LIMITS AND DERIVATIVES

Idea of limit & Limit of polynomial and rational function

SUBJECT : MATHEMATICS
CHAPTER NUMBER:13
CLASS NUMBER:01

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Idea of limit & Limit of polynomial and rational function

Definition of Limit:- If $f(x)$ approaches to a real number ℓ, when x approaches to a (through lesser or greater values to a) i.e if $f(x)\rightarrow \ell$, when $x\rightarrow a$, then ℓ is called limit of the function f(x) . In symbolic form, it can be written as $\lim_{x\rightarrow a} f(x)=\ell$.

Algebra of Limits:-

Sometimes two or more functions involving algebraic operations such as addition, subtraction, multiplication and division are given, and then to find the limit of these functions involving algebraic operations, we use the following theorem.

Let f and g be two real functions with common domain D, such that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exists. Then

(a) Limit of sum of two functions is sum of the limits of the functions.

i.e $\lim_{x\to a} (f+g)(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

(b) Limit of difference of two functions is difference of the limits of the function

i.e $\lim_{x\to a} (f-g)(x) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

(c) Limit of product of a constant and one function is the product of that constant and limit of a function,

i.e $\lim_{x\to a} \left[c.f(x)\right] = c \lim_{x\to a} f(x)$, where c is a constant.

(d) Limit of product of two functions is product of the limits of the function, i.e

$$\lim_{x\to a} \left[f(x).g(x)\right] = \lim_{x\to a} f(x).\lim_{x\to a} g(x)$$

(e) Limit of quotient of two functions is quotient of the limits of the functions, i.e

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \text{where } \lim_{x\to a} g(x) \neq 0$$

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Limits Of Polynomial Function

A function f is said to be a polynomial function, if f(x) is zero function or if

$f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$, where a_i's are real numbers and $a_n \neq 0$.

Method to find limit of a polynomial:-

To find the limit of given polynomial, we use the algebra of limits and then put the limit and simplify. It can be understand in the following way.

We know that, $\lim_{x \to a} x = a$. Then $\lim_{x \to a} x^2 = \lim_{x \to a}(x.x) = \lim_{x \to a} x . \lim_{x \to a} x = a.a = a^2$

Similarly, $\lim_{x \to a} x^n = a^n$

Example:-1

Evaluate the limits $\lim_{x\to 3}\left(4x^3-2x^2-x+1\right)$

Solution:-

$$\lim_{x\to 3}\left(4x^3-2x^2-x+1\right)$$

$$=4\lim_{x\to 3}x^3-2\lim_{x\to 3}x^2-\lim_{x\to 3}x+\lim_{x\to 3}1$$

$$=4(3)^3-2(3)^2-3+1=108-18-2=88$$

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Limits Of Rational Functions

A function f is said to be a rational function, if $f(x)=\frac{g(x)}{h(x)}$ where g(x) and h(x) are polynomial functions such that $h(x)\neq 0$.

Then, $\lim_{x\to a} f(x)=\lim_{x\to a}\frac{g(x)}{h(x)}$

$$=\frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)}=\frac{g(a)}{h(a)}$$

However, if h (a) = 0, then there are two cases arise,

(i) $g(a)\neq 0$ (ii) $g(a)=0$

In the first case, we say that the limit does not exist. In the second case, we can find limit. Limit of a rational function can be finding with the help of following methods.

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Direct Substitution Method:-

In this method, we substitute the point, to which the variable tends to in the given limit. If it gives us a real number, then the number so obtained is the limit of the function and if it does not give us a real number, then use other methods.

Example:-

Find the limits of the following

(i) $\lim_{x \to 2} \frac{x^2 - 4}{x + 3}$ (ii) $\lim_{x \to -1} \frac{(x-1)^2 + 3x^2}{(x^4+1)^2}$

Solution:-

(i) $\lim_{x \to 2} \frac{x^2 - 4}{x + 3} = \frac{4-4}{2+3} = \frac{0}{5} = 0$

(ii) $\lim_{x \to -1} \frac{(x-1)^2 + 3x^2}{(x^4+1)^2} = \frac{(-1-1)^2 + 3(-1)^2}{((-1)^4+1)^2} = \frac{(-2)^2 + 3(1)}{(1+1)^2} = \frac{4+3}{2^2} = \frac{7}{4}$

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Factorisation Method:-

Let $\lim_{x \to a} \frac{f(x)}{g(x)}$ reduces to the form $\frac{0}{0}$, when we substitute x = a. Then we factorise f(x) and g(x) and then cancel out the common factor to evaluate the limit.

Method to determine the limit by using factorisation method:-

Step – I write the given limit as $\lim_{x \to a} \frac{f(x)}{g(x)}$

If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$, then go to next step, otherwise use direct substitution method.

Step – II Factorise f(x) and g(x), such that $(x - a)$ is a common factor and write given limit as $\lim_{x \to a} \frac{(x-a)f_1(x)}{(x-a)g_1(x)}$

Step – III Cancel the common factor(s) then limit obtained in step III becomes $\lim_{x \to a} \frac{f_1(x)}{g_1(x)}$

Step – IV Use direct substitution method to obtain limit.

Example:- 1

Evaluate $\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$

Solution:-

On putting $x = \frac{1}{2}$, we get the form $\frac{0}{0}$. So, let us first factorise it

Consider, $\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = \lim_{x \to \frac{1}{2}} \frac{(2x+1)(2x-1)}{(2x-1)}$

$= \lim_{x \to \frac{1}{2}} (2x+1) = 2\left(\frac{1}{2}\right) + 1 = 2$

Example:- 2

Evaluate $\lim_{x \to 2}\left[\frac{x^2-4}{x^3-4x^2+4x}\right]$

Solution:-

On putting x = 2, we get the form $\frac{0}{0}$ so, let us first factorise it.

Consider, $\lim_{x \to 2}\frac{x^2-4}{x^3-4x^2+4x}=\lim_{x \to 2}\frac{(x+2)(x-2)}{x(x-2)^2}$

$=\lim_{x \to 2}\frac{(x+2)}{x(x-2)}=\frac{2+2}{2(2-2)}=\frac{4}{0}$ which is not defined.

