Cartesian Product Of Sets

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02
CHAPTER NAME : RELATIONS AND FUNCTIONS

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: **1800 120 2316**
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Learning outcomes

Students will be able to

- **return the elements of the Cartesian product of two sets as set of ordered pairs,**
- **apply standard set operations such as union and intersection to Cartesian product questions,**
- **understand that Cartesian products are non-commutative and non-associative,**
- **represent Cartesian products using arrow diagrams and Cartesian diagrams.**

Ordered pair

An ordered pair consists of two numbers that are written in the fixed order.

So, we can define an ordered pair as the pair of elements that occur in a particular order and are enclosed in parentheses.

The ordered pair (2, 5) means a pair of two integers, strictly in the order with 2 at the first place called the abscissa and 5 at the second place called the ordinate.

The ordered pair (2, 5) is not equal to the ordered pair (5, 2) because (2, 5) ≠ (5, 2). Therefore, in a pair, the order of elements is important.

Equality of Ordered Pairs:

Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

Example: If , (a - 3, b + 2) = (4, -2), find the values of a and b.
a - 3 = 4 and b + 2 = -2.
Therefore, a = 7 and b = - 4.

Introduction to Cartesian Product

Given two non-empty sets A and B.

The Cartesian product $A \times B$ is the set of all ordered pairs of elements from A and B,

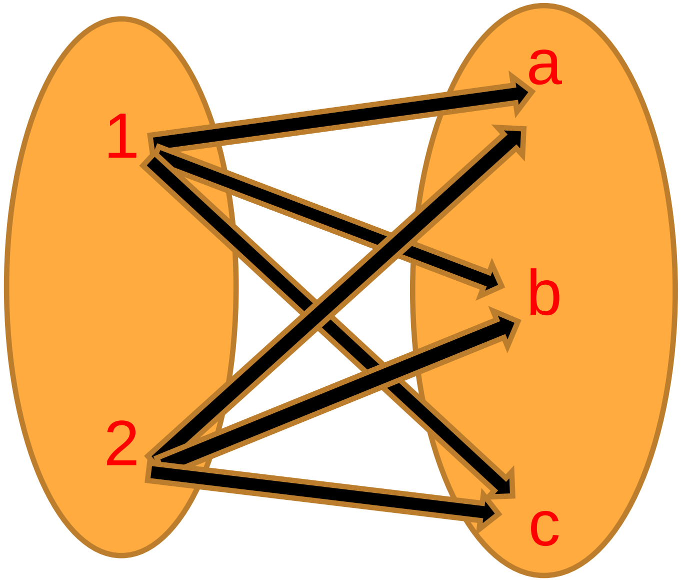
i.e., $A \times B = \{ (a, b) : a \in A, b \in B \}$

Example of Cartesian product

Example: Let A ={1,2} and B ={ a, b, c}.
Find A × B.

Solution:

A× B = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}



Notes

- If either A or B is an empty set, then , $A \times B = \emptyset$
- If A and B are non-empty sets and either A or B is an infinite set, then so is $A \times B$
- if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.
- In general, $A \times B \neq B \times A$
- $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$.

Here (a, b, c) is called an ordered triplet.

- $A \times (B \cap C) = (A \times B) \cap (A \times C)$ and
  $A \times (B \cup C) = (A \times B) \cup (A \times C)$

- **The Cartesian product R × R ={(x, y) : x, y ∈ R} represents the coordinates of all the points in two dimensional space.**

- **The Cartesian product R × R × R ={(x , y, z) : x, y, z ∈ R} represents the coordinates of all the points in three-dimensional space.**

Example :

If P = {a, b} and Q = {x, y}, find P × Q and Q × P.
Are these two products equal ?

Solution:

P × Q = {(a, x), (a, y), (b, x), (b, y)} and
Q × P = {(x , a), (x, b), (y, a), (y, b)}

The pair (a, x) is not equal to the pair (x, a).
Therefore P × Q ≠ Q × P.

Example :

Let $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{4, 5\}$

Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Solution:

$B \cap C = \{4\}$.

Therefore, $A \times (B \cap C) = \{1,2\}\times\{4\} = \{(1,4), (2,4)\}$………(1)

Also, $A\times B = \{(1, 3),(1,4), (2,3),(2,4)\}$,

$A\times C = \{(1, 4),(1,5), (2,4), (2,5)\}$…………………(2)

Therefore, $(A \times B) \cap (A \times C) = \{(1,4), (2,4)\}$

Hence, from (1) & (2), we get, $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Some Properties of Cartesian product of sets

(a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(c) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

(d) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

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RELATIONS

Definition of Relation, Pictorial diagrams

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02
CHAPTER NAME : RELATIONS AND FUNCTIONS

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Learning outcomes:

Students will be able to

- **understand the meaning of a relation between two sets or within a set itself,**
- **use an arrow diagram or Cartesian diagrams to represent a given relation between two sets,**
- **write down the ordered pairs that show the relationship between two sets given an arrow diagram, a Cartesian diagram, a rule, or a table,**
- **form ordered pairs, an arrow diagram, or an equation (rule) based on the described relationship shown,**

Relation

Relation: A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product A × B.

The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

The second element is called the image of the first element.

Example

Consider an example of two sets, A = {2, 5, 7, 8, 9, 10, 13} and B = {1, 2, 3, 4, 5}.

The Cartesian product A × B has 30 ordered pairs such as

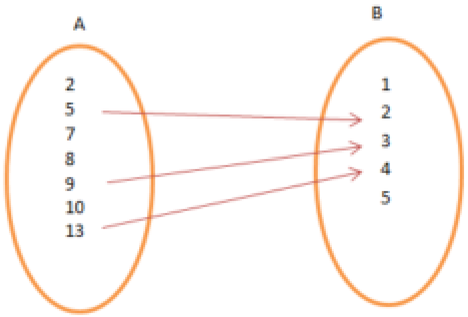
A × B = {(2, 3), (2, 5)…(10, 12)}.

From this, we can obtain a subset of A × B, by introducing a relation R between the first element and the second element of the ordered pair (x, y) as

R = {(x, y): x = 4y – 3, x ∈ A and y ∈ B}

Then, R = {(5, 2), (9, 3), (13, 4)}.

(Arrow representation of the Relation R)



Relation on a set

Definition: A relation on the set A is a relation from A to A.

In other words, a relation on the set A is a subset of A×A.

Example: Let A = {1, 2, 3, 4}. Which ordered pairs are in the relation R = {(a, b) | a < b} ?

Relation on a set

Solution:
R={(1,2),(1,3),(1,4),(2.3),(2,4),(3,4)}

R	1	2	3	4
1		*	*	*
2			*	*
3				*
4				

Number of Relations

For two non-empty set, A and B.

If the number of elements in A is h i.e., n(A) = m & that of B is k i.e., n(B) = n,

then the number of ordered pair in the Cartesian product will be n(A × B) = mn

.

The total number of relations is 2^{mn}.

Types of Relations

Universal Relation

A relation R on a set A is a universal relation if each element of A is related to every element of A, i.e., $R = A \times A$.

