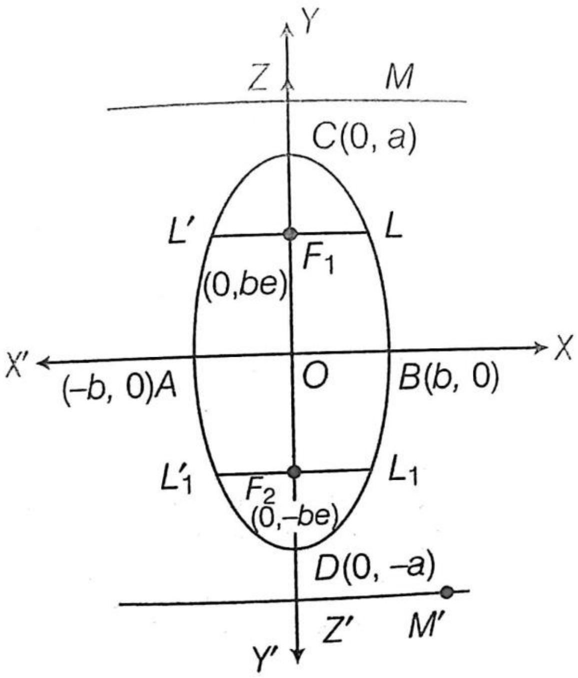
$\Rightarrow \frac{\sqrt{(x-c)^2+y^2}}{\left(\frac{a^2}{c}-x\right)} = \frac{c}{a}$, the ratio of the distance of $P(x, y)$ from the focus $(c, 0)$ and the distance of $P(x, y)$ from the line $x = \frac{a^2}{c}$ is a directrix w.r.t. the focus$(c, 0)$.Similarly, $x = -\frac{a^2}{c}$ s a directrix w.r.t. the focus $(-c, 0)$. Thus the equation of the directrixes are $x = \pm\frac{a^2}{c}$.

Properties

8. Eccentricity, $e = \frac{c}{a} < 1.$
9. If the foci are along the y-axis at $F(0, c), F'(0, -c)$, then the equation of the ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$

   The vertices are at $A(0, a), A'(0, -a)$.
   The end points of the minor axis are at $B(b, 0), B'(-b, 0)$.
   The end points of the latera recta are at $\left(\pm\frac{b^2}{a}, \pm c\right)$.
   The equation of major axis is $x = 0$. The equation of minor axis is $y = 0$.
   The equations of the latera recta are $y = \pm c$.
   The equations of the directrixes are $y = \pm\frac{a^2}{c}$.
   Centre is at $(0,0)$.
10. If $a = b$, then the equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ becomes $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1, i.e. x^2 + y^2 = a^2$, which represent a circle and known as auxiliary circle of the ellipse.

Diagram



Formation of Equation of an Ellipse from given conditions

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Examples

1. Find the coordinates of the centre, foci, ends of the major and minor axis, endpoints of latera recta, equation of major axis, minor axis, directrices, length of each latus rectum, and eccentricities of the ellipse $9x^2 + 4y^2 = 36$.

2. Find the equation of ellipse whose vertices are $(\pm 13,0)$ and foci are at $(\pm 5,0)$.

3.Find the equation of an ellipse with a major axis along the x-axis and passing through the points $(4,3)$ and $(-1,4)$.

Examples

4.If the eccentricity of the ellipse is $\frac{5}{8}$ and the distance between the foci is 10, then find the latus rectum of the ellipse.

5. If the length of the latus rectum of the ellipse is equal to half of the minor axis, find its eccentricity.

6. Find the equation of the ellipse whose axes are along the coordinate axes, vertices are $(0, \pm 10)$ and eccentricity,$e = \frac{4}{5}$.