$\therefore \lim_{x \to 2}\left[\frac{x^2-4}{x^3-4x^2+4x}\right]$ does not exist.

Rationalisation Method:-

If we get $\frac{0}{0}$ form and numerator or denominator or both have radical sign, then we rationalise the numerators or denominator or both by multiplying their to remove $\frac{0}{0}$ form and then find limit by direct substitution method.

Example:- 1

Evaluate $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$

Solution:-

When x = 0, then the expression $\frac{\sqrt{2+x} - \sqrt{2}}{x}$ becomes of the form $\frac{0}{0}$. So we will rationalising the numerator by multiplying and dividing its conjugate i.e $\sqrt{2+x} + \sqrt{2}$

$$\therefore \lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} = \lim_{x \to 0} \frac{\left(\sqrt{2+x} - \sqrt{2}\right)\left(\sqrt{2+x} + \sqrt{2}\right)}{x\left(\sqrt{2+x} + \sqrt{2}\right)}$$

$$= \lim_{x \to 0} \frac{2+x-2}{x\left(\sqrt{2+x} + \sqrt{2}\right)} = \lim_{x \to 0} \frac{1}{\sqrt{2+x} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$$

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LIMITS AND DERIVATIVES

Evaluation of Limit by Using standard Rules

SUBJECT : MATHEMATICS
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Evaluation of Limit by Using standard Rules

By using Some standard Limits:-

If the given limit is of the form $\lim_{x \to a} \frac{x^n - a^n}{x - a}$, then we can find the limit directly by using the following theorem.

Theorem:- Let n be any positive integer. Then $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$.

Proof:- We known that $x^n - a^n = (x-a)\left(x^{n-1} + ax^{n-2} + ... + a^{n-2}x + a^{n-1}\right)$

On dividing both sides by $(x-a)$, we get $\frac{x^n - a^n}{x - a} = x^{n-1} + ax^{n-2} + ... + a^{n-2}x + a^{n-1}$

Thus, $\lim_{x \to a} \frac{x^n - a^n}{x - a} = \lim_{x \to a}\left(x^{n-1} + ax^{n-2} + ... + a^{n-2}x + a^{n-1}\right)$

$= a^{n-1} + a\left(a^{n-2}\right) + ... + a^{n-2}a + a^{n-1}$

$= a^{n-1} + a^{n-1} + ... + a^{n-1} + a^{n-1}$

$= na^{n-1}$

Example:-1

Evaluate $\lim_{x\to 2}\frac{x^{10}-1024}{x-2}$

Solve:- When x = 2, the expression $\frac{x^{10}-1024}{x-2}$ become of the form $\frac{0}{0}$

Now, $\lim_{x\to 2}\frac{x^{10}-1024}{x-2}=\lim_{x\to 2}\frac{x^{10}-2^{10}}{x-2}=10\times 2^{10-1}=5120$

Example:-2

Evaluate $\lim_{x\to 2}\frac{x-2}{\sqrt[3]{x}-\sqrt[3]{2}}$

Solution:-

$$\lim_{x\to 2}\frac{x-2}{x^{1/2}-2^{1/3}}=\frac{1}{\lim_{x\to 2}\frac{x^{1/3}-2^{1/3}}{x-2}}=\frac{1}{\frac{1}{3}\left(2^{1/3-1}\right)}=\frac{1}{\frac{1}{3}\times\left(2^{-2/3}\right)}=3\left(2^{2/3}\right)$$

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Example:- 3

Evaluate $\lim_{x\to 2}\frac{x^5-32}{x^3-8}$

Solution:-

$$\lim_{x\to 2}\frac{x^5-32}{x^3-8} = \lim_{x\to 2}\frac{x^5-2^5}{x^3-2^3} = \lim_{x\to 2}\frac{\frac{x^5-2^5}{x-2}}{\frac{x^3-2^3}{x-2}} = \lim_{x\to k2}\frac{x^5-2^5}{x-2} \div \lim_{x\to 2}\frac{x^3-2^3}{x-2}$$

$$= 5\times 2^{5-1} \div 3\times 2^{3-1}$$

$$= 5\times 2^4 \div 3\times 2^2 = \frac{5\times 2^4}{3\times 2^2} = \frac{5}{3}\times 2^2 = \frac{20}{3}$$

Example:-4

Evaluate $\lim_{x\to 2}\frac{(2x+4)^{1/3}-2}{x-2}$

Solve:-

Put $2x+4=y$, then $y\to 8$ as $x\to 2$

$$\therefore \lim_{x\to 2}\frac{(2x+4)^{1/3}-2}{x-2}=\lim_{y\to 8}\frac{y^{1/3}-2}{\frac{y-4}{2}-2}$$

$$=2\lim_{x\to 8}\frac{y^{1/3}-(8)^{1/3}}{y-4-4}=2\lim_{x\to 8}\frac{y^{1/3}-8^{1/3}}{y-8}$$

$$=2.\frac{1}{3}(8)^{\frac{1}{3}-1}$$

$$=2.\frac{1}{3}\left(2^{3}\right)^{-\frac{2}{3}}=\frac{2}{3}.(2)^{-2}=\frac{2}{3}\times\frac{1}{4}=\frac{1}{6}$$

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Limits of Trigonometric Functions

Three important limits are

(i) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(ii) $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

(iii) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

Where x is measured in radian

Method to determine the limit of trigonometric Function:-

Step – I First, check that given variable tends to zero or not. If yes, then go to step II, otherwise put $x = a + h$ in the given function such that as $x \to a, \text{then } h \to 0$

Step – II Put the limit in given function, if $\frac{0}{0}$ form is obtained, then we go to next step. Otherwise, we get the required answer.

Step-III Simplify the numerator and denominator to eliminate those factors which becomes 0 on putting the limit.

Step – IV Now, convert the result obtained in step III, in the form of $\frac{\sin\theta}{\theta}$ or $\frac{\tan\theta}{\theta}$.

Step – V Substitute the value of standard limit of trigonometric function as obtained in step IV and simplify it.