Empty relation and Universal relation are sometimes called trivial relation.

Empty Relation

If no element of set A is related or mapped to any element of A, then the relation R on A is an empty relation, i.e, R = Φ.

Think of an example of set A consisting of students of RD Women's college.

R is a relation on A defined as "is brother of"
Here is an empty relation.

Identity Relation

In Identity relation, every element of set A is related to itself only.

I = {(a, a), a∈ A}.

For example, If we throw two dice, we get 36 possible outcomes, (1, 1), (1, 2), … , (6, 6).

If we define a relation as R= {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}, it is an identity relation.

Representation of a Relation

- **A relation may be represented algebraically either by Roster method or by Set- builder method**

- **An arrow diagram is a visual representation of a relation.**

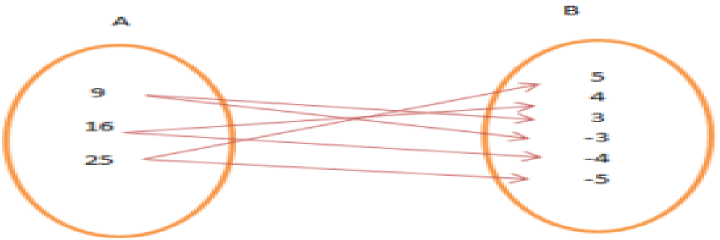
Example on Representation:

Consider an example of two sets A = {9, 16, 25} and B = {5, 4, 3, -3, -4, -5}.

The relation is that the elements of A are the square of the elements of B.

In set-builder form, R = {(x, y): x is the square of y, x ∈ A and y ∈ B}.

In roster form, R = {(9, 3), (9, -3), (16, 4), (16, -4), (25, 5), (25, -5)}.



(Arrow representation of the Relation R)

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RELATIONS

Domain, Co-domain and Range

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02
CHAPTER NAME : RELATIONS AND FUNCTIONS

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: **1800 120 2316**
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Learning outcomes

Students will able to

- **represent relations using diagrams, sets of ordered pairs, equations, and graphs,**
- **define and find a domain, a co-domain, and a range of discrete relations (as sets),**
- **find inverse of a relation.**

Domain and Range of a Relation

- **The set of all first components of the ordered pairs belonging to R is called the domain of R.**

- **Thus, Dom(R) = {a ∈ A: (a, b) ∈ R for some b ∈ B}.**

- **The set of all second components of the ordered pairs belonging to R is called the range of R.**

Thus, range of R = {b ∈ B: (a, b) ∈R for some a ∈ A}.

Contd…..

Therefore, Domain (R) = {a : (a, b) ∈ R} and Range (R) = {b : (a, b) ∈ R}

Note:

The domain of a relation from A to B is a subset of A.

The range of a relation from A to B is a subset of B.

Example

If A = {2, 4, 6, 8)   B = {5, 7, 1, 9}.

Let R be the relation ‘is less than’ from A to B.

Find Domain (R) and Range (R).

Solution

R = {(4, 5), (4, 7), (4, 9), (6, 7), (6, 9), (8, 9) (2, 5) (2, 7) (2, 9)}

Domain (R) = {2, 4, 6, 8}

Range (R) = {1, 5, 7, 9}

Example

Let A = {2, 3, 4, 5} and B = {8, 9, 10, 11}.

Let R be the relation ‘is factor of’ from A to B.

(a) Write R in the roster form. Also, find Domain and Range of R.

(b) Draw an arrow diagram to represent the relation.

Solution

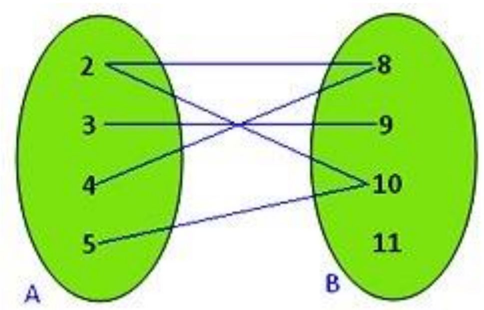
(a) **Clearly, R consists of elements (a, b) where a is a factor of b.**

Therefore, Relation (R) in the roster form is R = {(2, 8); (2, 10); (3, 9); (4, 8), (5, 10)}

Therefore, Domain (R) = Set of all first components of R = {2, 3, 4, 5} and Range (R) = Set of all second components of R = {8, 10, 9}

Here co-domain is B= {8, 9, 10, 11}.

(b) The arrow diagram representing R is as follows:



The inverse of a Relation:

Let R be a relation from a set A to another set B.

Then R is of the form R= {(x, y): x ∈ A and y ∈ B}.

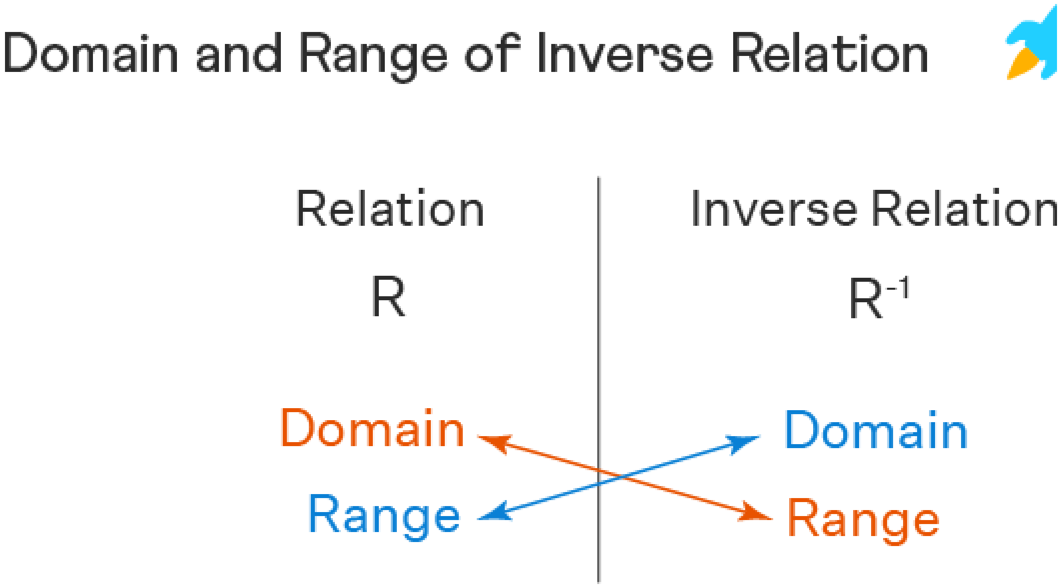
The inverse relationship of R is denoted by R^{-1} and its formula is
R^{-1} = {(y, x): y ∈ B and x ∈ A}.

i.e., If R is from A to B, then R^{-1} is from B to A.
Thus, if R is a subset of A x B, then R^{-1} is a subset of B x A.