7. Find the equation of the ellipse, whose length of the major axis is 20 and foci are $(0, \pm 5)$.

Equation of a Hyperbola in different forms

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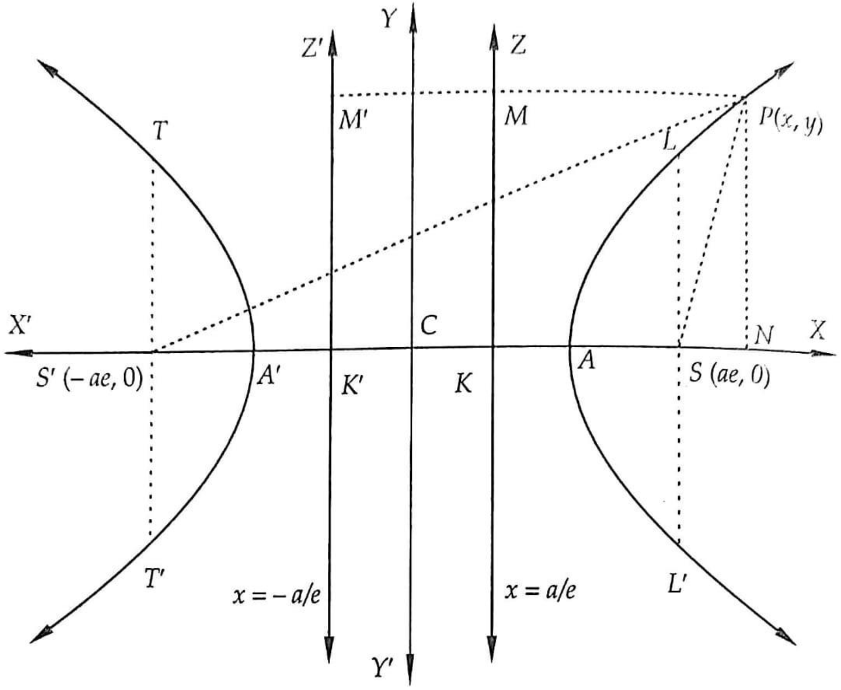
The Hyperbola

The locus of a point which moves on the plane in such a way that the difference of its distance from two fixed points remains constant is called a hyperbola.

Each of the fixed point is called a focus.

Equation of a Hyperbola:



Equation of a Hyperbola:

Let the foci of the hyperbola are along the x-axis at $F(c, 0)$, and $F'(-c, 0)$.

If $P(x, y)$ be any point on the hyperbola, then $|PF'| - |PF| = constant$

$\Rightarrow \sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = \pm 2a$

After simplification, we get $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ....(1) $\quad | \because b^2 = c^2 - a^2$

Properties:

1. The hyperbola equation (1) meets the x-axis at $A(a, 0)$, and $A'(-a, 0)$. These two points are called vertices. The line segment $A - A'$ is called the transverse axis and its length is 2a.

   The equation of transverse axis is $y = 0$.
2. The hyperbola (1) meets the y-axis at imaginary points $B(0, b)$ and $B'(0, -\text{b})$. The line segment BB' is called the conjugate axis and its length is 2b.

   The equation of conjugate axis is $x = 0$.
3. There is no restriction for the relative values of a and b. The foci always lies on the transverse axis.
4. The point of intersection of the transverse and conjugate axis is called the centre and here centre is $O(0,0)$.
5. The chords LR and $L'R'$ are called latera recta. The end-points

   of the latera recta are at $\left(\pm c, \pm \frac{b^2}{a}\right)$.

Properties:

The length of each latus rectum is $\frac{2b^2}{a}$. The equations of the latera recta are $x = \pm c$.

7. Eccentricity, $e = \frac{c}{a} > 1$.

8. If the foci are on the y-axis at $(0, c)$ and $(0, -c)$, then the equation of the hyperbola is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

The vertices are at $A(0, a), A'(0, -a)$.

The end points of the conjugate axis are at B$'(-b, 0)$ and $B(b, 0)$.

The end-points of the latera recta are at $\left(\pm\frac{b^2}{a}, \pm c\right)$.

The equation of the transverse axis is $x = 0$ and the equation of conjugate axis is y

The equation of latera recta is $y = \pm c$.

The equation of directrixes is $y = \pm\frac{a^2}{c}$.

Rectangular and Conjugate Hyperbola

Rectangular Hyperbola:

If $a = b$, then the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, becomes $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$, $i.e. x^2 - y^2 = a^2$,which is the equation of a rectangular or an **equilateral** hyperbola. The eccentricity of a rectangular hyperbola is $\sqrt{2}$.

Conjugate Hyperbolas:

The hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1$, $i.e.$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ are conjugate of each other. If e_1 and e_2 are the eccentricities of the conjugate hyperbolas, then $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$.