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Example:- 1

Evaluate $\lim_{\theta \to 0} \theta \operatorname{cosec} \theta$

Solution:-

$$\lim_{\theta \to 0} \theta \operatorname{cosec} \theta = \lim_{\theta \to 0} \frac{\theta}{\sin \theta}$$

$$= \lim_{\theta \to 0} \frac{1}{\frac{\sin \theta}{\theta}} = \frac{1}{1} = 1$$

Example:- 2

Evaluate $\lim_{x \to 0} \frac{\sin 3x}{5x}$

Solution:-

$$\lim_{x \to 0} \frac{\sin 3x}{5x} = \lim_{x \to 0} \frac{\sin 3x}{5x \times \frac{3}{3}} = \lim_{x \to 0} \frac{3}{5} . \frac{\sin 3x}{3x} = \frac{3}{5} . \lim_{x \to 0} \frac{\sin 3x}{3x}$$

$$= \frac{3}{5} \times 1 = \frac{3}{5}$$

Example:- 3

Evaluate $\lim\limits_{x\to 0}\left(\frac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}\right)$

Solution :-

$$\lim_{x\to 0}\left(\frac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}\right)$$

$$=\lim_{x\to 0}\left(\frac{2\sin 4x\cos 3x}{2\cos 4x\sin x}\right)$$

$$=\lim_{x\to 0}\left(\frac{\sin 4x\cos 2x}{\cos 4x\sin x}\right)$$

$$=\lim_{x\to 0}\left\{\frac{\sin 4x}{4x}\times\frac{x}{\sin x}\times\cos 2x\times\frac{1}{\cos 4x}\times 4\right\}$$

$$=4\times\lim_{4x\to 0}\frac{\sin 4x}{4x}\times\lim_{x\to 0}\left(\frac{x}{\sin x}\right)\times\lim_{2x\to 0}\cos 2x\times\frac{1}{\lim\limits_{4x\to 0}\cos 4x}=\left(4\times 1\times 1\times 1\times\frac{1}{1}\right)=4$$

Example:- 4

Evaluate $\lim_{x\to 0}\frac{\tan x^{o}}{x^{o}}$

Solution :-

$$\lim_{x\to 0}\frac{\tan x^{o}}{x^{o}}=\lim_{x\to 0}\frac{\tan\frac{\pi x}{180}}{\frac{\pi x}{180}}=1$$

Example:- 5

Evaluate $\lim_{x\to 0}\frac{1-\cos 4x}{x}$

Solution:-

$$\lim_{x\to 0}\frac{1-\cos 4x}{x}$$

$$=\lim_{x\to 0}\frac{2\sin^{2}2x}{x}\times\frac{x}{x}$$

$$=\lim_{x\to 0}2\left(\frac{\sin 2x}{2x}\right)^{2}\times 4x=2\times 1\times 0=0$$

Example:- 6

Evaluate $\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x^3}$

Solution:- $\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x^3}$

$$=\lim_{x\to 0}\frac{\frac{\sin 2x}{\cos 2x}-\sin 2x}{x^3}$$

$$=\lim_{x\to 0}\frac{\sin 2x-\sin 2x.\cos 2x}{x^3.\cos 2x}$$

$$=\lim_{x\to 0}\frac{\sin 2x\left(1-\cos 2x\right)}{x^3.\cos 2x}$$

$$=\lim_{x\to 0}\frac{\tan 2x}{x}\times\lim_{x\to 0}\frac{2\sin^2 x}{x^2}$$

$$=2.\lim_{x\to 0}\frac{\tan 2x}{2x}\times 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2$$

$$=2\left(1\right)\times 2\left(1\right)^2=4$$

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Example:- 7

Evaluate $\lim\limits_{x \to \pi} \frac{\sin x}{\pi - x}$

Solution:-Put $\pi - x = y\ ,\ y \to 0\ as\ x \to \pi$

Therefore $\lim\limits_{x \to \pi} \frac{\sin x}{\pi - x}$

$$= \lim_{y \to 0} \frac{\sin(\pi - y)}{y}$$

$$= \lim_{y \to 0} \frac{\sin y}{y}$$

=1

Evaluation of Trigonometric Limits By Factorisation

Sometimes, trigonometric limits can be evaluated by factorisation method.

Example:- 8

Evaluate $\lim\limits_{x\to\pi/6} \dfrac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$

Solution:-

$$\lim_{x\to\pi/6} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$$

$$= \lim_{x\to\pi/6} \frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2}$$

$$= \lim_{x\to\pi/6} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2}$$

$$= \lim_{x\to\pi/6} \frac{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}{(\operatorname{cosec} x - 2)}$$

$$= \lim_{x\to\pi/6} (\operatorname{cosec} x + 2)$$

$$= \operatorname{cosec}\frac{\pi}{6} + 2 = 2 + 2 = 4$$

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LIMITS AND DERIVATIVES

Limit of exponential and logarithmic functions

SUBJECT : MATHEMATICS
CHAPTER NUMBER:13
CLASS NUMBER:03

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Limits Of Exponential Functions

A function of the form of $f(x)=e^x$ is called exponential function

To find the limit of a function involving exponential function, we use the following theorem.

Theorem $\lim_{x\to 0}\frac{e^x-1}{x}=1$

Method to find the limit of exponential functions:-

If gives function has exponential term, then we convert the given theorem in the form of $\frac{e^x-1}{x}$ and then use the theorem $\lim_{x\to 0}\frac{e^x-1}{x}=1$.