Inverse Relation Examples

- Have a look at the following relations and their inverse relations on two sets A = {a, b, c, d, e} and B = {1, 2, 3, 4, 5}.
- If R = {(a, 2), (b, 4), (c, 1)} ⇔ R^{-1} = {(2, a), (4, b), (1, c)}
- If R = {(c, 1), (b, 2), (a, 3)} ⇔ R^{-1} = {(1, c), (2, b), (3, a)}
- If R = {(b, 3), (c, 2), (e, 1)} ⇔ R^{-1} = {(3, b), (2, c), (1, e)}

Domain and range of inverse relation



The domain of R^{-1} = the range of R

and

The range of R^{-1} = the domain of R.

Example

Let R={(a,b) : a,b ∈N and a+3b=12}.

Find the domain , range of R and R^{-1}

R = {(9 , 1) , (6 , 2) , (3 , 3)} -1

Domain of R = { 9 , 6 , 3 }

Range of R = { 1 , 2 , 3 }

R^{-1} = {(1 , 9) , (2 , 6) , (3 , 3)}

Assignment

1. **Let $A = \{3, 4, 5, 6\}$ $B = \{1, 2, 3, 4, 5, 6\}$ Let $R = \{(a, b) : a \in A, b \in B \text{ and } a < b\}$.**
   Write R in the roster form. Find its domain and range.

2. **Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ Let R be A relation on A defined by $R = \{a, b\} : a \in A, b \in A,$ a is a multiple of b}. Find R, domain of R, range of R.**

3. **Determine the range and domains of the relation R defined by $R = \{(x - 1), (x + 2) : x \in (2, 3, 4, 5)\}$**

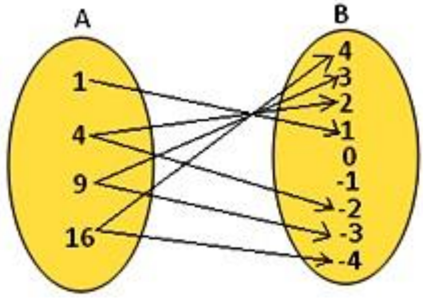
4. **Let $A = \{1, 2, 3, 4, 5, 6\}$ Define a relation R from A to A by R $\{(x, y) : y = x + 2\}$**
   - **Depict this relation using an arrow diagram.**
   - **Write down the domain and range of R**

5. The adjoining figure shows a relation between the set A and B. Write this relation in

(i) Set builder form.

(ii) Roster form.

(iii) Find domain and range.



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FUNCTIONS

Pictorial Representation, Domain and Range

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02
CHAPTER NAME : RELATIONS AND FUNCTIONS

Learning outcomes

Students will be able to

- **define a function,**
- **determine whether a relation is a function from a relation diagram, a table, and a set of ordered pairs,**
- **determine whether a relation is a function from its schematic descriptions or equation,**
- **determine equations that can be defined as functions,**
- **represent functions using diagrams, sets of ordered pairs, equations, and graphs,**
- **define and find a domain, a co-domain, and a range of discrete functions (as sets).**

FUNCTIONS

- A function f from a set A to a set B is a special relation in which, every element of set A has unique image in set B.

- The function f from A to B is denoted by f : A $\rightarrow$ **B**

- If, f(a) = b, then ‘b’ is called the image of ‘a’ under f and ‘a’ is called the pre image of ‘b’ under f

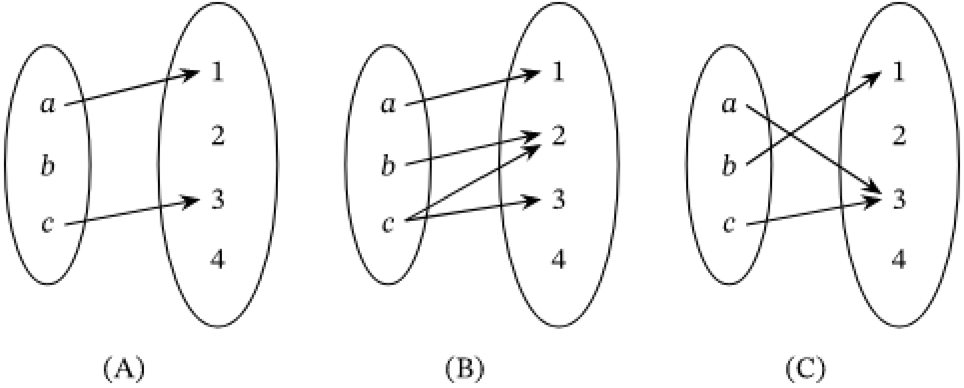
Representation of function

A function can be represented by three methods:

- By mapping (Diagrammatic)
- By Algebraic method
- In the form of order pairs

Diagrammatic representation

It shows the graphical aspect of the relation of elements of A and B.
First two cases are not functions but the last represents a function



Algebraic method

It shows the relation between the elements of two sets in the form of two variables x and y ,where x is independent and y is dependent variable.

If A and B are two given sets with A={1,2,3} and B={ 5,7,9}
Then f: $A \rightarrow B$ given by $y = 2x+3$ or $f(x) = 2x+3$

In the form of order pairs

A function f: A→B can be expressed as a set of order pairs in which the first element of every ordered pair is in A and second element is in B

So f is set of order pairs (a,b) such that

- a is an element of A
- B is an element of B
- Two ordered pairs should not have the same first elements

Determining Whether a Relation Is a Function from a Set of Ordered Pairs

- Which of the following relations represents a function?
- Relation R= (4,12)(4,15)(5,18)(5,21)(6,24)

- Relation B=(4,12)(5,15)(6,18)(7,21)(8,24)

Number of Functions

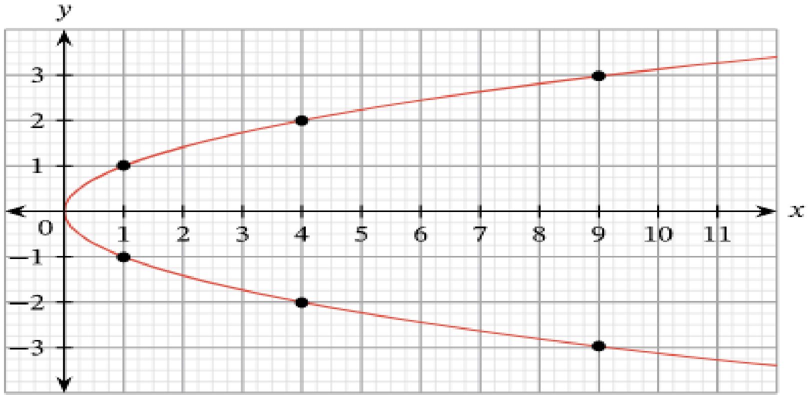
Suppose A and B are two sets with n(A)=m and n(B)= n.
Then number of functions fro A to B is ……….?

n

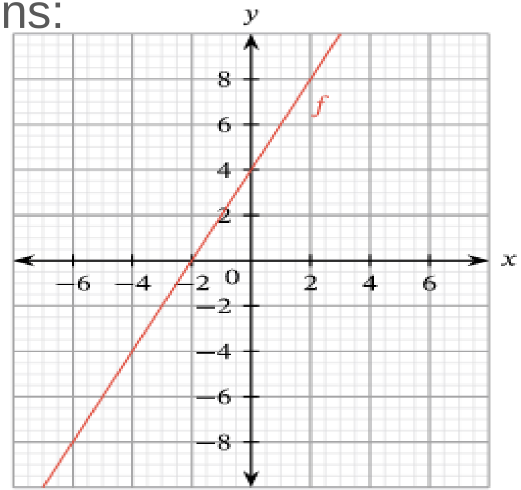
Vertical Line test

- The graph of a function can only have at most one point for each x-coordinate.
- Another way of saying this is every vertical line $x=a$ can only intersect the graph of a function at most once.
- If a vertical line $x=a$ intersects a graph more than once, then it is not the graph of a function.