Formation of Equation of a Hyperbola from given conditions

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Examples

1. Find the centre, foci, endpoints of the transverse and conjugate axis, endpoints of latera recta, equations of the transverse and conjugate axis, equations of directrices, length of transverse axis, conjugate axis, latus rectum and eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.
2. Find the equation of the hyperbola with foci$(0, \pm 3)$ and vertices $\left(0, \pm \frac{\sqrt{11}}{2}\right)$.
3. Find the equation of hyperbola here foci are $(0, \pm 12)$ and the length of the latus rectum is 36.
4. Find the equation of hyperbola whose length of the latus rectum is 8 and eccentricity is $\frac{3}{\sqrt{5}}$.

Examples

5. Find the coordinates of the vertices, the foci, the eccentricity and the equation of the directrices of the hyperbola $9y^2 - 16x^2 = 144$.

6. Find the equation of the hyperbola with vertices at $(0, \pm 7)$ and $e = \frac{4}{3}$.

7. Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.

8. Find the eccentricity of the hyperbola whose length of the latus rectum is 8 and the conjugate axis is equal to half of its distance between the foci.

Word Problems related to Conic

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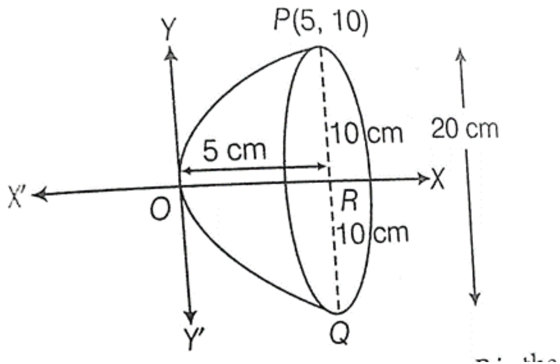
Examples

1. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus.

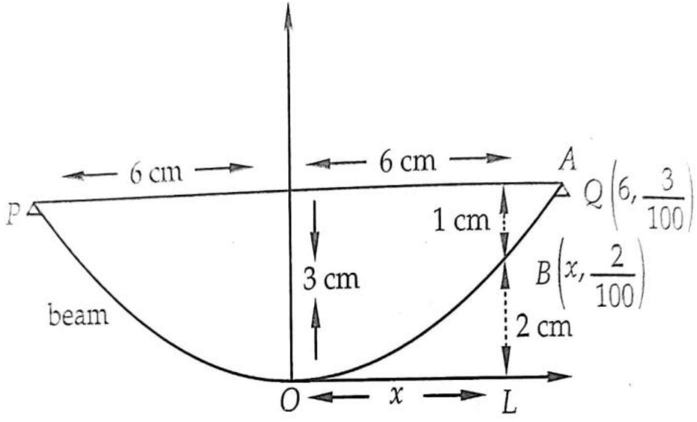


Examples

2. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.

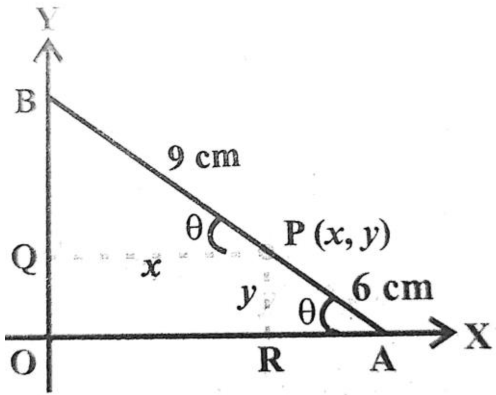
Examples

3. A beam is supported at its ends by supports which are 12 meters apart. Since the load connected at its center, there is a deflection of 3 cm at the center and the deflected beam is in the shape of a parabola. How far from the center is the deflection 1 cm?



Examples

4. A rod AB of length 15 cm rests in between two coordinate axes in such a way that the endpoint A lies on the $x - axis$ and the endpoint B lies on the $y - axis$. A point $P(x, y)$ is taken on the rod in such a way that $AP = 6cm$. Show that the locus of P is an ellipse.



Examples

5. An arch is in the form of a semi – ellipse. It is 8m wide and 2 m high at the centre. Find the height of the arc at a point $1.5m$ from one end.

6. A debate on national integration was organised and seating arrangement was done in the form of a parabola, represented by y^2=12x. The last two participants are sitting at corners which represent the end points of latus rectum of a parabola. What is the distances between these two participants.

7. A ball thrown upwards reaches a maximum height of 4m at a distance of 0.5 m from the initial point . Find the height of the ball above the horizontal line passing through the initial point at a distance of 0.75 m from the initial point.

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