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Example:- 1

Find the value of $\lim_{x \to 0} \frac{e^{3x} - 1}{x}$

Solution:-

$$\lim_{x \to 0} \frac{e^{3x} - 1}{x} = \lim_{x \to 0} \frac{e^{3x} - 1}{x} \times \frac{3}{3}$$

$$= 3 \lim_{x \to 0} \frac{e^{3x} - 1}{3x} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Let h = 3x. Then $h \to 0$ as $x \to 0$

Now, from eq. (1) we get

$$3 \lim_{x \to 0} \frac{e^{3x} - 1}{x} = 3 \lim_{h \to 0} \frac{e^{h} - 1}{h} = 3 \times 1 = 3$$

Example:- 2

Evaluate $\lim_{x \to 3} \frac{e^{x} - e^{3}}{x - 3}$

Solution:-

We have $\lim_{x \to 3} \frac{e^{x} - e^{3}}{x - 3}$

On put h = x – 3 we get

$$\lim_{x \to 3} \frac{e^{x} - e^{3}}{x - 3} = \lim_{h \to 0} \frac{e^{h+3} - e^{3}}{h}$$

$$= \lim_{h \to 0} \frac{e^{h} e^{3} - e^{3}}{h}$$

$$= e^{3} \lim_{h \to 0} \frac{e^{h} - 1}{h} = e^{3} \times 1 = e^{3}$$

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Theorem:- $\lim_{x\to 0}\frac{\log_e(1+x)}{x}=1$

Note:- $\lim_{x\to 0}\frac{\log_e(1-x)}{-x}=1$

Corollary:-

(i) $\lim_{x\to 0}\frac{\log_e(1-x)}{-x}=1$

(ii) $\lim_{x\to 0}\frac{a^x-1}{x}=\log a$

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Limits Of Logarithmic Functions

Theorem:- $\lim_{x\to 0}\frac{\log_e(1+x)}{x}=1$

Note:- $\lim_{x\to 0}\frac{\log_e(1-x)}{-x}=1$

Corollary:-

(i) $\lim_{x\to 0}\frac{\log_e(1-x)}{-x}=1$ (ii) $\lim_{x\to 0}\frac{a^x-1}{x}=\log a$

Method to find the limit of logarithmic Function:-

If given function involves logarithmic function, then we convert the given function in the form of $\frac{\log_e(1+x)}{x}$ and then use the theorem $\lim_{x\to 0}\frac{\log_e(1+x)}{x}=1$

Example:- 3

Evaluate $\lim_{x\to 0}\frac{\log_e(1+2x)}{x}$

Solve:- We have, $\lim_{x\to 0}\frac{\log_e(1+2x)}{x}\times\frac{2}{2}$

$$=2\lim_{x\to 0}\frac{\log_e(1+2x)}{2x}$$

On putting h = 2x we get

$$=2\lim_{x\to 0}\frac{\log_e(1+2x)}{x}=2\lim_{h\to 0}\frac{\log_e(1+h)}{h}=2\times 1=2$$

Example:- 4

Evaluate $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}$

Solution:-

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}$$

On multiplying numerators and denominator by $\sqrt{1+x}+1$ we get

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}\times\frac{\sqrt{1+x}+1}{\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{1+x-1}{\left(\sqrt{1+x}+1\right)\log(1+x)}$$

$$=\lim_{x\to 0}\frac{x}{\left(\sqrt{1+x}+1\right)\log(1+x)}$$

$$=\frac{1}{\left(\sqrt{1+0}+1\right)}\lim_{x\to 0}\frac{1}{\frac{\log(1+x)}{x}}=\frac{1}{1+1}\times 1=\frac{1}{2}$$

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Example:- 5

Evaluate $\lim_{x\to 0}\frac{2^x-1}{\sqrt{1+x}-1}$

Solution:-

$$\lim_{x\to 0}\frac{2^x-1}{\sqrt{1+x}-1}$$

$$=\lim_{x\to 0}\frac{2^x-1}{\sqrt{1+x}-1}\times\frac{\left(\sqrt{1+x}+1\right)}{\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{2^x-1}{x}\times\left\{\sqrt{1+x}+1\right\}$$

$$=\lim_{x\to 0}\frac{2^x-1}{x}\times\lim_{x\to 0}\left(\sqrt{1+x}+1\right)$$

$$=(\log 2)\times 2=2\log 2$$

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LIMITS AND DERIVATIVES

Existence of Limits

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EXISTENCE OF LIMITS

Concept Of Left Hand And Right Hand Limits

Left hand limit:- A real number ℓ_1 is the left hand limit of function $f(x)$ at x = a, if the values of f(x) can be made as close as ℓ_1 at points closed to a and on the left of a. Symbolically, it is written as $LHL = \lim_{x \to a^-} f(x) = \ell_1$.

In other words, we can say that $LHL = \lim_{x \to a^-} f(x) = \ell_1$ is the expected value of f at x = a, when we have the values of f near x to the left of a. This value is called the left hand limit of f(x) at a.

Right hand limit:- A real number ℓ_2 is the right hand limit of function f(x) at x = a, if the values of f(x) can be made as close as ℓ_2 at point closed to *a* and on the right of a. Symbolically, it is written as $RHL = \lim_{x \to a^+} f(x) = \ell_2$. In other words, we can say that, $RHL = \lim_{x \to a^+} f(x) = \ell_2$ is the expected value of f at x = a, when we have the values of f near x to the right of a. This value is called the right hand limit of f(x) at a.

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Existence of Limit:-

If the right hand limit and left hand limit coincide (i.e same), then we say that limit exists and their common value is called the limit of f(x) at x = a and denoted it by $\lim_{x \to a} f(x)$.

Method to solve the left hand and right hand limits of a function:-

With the help of following steps, we can find the left hand and right hand limits of a function easily.

Step – I For left hand limit, write the given function as $\lim_{x \to a^-} f(x)$ and for right hand limit, write the given function as $\lim_{x \to a^+} f(x)$.

Step – II For left hand limit, put $x = a - h$ and change the limit $x \to a^-$ by $h \to 0$. Then, limit obtained from step I is $\lim_{h \to 0} f(a - h)$. Similarly, for right hand limit, put $x = a + h$ and change the limit $x \to a^+$ by $h \to 0$. Then, limit obtained from step I is $\lim_{h \to 0} f(a + h)$.

Step – III Now, simplify the result obtained in step – **II** i.e $\lim_{h \to 0} f(a - h)$ or $\lim_{h \to 0} f(a + h)$.