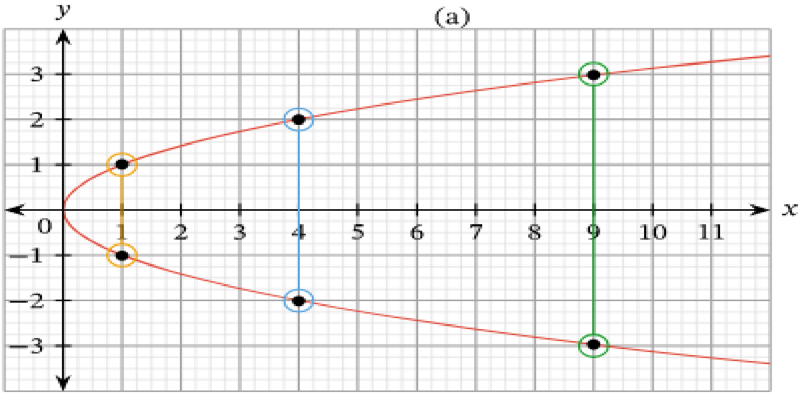
Check whether the following are functions:

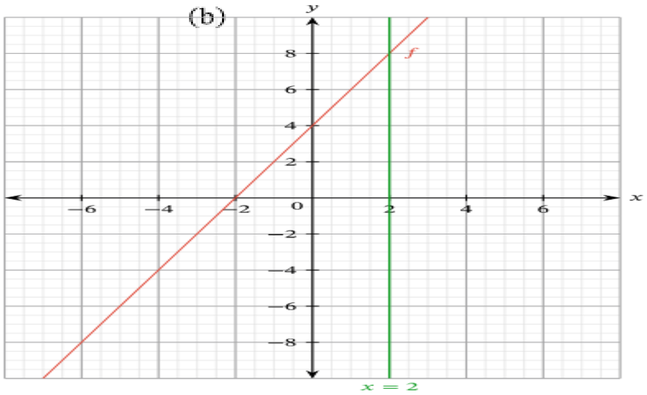


(a)



(b)





Domain, Co-domain, and Range of a Function:

Here we will discuss about domain, co-domain and range of function.
Let f: A → B (f be function from A to B), then

● Set A is known as the domain of the function 'f'

● Set B is known as the co-domain of the function 'f'

● Set of all f-images of all the elements of A is known as the range of f. Thus, range of f is denoted by f(A).

Example:

3. Let A = {1, 2, 3, 4} and B = {0, 3, 6, 8, 12, 15}

Consider a rule $f(x) = x^2 - 1, x \in A$, then

(a) show that f is a mapping from A to B.
(b) draw the arrow diagram to represent the mapping.
(c) represent the mapping in the roster form.
(d) write the domain and range of the mapping.

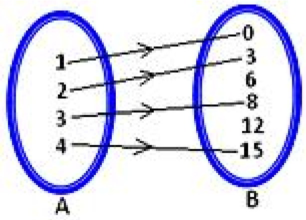
Solution:

a) Using f (x) = x^2 - 1, x ∈ A we have f(1) = 0, f(2) = 3, f(3) = 8, f(4) = 15

We observe that every element in set A has unique image in set B.

Therefore, f is a mapping from A to B.

b)



c) Mapping can be represented in the roster form as

f = {(1, 0); (2, 3); (3, 8); (4, 15)}

(d) Domain (f) = {1, 2, 3, 4} Range (f) = {0, 3, 8, 15}

Equal Functions:

Let A and B be sets and $f:A \to B$ and $g:A \to B$ be functions. We say that f and g are *equal* and write f=g if $f(a)=g(a)$ for all $a \in A$.

i.e.

f=g

If Dom f = Dom g

Co-dom f = Co-dom g

$f(a) = g(a)$ for all $a \in A$ i.e Rang f= Rng g

Check whether the following functions are equal?

f, g : R→R given by

i) $f(x) = 1$, $g(x) = \frac{x}{x}$

i) $f(x) = 1$, $g(x) = \sin^2 x + \cos^2 x$

i) $f(x) = \log(x^2)$, $g(x) = 2\log(x)$

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FUNCTIONS

Real Valued Function , Domain and Range

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02 (Period – 05)
CHAPTER NAME : RELATIONS AND FUNCTIONS

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Learning outcomes

In this module we are going to learn about

- Real functions
- Domain, Co-domain and Range of a Function
- Procedures to find domain and range of functions

Real functions

A function $f: A \to B$ is called a real-valued function if B is a subset of R (set of real numbers)

If both A and B are subsets of R , then f is called a real function.

Example: $f: R \to R$ given by $f(x) = x^2 + 3x + 7$ is a real function.

Examples:

1. If f is a real function defined by $f(x) = \frac{x-1}{x+1}$, then prove that $f(2x) = \frac{3\,f(x)+1}{f(x)+3}$
2. If $f(x) = x + \frac{1}{x}$, prove that $[f(x)]^3 = f(x^3) + 3\,f\left(\frac{1}{x}\right)$.
3. If for non –zero, $a\,f(x) + b\,f\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then find $f(x)$.

Domain and range of a function

What is Domain?

Domain is the set of all possible input values (*x*-values) for which f(x) is well defined.

What is Range?

Range: is the set of all possible output values (*y*-values)

Example: Find the domain of the following functions:

$(i)\ f(x) = \dfrac{1}{x+2}$

Sol: Here $f(x)$ assumes real values for all real values of x except for the values of x satisfying $x + 2 = 0\ i.e. x = -2$.

Hence $dom - f = R - \{-2\}$

$(ii) f(x) = x^2 + 2x + 7$

Sol: Here $f(x)$ is real or can be defined for all real values of x.

So $dom - f = R$.

Example

f(x) = 2x – 5

there would be no restrictions on this, so the domain is All Real Numbers

$$g(x) = \frac{1}{x - 2}$$

**a denominator cannot equal 0, so x ≠ 2. The domain is {x | x ≠ 2}*

h(x) = √x + 6

**you cannot take the square root of a negative number, so x must be ≥ -6. The domain is {x | x ≥ -6}*

Your Turn…

Find the domain of each function :

$f(x) = x^2 + 2$

$g(x) = \sqrt{x} - 1$

$h(x) = \dfrac{1}{x + 5}$

Range

The range of a real function $y = f(x)$ is the set of all real values taken by $f(x)$,when x belongs to domain.