Example:- 1

Suppose the function is defined by $f(x)=\begin{cases}\dfrac{|x-3|}{x-3}, & \text{if } x\neq 3\\ 0, & \text{if } x=3\end{cases}$

(i) Find the left hand limit of f(x) at x = 3

(ii) Find the right hand limit of f(x) at x = 3

Solution:-

(i) Given, $f(x)=\begin{cases}\dfrac{|x-3|}{x-3}, & \text{if } x\neq 3\\ 0, & \text{if } x=3\end{cases}$

$\therefore$ Left hand limit at x = 3 is

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}\frac{|x-3|}{x-3} \text{ .... (1)}$$

On putting $x=3-h$ and changing the limit $x\to 3^-$ by $h\to 0$ in eq. (i) we get

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}\frac{|x-3|}{x-3}=\lim_{h\to 0}\frac{|h|}{-h}$$

$$\Rightarrow \lim_{x\to 3^-} f(x)=\lim_{h\to 0}\frac{h}{(-h)}=-1$$

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(ii) Right hand limit at x = 3 is

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{|x-3|}{x-3} \text{ .... (ii)}$$

On putting $x = 3 + h$ and changing the limit $x \to 3^+$ by $h \to 0$ in equation (ii) we get

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{|x-3|}{x-3} = \lim_{h\to 0} \frac{|h|}{h}$$

$$\Rightarrow \lim_{x\to 3^+} f(x) = \lim_{h\to 0} \frac{|h|}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

Example:-2

Evaluate the left hand and right hand limits of the following functions at x = 2,

$f(x)=\begin{cases}2x+3, & \text{if } x\le 2\\ x+5, & \text{if } x>2\end{cases}$. Does $\lim_{x\to 2} f(x)$ exist?

Solution:-

Given $f(x)=\begin{cases}2x+3, & \text{if } x\le 2\\ x+5, & \text{if } x>2\end{cases}$

LHL = $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2x+3$

$= \lim_{h\to 0}\left[2(2-h)+3\right] = 2(2-0)+3$

[Putting $x=2-h$ and when $x\to 2^-$, then $h\to 0$]

$= 4+3 = 7$

RHL = $\lim_{x\to 2^+} f(x)$

$= \lim_{x\to 2^+}(x+5)$

$= \lim_{h\to 0}(2+h+5) = 2+0+5 = 7$

Putting x = 2 + h and $x\to 2^+$, then $h\to 0$

LHL of $f(\text{at } x=2)$ = RHL of $f(\text{at } x=2)$

$\lim_{x\to 2} f(x)$ exists and it is equal to 7.

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LIMITS AND DERIVATIVES

Define differentiation and its geometical representation

SUBJECT : MATHEMATICS
CHAPTER NUMBER:13
CLASS NUMBER:05

DERIVATIVE AT A POINT

Suppose *f* is a real valued function and *a* is a point in its domain. Then, derivative of *f* at *a* is defined by $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ provided this limit exists

The derivative of f(x) at a is denoted by $f'(a)$

Example:- 1

Find the derivative of $f(x)=4x+5$ at x = 3.

Solution:-

Given $f(x)=4x+5$. We know that, at x = a, $f'(a)=\lim_{h\to0}\frac{f(a+h)-f(a)}{h}$

$$\therefore f'(3)=\lim_{h\to0}\frac{f(3+h)-f(3)}{h}$$

$$=\lim_{h\to0}\frac{4(3+h)+5-(4\times3+5)}{h}$$

$$=\lim_{h\to0}\frac{12+4h+5-17}{h}=\lim_{h\to0}\frac{4h}{h}=4$$

GEOMETRICAL MEANING OF DERIVATIVES

Consider a graph of a function $y = f(x)$:

From Fig.1 we see, that for any two points A and B of the function graph:

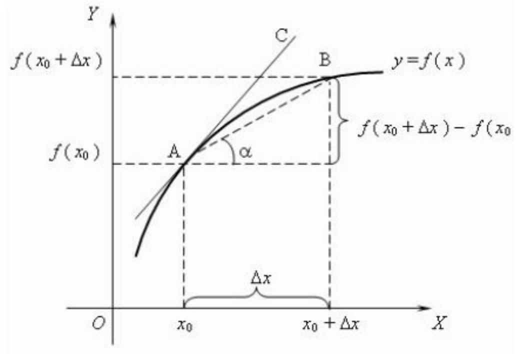

Fig. 1

$$\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \tan \alpha$$

where α- a slope angle of the secant AB.

So, the difference quotient is equal to a secant slope. If to fix the point A and to move the point B towards A, then Δx will

unboundedly decrease and approach 0, and the secant AB will approach the tangent AC. Hence, a limit of the difference quotient is equal to a slope of a tangent at point A. Hence it follows: *a derivative of a function at a point is a slope of a tangent of this function graph at this point.*

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LIMITS AND DERIVATIVES

Differentiation from first principle

SUBJECT : MATHEMATICS
CHAPTER NUMBER:13
CLASS NUMBER:06

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First Principle Of Derivative

Suppose f is a real valued function, the function defined by $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, wherever the limit exists, is defined to be the derivative of f at x and is denoted by $f'(x)$. This definition of derivative is called the first principle of derivative.

Thus, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Sometimes $f'(x)$ is denoted by $\frac{d}{dx}\left[f(x)\right]$ or if $y=f(x)$ then it is denoted by $\frac{dy}{dx}$ and referred to as derivative of $f(x)$, or y with respect to x. It is also denoted by $D\left[f(x)\right]$.

Note:-

Derivative of at x = a is also given by substituting x = a in $f'(x)$ and it is denoted by

$\frac{d}{dx}f(x)\Big|_a$ or $\frac{df}{dx}\Big|_a$ or $\left(\frac{df}{dx}\right)_{x=a}$.

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Example:- 1

Find the derivative of $f(x)=\frac{1}{x}$ from first principle.

Solution:-

We have $f(x)=\frac{1}{x}$

By using first principle, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$\therefore f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right]=\lim_{h\to 0}\frac{1}{h}\left[\frac{-h}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\left[\frac{-1}{x(x+h)}\right]=\frac{-1}{x^2}$$

Example:-2

Find the derivative of e^x, using first principle.

Solution:-

Let $f(x)=e^x$. By using first principle of derivative, we have

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$$

$$\lim_{h\to 0}\frac{e^x\left(e^h-1\right)}{h}=e^x\lim_{h\to 0}\frac{\left(e^h-1\right)}{h}=e^x\times 1=e^x$$

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Example :- 3

Find the derivative of the function $\log x$, by using first principle.

Solution:-

Let $f(x) = \log x$

By using first principle of derivative, we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\log(x+h) - \log x}{h}$$

$$= \lim_{h \to 0} \frac{\log\left(\frac{x+h}{x}\right)}{h} = \lim_{h \to 0} \frac{\log\left(1 + \frac{h}{x}\right)}{\frac{h}{x}} \times \frac{1}{x}$$

$$= 1 \times \frac{1}{x} = \frac{1}{x}$$

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Example:-4

Find the derivative of the following function by using first principle.

(i) sin x (ii) sec x (iii) tanx

Solution:-

(i) Let $f(x)=\sin x$

By using first principle of derivative, we have

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$=\lim_{h\to 0}\frac{2\cos\left(\frac{x+h+x}{2}\right).\sin\left(\frac{x+h-x}{2}\right)}{h}$$

$$\left[\therefore \sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\times\sin\left(\frac{C-D}{2}\right)\right]$$

$$=\lim_{h\to 0}\frac{2\cos\left(x+\frac{h}{2}\right).\sin\frac{h}{2}}{h}$$

$$=\lim_{h\to 0}\cos\left(x+\frac{h}{2}\right).\lim_{\frac{h}{2}\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$=\lim_{h\to 0}\cos\left(x+\frac{h}{2}\right)\times 1$$

$$=\cos(x+0)=\cos x$$

$$\frac{d}{dx}(\sin x)=\cos x$$

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(ii) Let $f(x) = \sec x$

By using first principle of derivative, we have

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\therefore f'(x) = \lim_{h\to 0}\frac{\sec(x+h)-\sec x}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1}{\cos(x+h)}-\frac{1}{\cos x}}{h}$$

$$= \lim_{h\to 0}\frac{\cos x-\cos(x+h)}{h\times \cos x.\cos(x+h)}$$

$$= \lim_{h\to 0}\left[\frac{-2\sin\left(\frac{x+x+h}{2}\right).\sin\left(\frac{x-x-h}{2}\right)}{h.\cos x.\cos(x+h)}\right]$$

$$\left[\therefore \cos C-\cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$$

$$= \lim_{h\to 0}\left[\frac{-2\sin\left(x+\frac{h}{2}\right).\left(-\sin\frac{h}{2}\right)}{h.\cos x\cos(x+h)}\right]$$

$$= \lim_{h\to 0}\frac{\sin\left(x+\frac{h}{2}\right)}{\cos(x+h).\cos x}.\lim_{h\to 0}\frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= \frac{\sin x}{\cos^2 x}\times 1$$

$$= \frac{\sin x}{\cos x}.\frac{1}{\cos x} = \tan x.\sec x$$

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(iii) Let $f(x) = \tan x$

Then, by first principle of derivative, we get

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan(x+h) - \tan x}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x+h)\cos x - \sin x \cos(x+h)}{\cos(x+h)\cos x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x+h-x)}{\cos(x+h)\cos x}\right]$$

$$\left[\because \sin A \cos B - \cos A \sin B = \sin(A-B)\right]$$

$$= \lim_{h\to 0} \frac{\sinh}{h} . \lim_{h\to 0} \frac{1}{\cos(x+h)\cos x}$$

$$= 1 . \frac{1}{\cos(x+0)\cos x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

Hence, $f'(x)$ or $\frac{d}{dx}(\tan x) = \sec^2 x$

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LIMITS AND DERIVATIVES

Rules of sum, difference, product and quotient

SUBJECT : MATHEMATICS
CHAPTER NUMBER:13
CLASS NUMBER:07

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ALGEBRA OF DERIVATIVE

Algebra of Derivative of Functions:-

Let f and g be two functions such that their derivatives are defined in a common domain. Then,

(a) Derivative of sum of two functions is sum of the derivatives of the functions.

$$\frac{d}{dx}\left[f(x)+g(x)\right]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

(b) Derivative of difference of two functions is difference of the derivatives of the functions.

$$\frac{d}{dx}\left[f(x)-g(x)\right]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

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(c) Derivative of product of two functions is given by the following product rule.

$$\frac{d}{dx}\left[f(x).g(x)\right]=f(x)\frac{d}{dx}g(x)+g(x)\frac{d}{dx}f(x).$$

This is also known as Leibnitz product rule of derivative.

(d) Derivative of quotient of two functions is given by the following quotient rule.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\frac{d}{dx}f(x)-f(x)\frac{d}{dx}g(x)}{\left[g(x)\right]^2},g(x)\neq 0$$

Note:- $\frac{d}{dx}\left[c.f(x)\right]=c\frac{d}{dx}f(x)$

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Theorem :- Derivative of $f(x)=x^n$ is nx^{n-1} for any real number n.

Example:- 1

Differentiate $2x^3-4x^2+6x+8$ w.r.t x

Solution:-

Let $y=2x^3-4x^2+6x+8$

On differentiating both sides w.r.t x we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(2x^3-4x^2+6x+8\right)$$

$$=2\frac{d}{dx}\left(x^3\right)-4\frac{d}{dx}\left(x^2\right)+6\frac{d}{dx}(x)+\frac{d}{dx}(8)$$

$$=2\left(3x^2\right)-4(2x)+6(1)+0$$

$$=6x^2-8x+6$$

Example:-2

If $u = 7t^4 - 2t^3 - 8t - 5$, then find $\frac{du}{dt}$ at $t = 2$.