To find the range of $f(x)$, we may use the following algorithm.

Step-I: Put $y = f(x)$

Step –II: Solve the equation $y = f(x)$ for x. Let $x = g(y)$

Step-III: Find the values of y for which x is real.

The set of values of y is the range of f.

Find domain and range of following :

1. $f(x) = \frac{1}{\sqrt{x-5}}$
2. $f(x) = \frac{x-2}{3-x}$
3. $f(x) = \frac{x}{1+x^2}$
4. $f(x) = \frac{3}{2-x^2}$
5. $f(x) = \frac{1}{1-x^2}$
6. $f(x) = \frac{x^2}{1+x^2}$
7. $f(x) = \sqrt{4 - x^2}$

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FUNCTIONS

Different Real Functions and their Domain ,Range with Graphs

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02 (Period – 06)
CHAPTER NAME : RELATIONS AND FUNCTIONS

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Learning outcomes

- Define the domain and range of functions from graphs
- Find the domain and range of some algebraic functions

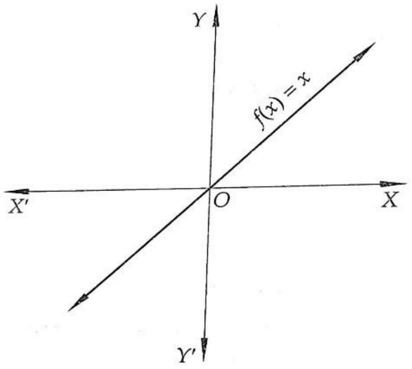
Some Real Functions and their Graphs:

1. Identity Function:

The function $f: R \to R$ defined by $f(x) = x$ for all $x \in R$ is called the identity function.

Here $dom - f = rng - f = R$.

The graph of the identity function is a straight line passing through the origin and equally inclined to the coordinate axes.

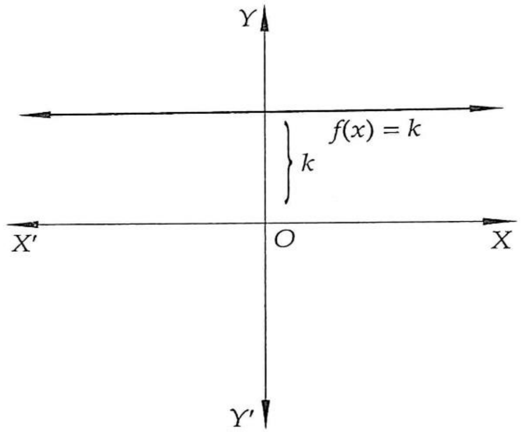


2. Constant Function:

- If k is a fixed real number, then a function $f(x) = k$ for all $x \in R$ is called a constant function.
- Here $domf = R$ and $rngf = \{ k \}$
- The graph of a constant function is a straight line parallel to $x - axis$, which is above or below $x - axis$ according to k is positive or negative .
- If $k = 0$,then the straight line is coincident to $x - axis$.

Graph of constant function

- $f(x) = k$ for all $x \in R$



3. Polynomial Function

- A function $f: R \rightarrow R$ defined by $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, where $n \in N$ and $a_0, a_1, \ldots, a_n$ are real constants and $a_0 \neq 0$ is called a polynomial function in x of degree n.
- If $n = 1, f(x) = a_o + a_1x, a_0 \neq 0$ is called a linear function.
- If $n = 2, f(x) = a_0 + a_1x + a_2x^2, a_2 \neq 0$ is called a quadratic function.
- Some of the examples of polynomial functions are here:

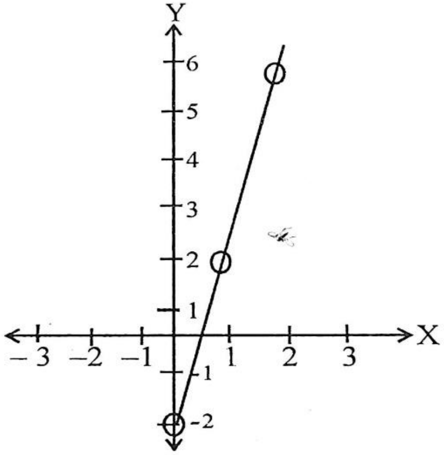
x^2+2x+1	quadratic
$3x-7$	Linear
$7x^3+x^2-2$	cubic

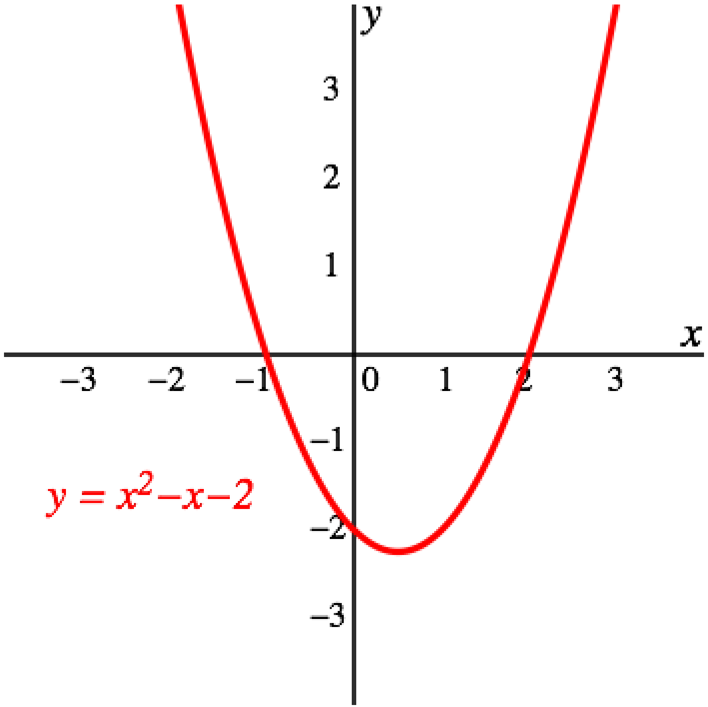
Types of Polynomial Functions

- There are various types of polynomial functions based on the degree of the polynomial. The most common types are:
- Zero Polynomial Function: $P(x) = a = ax^0$
- Linear Polynomial Function: $P(x) = ax + b$
- Quadratic Polynomial Function: $P(x) = ax^2+bx+c$
- Cubic Polynomial Function: ax^3+bx^2+cx+d
- Quartic Polynomial Function: $ax^4+bx^3+cx^2+dx+e$