Solution:-

We have $u = 7t^4 - 2t^3 - 8t - 5$

On differentiating both sides w.r.t x we get

$$\frac{du}{dt} = \frac{d}{dt}\left[7t^4 - 2t^3 - 8t - 5\right]$$

$$= 7\left(4t^3\right) - 2\left(3t^2\right) - 8(1) - 0$$

$$= 28t^3 - 6t^2 - 8$$

Now, $\left(\frac{du}{dt}\right)_{t=2} = 28(2)^3 - 6(2)^2 - 8$

$$= 224 - 24 - 8 = 192$$

Example :-3

Differentiate the following functions w.r.t x $(ax+b)(cx+d)^2$

Let $y=(ax+b)(cx+d)^2$

On differentiating both sides w.r.t x we get

$$\frac{dy}{dx}=(ax+b)\frac{d}{dx}(cx+d)^2+(cs+d)^2\frac{d}{dx}(ax+b)$$

$$=(ax+b)\frac{d}{dx}(c^2x^2+d^2+2cxd)+(cx+d)^2(a\times1+0)$$

$$=(ax+b)(c^2(2x)+0+2c\times1\times d)+(cx+d)^2\times a$$

$$=(ax+b)(2c^2x+2cd)+a(cx+d)^2$$

$$=(ax+b)2c(cx+d)+a(cx+d)^2$$

$$=(cx+d)\left[2c(ax+b)+a(cx+d)\right]$$

$$=(cx+d)(2acx+2bc+acx+ad)$$

$$=(cx+d)(3ax+2bc+ad)$$

Example:-4

Differentiate $\frac{x^2+3x-9}{x^2-9x+3}$ w.r.t. x

Solution:-

Let $y=\frac{x^2+3x-9}{x^2-9x+3}$

On differentiating both side w.r.t x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{x^2+3x-9}{x^2-9+3}\right]$$

$$=\frac{\left\{\left(x^2-9x+3\right)\left[\frac{d}{dx}\left(x^2+3x-9\right)\right]-\left(x^2+3x-9\right)\frac{d}{dx}\left(x^2-9x+3\right)\right\}}{\left(x^2-9x+3\right)^2}$$

$$=\frac{\left[\left(x^2-9x+3\right)\left(2x+3-0\right)-\left(x^2+3x-9\right)\times\left(2x-9+0\right)\right]}{\left(x^2-9x+3\right)^2}$$

$$=\frac{\left[\left(x^2-9x+3\right)\left(2x+3\right)-2x\left(x^2+3x-9\right)+9\left(x^2+3x-9\right)\right]}{\left(x^2-9x+3\right)^2}$$

$$=\frac{\left[2x\left(x^2-9x+3-x^2-3x+9\right)+\left(3x^2-27x+9+9x^2+27x-81\right)\right]}{\left(x^2-9x+3\right)^2}$$

$$=\frac{2x\left(-12x+12\right)+\left(12x^2-72\right)}{\left(x^2-9x+3\right)^2}$$

$$=\frac{-2x^2+24x+12x^2-72}{\left(x^2-9x+3\right)^2}$$

$$=\frac{-12x^2+24x-72}{\left(x^2-9x+3\right)^2}=\frac{-12\left(x^2-2x+6\right)}{\left(x^2-9x+3\right)^2}$$

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LIMITS AND DERIVATIVES

Derivative of Trigonometric functions

SUBJECT : MATHEMATICS
CHAPTER NUMBER:13
CLASS NUMBER:08

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DERIVATIVE OF TRIGONOMETRIC FUNCTIONS

To find the derivative of trigonometric functions, we use the algebra of derivative and the following formulae.

(a) $\frac{d}{dx}(\sin x) = \cos x$

(b) $\frac{d}{dx}(\cos x) = -\sin x$

(c) $\frac{d}{dx}(\tan x) = \sec^2 x$

(d) $\frac{d}{dx}(\sec x) = \sec x . \tan x$

(e) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(f) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x$

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Example:- 1

Find the derivative of following functions.

(a) $x^2 \cos x$ (b) $x^3 \sec x$ (c) $x \tan x$

Solution:-

(a) Let $y = x^2 \cos x$

On differentiating both sides w.r.t x we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 \cos x\right) = x^2 \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}\left(x^2\right)$$

$$= x^2(-\sin x) + \cos x(2x)$$

$$= 2x\cos x - x^2 \sin x$$

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(b) Let $y = x^3 \sec x$

On differentiating both sides w.r.t x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 \sec x\right)$$

$$\Rightarrow \frac{dy}{dx} = x^3 \frac{d}{dx}(\sec x) + \sec x \frac{d}{dx}\left(x^3\right)$$

$$= x^3 . \sec x \tan x + \sec x\left(3x^2\right)$$

$$= x^3 . \sec x \tan x + 3x^2 \sec x$$

(c) Let $y = x \tan x$

On differentiating both sides w.r.t x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(x \tan x)$$

$$\frac{dy}{dx} = x \frac{d}{dx}(\tan x) + \tan x \frac{d}{dx}(x)$$

$$= x \sec^2 x + \tan x(1)$$

$$= x \sec^2 x + \tan x$$

Example:-2

If $y=\left\{\frac{1-\tan x}{1+\tan x}\right\}$, then show that $\frac{dy}{dx}=\frac{-2}{1+\sin 2x}$.

Solution:-

We have, $y=\frac{1-\tan x}{1+\tan x}$

On differentiating both sides w.r.t x we get

$$\frac{d}{dx}=\frac{(1+\tan x).\frac{d}{dx}(1-\tan x)-(1-\tan x).\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$=\frac{(1+\tan x)\left(-\sec^2 x\right)-(1-\tan x)\left(\sec^2 x\right)}{(1+\tan x)^2}$$

$$=\frac{-2\sec^2 x}{(1+\tan x)^2}=\frac{-2}{\left(\cos^2 x\right)\left(1+\tan^2 x+2\tan x\right)}$$

$$=\frac{-2}{\left(\cos^2 x\right)\left\{1+\frac{\sin^2 x}{\cos^2 x}+\frac{2\sin x}{\cos x}\right\}}=\frac{-2}{(1+\sin 2x)}$$

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LIMITS AND DERIVATIVES

Differentiation using Chain rule

SUBJECT : MATHEMATICS
CHAPTER NUMBER:13
CLASS NUMBER:09

DERIVATIVE OF COMPOSITE FUNCTIONS

To study derivative of composition functions, we start with illustrative example, say we want find the derivative of f where $f(x) = (2x+1)^3$.

Now $\frac{df(x)}{dx} = \frac{d(2x+1)^3}{dx} = \frac{d\left(8x^3 + 12x^2 + 6x + 1\right)}{dx} = 24x^2 + 24x + 6 = 6(2x+1)^2$

We observe that , if we take $g(x) = 2x+1$ and $h(x) = x^3$

Then $f(x) = hog(x) = (2x+1)^3 \Rightarrow \frac{df(x)}{dx} = f'(x) = \frac{d\,hog(x)}{d\,g(x)} \times \frac{d\,g(x)}{dx}$

i.e. $\frac{d(2x+1)^3}{d(2x+1)} \times \frac{d(2x+1)}{dx} \Rightarrow f'(x) = 3 \times (2x+1)^2 \times 2 = 6(2x+1)^3$

The advantage of such observation is that it simplifies the calculation in finding the derivative.

CHAIN RULE

Let f be the real valued function which is composition of two functions $u \;\& v$. i.e. $f = uov$.

Where $u \;\& v$ are differentiable functions and uov is also differentiable function.

$\Rightarrow \dfrac{df}{dx} = \dfrac{d\,uov}{d\,x} = \dfrac{d\,uov}{d\,v} \times \dfrac{dv}{dx}$, Provided all the derivatives in the statement exists.

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Example:-1

Find $\frac{dy}{dx}$ of $y = \sin(x^2 + 1)$

Solution:- Given $y = \sin(x^2 + 1)$

$$\Rightarrow \frac{dy}{dx} = \frac{d(\sin(x^2 + 1))}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d(\sin(x^2 + 1))}{d(x^2 + 1)} \times \frac{d(x^2 + 1)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \cos(x^2 + 1) \times 2x$$

Example:2

Find $\frac{dy}{dx}$ of $y = \log(\tan x)$

Solution:- Given $y = \log(\tan x)$

$$\Rightarrow \frac{dy}{dx} = \frac{d(\log(\tan x))}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d(\log(\tan x))}{d \tan x} \times \frac{d \tan x}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\tan x} \times \sec^2 x$$

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Example:-3

Find $\frac{dy}{dx}$ of $y = e^{\sin(x^2)}$

Solution:- Given $y = e^{\sin(x^2)}$

$$\Rightarrow \frac{dy}{dx} = \frac{d(e^{\sin(x^2)})}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d(e^{\sin(x^2)})}{d\sin(x^2)} \times \frac{d\sin(x^2)}{d(x^2)} \times \frac{d(x^2)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = e^{\sin(x^2)} \times \cos(x^2) \times 2x$$

Example:-4

Find $\frac{dy}{dx}$ of $y = (x^2 + x + 1)^4$

Solution:- Given $y = (x^2 + x + 1)^4$

$$\Rightarrow \frac{dy}{dx} = \frac{d(x^2 + x + 1)^4}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d(x^2 + x + 1)^4}{d(x^2 + x + 1)} \times \frac{d(x^2 + x + 1)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 4(x^2 + x + 1)^3 \times (2x + 1)$$

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Example:-5

Find $\frac{dy}{dx}$ of $y = \frac{1}{\sqrt{a^2 - x^2}}$

Solution:-Given $y = \frac{1}{\sqrt{a^2 - x^2}}$

$$\Rightarrow \frac{dy}{dx} = \frac{d(a^2 - x^2)^{-1/2}}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d(a^2 - x^2)^{-1/2}}{d(a^2 - x^2)} \times \frac{d(a^2 - x^2)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{2}(a^2 - x^2)^{-3/2} \times (-2x)$$

$$= \frac{x}{(a^2 - x^2)^{\frac{3}{2}}}$$

Example:-6

Find $\frac{dy}{dx}$ of $y = \sin^3 x$

Solution:-Given $y = \sin^3 x$

$$\Rightarrow \frac{dy}{dx} = \frac{d(\sin^3 x)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d(\sin^3 x)}{d\sin x} \times \frac{d\sin x}{dx}$$

$$= 3\sin^2 x \times \cos x$$

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Example:-7

Find $\frac{dy}{dx}$ of $y = \log(\sec x + \tan x)$

Solution:-Given that $y = \log(\sec x + \tan x)$

$$\Rightarrow \frac{dy}{dx} = \frac{d(\log(\sec x + \tan x))}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d(\log(\sec x + \tan x))}{d(\sec x + \tan x)} \times \frac{d(\sec x + \tan x)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sec x + \tan x} \times \left(\sec x.\tan x + \sec^2 x\right) = \sec x$$

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Example:-8

Find $\frac{dy}{dx}$ of $y = e^{x\sin x}$

Solution:-Given that $y = e^{x\sin x}$

$$\Rightarrow \frac{dy}{dx} = \frac{d\left(e^{x\sin x}\right)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d\left(e^{x\sin x}\right)}{d(x\sin x)} \times \frac{d(x\sin x)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = e^{x\sin x} \times \{x\cos x + \sin x\}$$

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Example:-9

Find $\frac{dy}{dx}$ of $y = \cos(x^3).\sin(x^3)$

Solution:-Given $y = \cos(x^3).\sin(x^3)$

$$\Rightarrow \frac{dy}{dx} = \frac{d\left(\cos x^3 . \sin x^3\right)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d\left(\cos x^3 . \sin x^3\right)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \cos x^3 \frac{d\left(\sin x^3\right)}{dx^3} \times \frac{dx^3}{dx} + \sin x^3 \frac{d\left(\cos x^3\right)}{dx^3} \times \frac{dx^3}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \left(\cos x^3\right)\left(\cos x^3\right)3x^2 - \left(\sin x^3\right)\left(\sin x^3\right)3x^2$$

$$\Rightarrow \frac{dy}{dx} = \left(\cos x^3\right)^2 3x^2 - \left(\sin x^3\right)^2 3x^2$$

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Example:-10

Find $\frac{dy}{dx}$ of $y = \frac{\sin(ax+b)}{\cos(cx+d)}$

Solution:-Given that $y = \frac{\sin(ax+b)}{\cos(cx+d)}$

$$\Rightarrow \frac{dy}{dx} = \frac{d\left\{\frac{\sin(ax+b)}{\cos(cx+d)}\right\}}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(cx+d)\times\frac{d(\sin(ax+b))}{d(ax+b)}\times\frac{d(ax+b)}{dx} - \sin(ax+b)\times\frac{d(\cos(cx+d))}{d(cx+d)}\times\frac{d(cx+d)}{dx}}{\cos^2(cx+d)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(cx+d)\times\cos(ax+b)\times a + \sin(ax+b)\times\sin(cx+d)\times c}{\cos^2(cx+d)}$$

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THANKING YOU

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