Graph of polynomial functions

- $f(x) = 4x - 2$

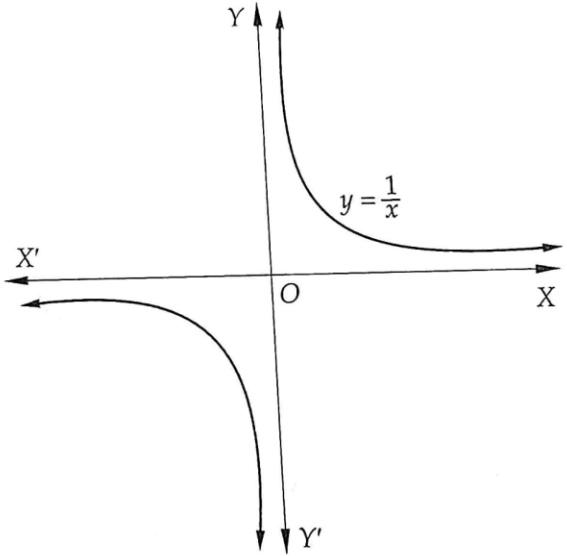




4. Rational Function

- A function which can be expressed as the quotient of two polynomial functions is called rational function $i.e., r(x) = \frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are polynomial functions of x defined in a domain and $g(x) \neq 0$, is a rational function.
- For example, $f(x) = \frac{1}{x} (x \neq 0)$, $\frac{x^3+2x+3}{x^2+x+1}, \frac{2x+1}{x^2+4}$ are rational functions.

Graph of $f(x) = \frac{1}{x}$

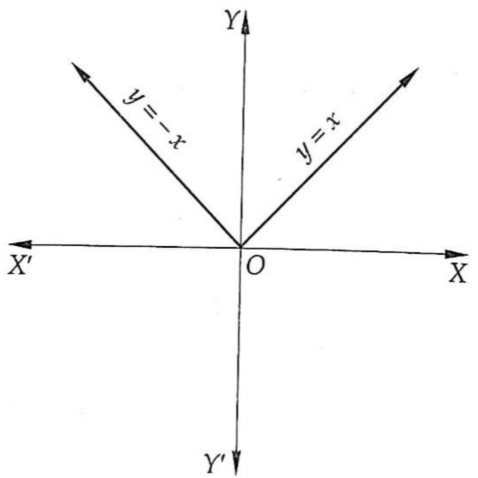


5. Modulus Function:

-
- The function $f(x)$ defined by $f(x) = |x| = \begin{cases} x, & when\, x \geq 0 \\ -x, & when\, x < 0 \end{cases}$ is called the modulus function or absolute value function.
- Here $dom f = R$ and $rng f = R^+ = \{x \in R : x \geq 0\}$.

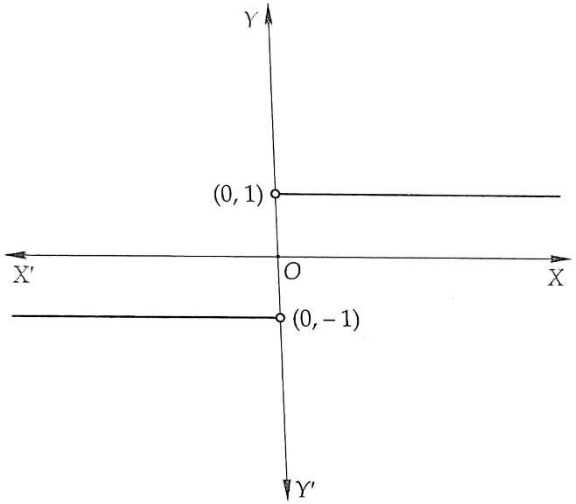
Graph of modulus function

- The graph consists of two half-lines, one in the first quadrant bisecting the axes and
- the other in the second quadrant, bisecting the axes where the origin is included in the graph.



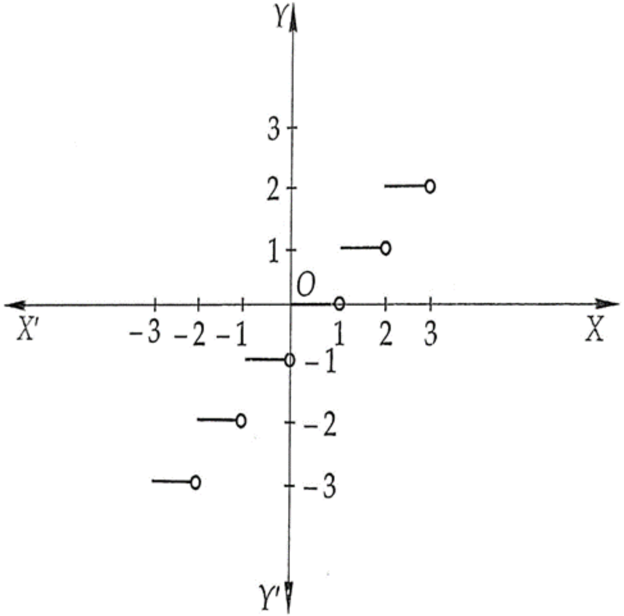
6. Signum Function

- The function $f: R \to R$ defined by $f(x) = \begin{cases} \frac{|x|}{x}, x \neq 0 \\ 0, \ x = 0 \end{cases} = \begin{cases} 1, if \ x > 0 \\ 0, if \ x = 0 \\ -1, if \ x < 0 \end{cases}$ is called the signum function.
- It is also denoted by $sgn\ (x)$.
- Here $dom f = R$ and $rng\ f = \{-1, 0, 1\}$.



7.Greatest Integer Function

- A function $f: R \to R$ be defined by $f(x) = [x]$ is called the greatest integer function,
- where $[x] = n, n \leq x < n+1, n \in Z$.
- Here $dom - f = R$ and $rng - f = Z$



Examples

- $[2.5]$=2, $[-3.7]$ = -4

If f be a real-valued function defined by $f(x) = x - [x]$, then compute

$(i)\ f(2.5) = 2.5 - \ [2.5] = 2.5 - 2 = 0.5$

$(ii) f(-3.7) = -3.7\ -\ [-3.7] = -3.7\ - (-4) = -3.7 + 4 = 0.3$

Questions

1. Draw the graph of the following functions. Also, determine their domain and range.

$$(i)\ f(x) = 2 \qquad (ii) f(x) = -2 \qquad (iii) f(x) = |x-3|$$

2. Let $f: R \to R$ defined by $f(x) = 1 - x^2$ for all $x \in R^+$ .Find its domain and range. Also, draw its graph.

3. Draw the graph of $f(x) = \frac{x^3}{2}$ .Also, find its domain and range.

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FUNCTIONS

Sum, Difference, Product and Quotient of Functions

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02
CHAPTER NAME : RELATIONS AND FUNCTIONS

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Learning outcomes

- We will define the sum, difference, product and quotient of functions
- To consider their domains and ranges where possible.

Algebra of Real Functions

Let $f: A \to R$ and $g: B \to R$ be two real functions. Let $D = A \cap B$. Then, we define addition, subtraction, multiplication, and quotient of two real functions as follows:

(i) Addition: $(f + g): D \to R$ be defined by $(f + g)(x) = f(x) + g(x)$ for all $x \in D$.

(ii)Subtraction: $(f - g): D \to R$ be defined by $(f - g)(x) = f(x) - g(x)$ for all $x \in D$.

(iii) Scalar Multiplication : $(\alpha f): A \to R$ be defined by $(\alpha f)(x) = \alpha f(x)$ for all $x \in A$.

Algebra of Real Functions contd…

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(iv) Multiplication: $(f.g): D \rightarrow R$ be defined by $(f.g)(x) = f(x).g(x)$ for all $x \in D$.

(v)Reciprocal : $\left(\frac{1}{f}\right): C \rightarrow R$ be defined by $\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)}$

for all $x \in C = A - \{x: f(x) = 0\}$

(vi) Quotient: $\left(\frac{f}{g}\right): D_1 \rightarrow R$ be defined by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

for all $x \in D_1 = D - \{x: g(x) = 0\}$

Example

Let $f, g: R \to R$ be defined by $f(x) = 2x + 5, g(x) = x + 3$, find $f \pm g,\ 2f,\ \frac{f}{g}, \frac{1}{f}, \frac{1}{g}$.

Sol: $(f + g)(x) = f(x) + g(x) = 2x + 5 + x + 3 = 3x + 8$

$(f - g)(x) = f(x) - g(x) = 2x + 5 - x - 3 = x + 2$

$(2f)(x) = 2\, f(x) = 2(2x + 5) = 4x + 10$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x + 5}{x + 3}, x \neq -3\ i.e., x \in R - \{-3\}$$

$$\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)} = \frac{1}{2x + 5}\ \forall\ x \in R - \left\{-\frac{5}{2}\right\}$$

$$\left(\frac{1}{g}\right)(x) = \frac{1}{g(x)} = \frac{1}{x + 3}\ \forall x \in R - \{-3\}$$

Question

1. Let f and g be real functions defined by $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{4-x^2}$. Then, find each of the following functions:

$(i) f+g \quad (ii)\ fg \quad (iii) \frac{f}{g} \quad (iv)\ ff$

2. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined over the set of non – negative real numbers.

Find $(f+g)(x), (f-g)(x), (fg)(x)$ and $\left(\frac{f}{g}\right)(x)$.

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FUNCTIONS

Concept of Exponential and Logarithmic Functions

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 02
CHAPTER NAME : RELATIONS AND FUNCTIONS

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Learning Objectives

- Exponential functions.
- Properties of exponentials.
- Logarithmic functions.
- Properties of logarithms.
- Exponential and logarithmic equations.
- Applications.

Exponential Function

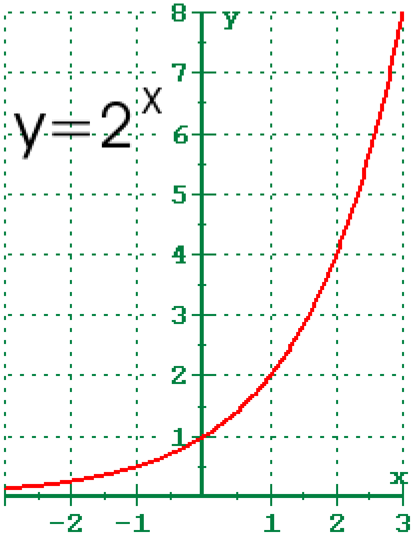
- An exponential function is of the form $f(x) = \ a^x \ (a > 0, a \neq 1)$, and x is any real number.
- Domain = R.
- Range (0, ∞).
- Euler number e = 2.71828 is the base of the natural exponential function e^x.
- Growth function = a^x if a> 1.
- Decay function = a^x if 0 <a < 1.

Graphs of Exponential Functions

Case – I: When $a > 1$.

We get the values of $y = f(x) = a^x$ increase as the values of x increase.

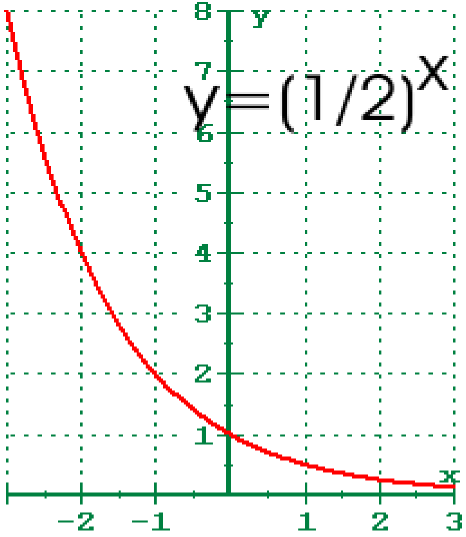
- The graph passes through the point (0,1)
- The domain is all real numbers.
- The range is y>0.
- The graph is increasing.
- The graph is asymptotic to the x-axis as x approaches negative infinity.
- The graph increases without bound as x approaches positive infinity.
- The graph is continuous.



Case II: When $0 < a < 1$

In this case, the values of $y = f(x) = a^x$, 0<a<1 and $y > 0$ for all $x \in R$.

- The graph passes through the point (0,1)
- The domain is all real numbers
- The range is y>0.
- The graph is decreasing
- The graph is asymptotic to the x-axis as x approaches positive infinity
- The graph increases without bound as x approaches negative infinity
- The graph is continuous
- The graph is smooth



Properties:

Let $f(x) = a^x, (a > 0, a \neq 1)$

$(i)\ f(x+y) = a^{x+y} = a^x . a^y = f(x). f(y)$

$$(ii) f(x-y) = a^{x-y} = \frac{a^x}{a^y} = \frac{f(x)}{f(y)}$$

$(iii)\{f(x)\}^y = (a^x)^y = a^{xy} = f(xy)$

$(iv)\ f(x) = 1 \Leftrightarrow x = 0$

$(v) f(x)f(-x) = f(0) = 1$

Logarithmic function

A **logarithmic function** is a function of the form
which is read " *y* equals the log of *x*, base *b*" or " *y* equals the log, base *b*, of *x*."

In both forms, $x > 0$ and $b > 0$, $b \neq 1$. There are no restrictions on *y*.

$$y = \log_b x \quad x > 0, \text{ where } b > 0 \text{ and } b \neq 1,$$

$$y = \log_b x \quad \text{is equivalent to} \quad x = b^y$$

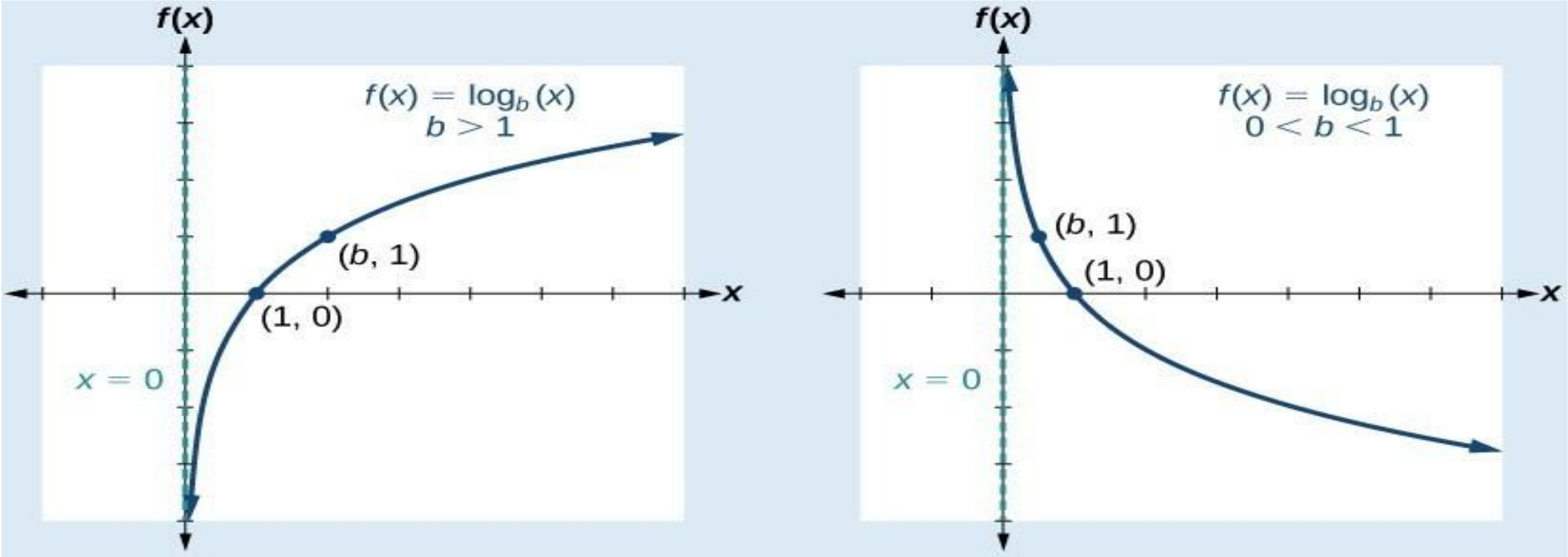
the base remains the base

Graph of Logarithm functions

A function $f:(0,\infty)\to R$ defined by $f(x)=\log_b x\ (b>0, b\neq 1)$ is called a logarithmic function.

Here $domf = R_+$ and $rngf = R$.

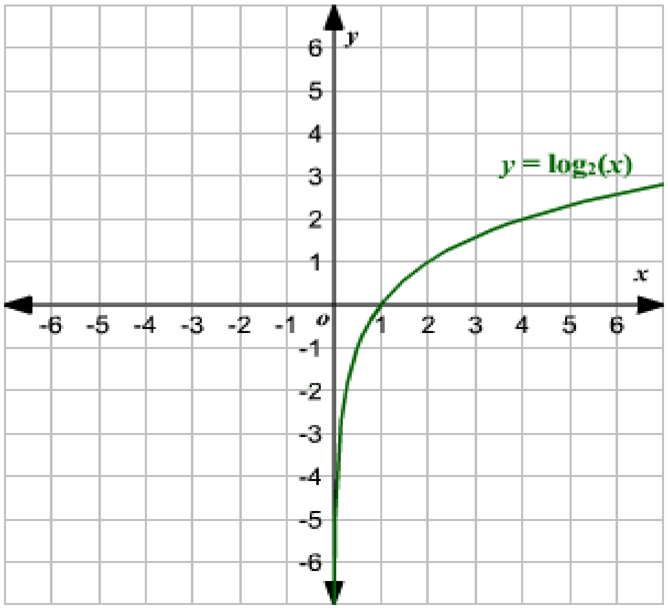
As b>0 and b$\neq 1$, so we have the following cases.

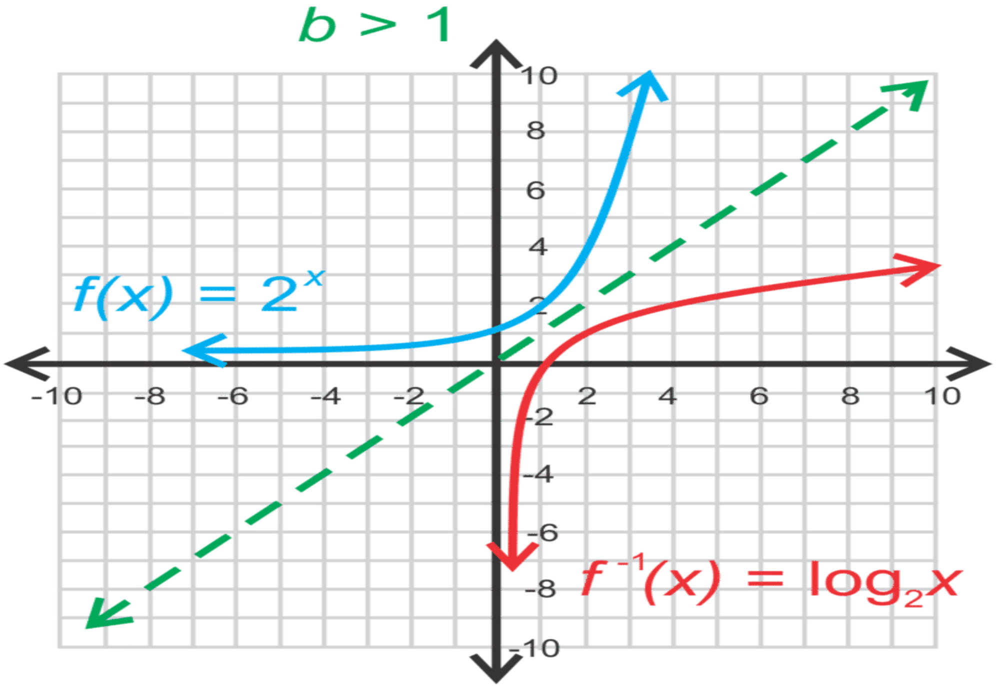


Graph of logarithm function

- The graph passes through the point (1,0)
- The domain is all positive real numbers.
- The range is set of real number.
- The graph is increasing if $b>1$ and decreasing if $0<b<1$.
- The graph is asymptotic to the y-axis as x approaches infinity.
- The graph is continuous.

Graph of Logarithm function and its comparison with the corresponding exponential function





Properties of logarithm function

(i) $\log_a 1 = 0$, where $a > 0, a \neq 1$

(ii) $\log_a a = 1$, where $a > 0, a \neq 1$

(iii) $\log_a(xy) = \log_a|x| + \log_a|y|$, where $a > 0, a \neq 1$ and $xy > 0$

(iv) $\log_a\left(\frac{x}{y}\right) = \log_a|x| - \log_a|y|$, where $a > 0, a \neq 1$ and $\frac{x}{y} > 0$

(v) $\log_a(x^n) = n \log_a|x|$,

Questions

1. If $f(x) = 2^x$, then what is the range of f?

2. Fill in the blank. If $f(x) = \log_2 x$, then $f(x) < 0$ for x lies in the interval ____

3. Find the domain of $\log_e(x+1)$.

4. Find the domain of $e^x + 2^x$.

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