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Changing your Tomorrow

Introduction to Sequences and Series

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 09
CHAPTER NAME : SEQUENCES AND SERIES

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Learning Objectives:

- Students will be able to learn about sequences and series.
- Students will be able to learn about arithmetic progression and arithmetic mean.
- Students will be able to learn about geometric progression and geometric mean.
- Students will be able to learn the relation between AM and GM.
- Students will be able to learn the infinite GP and how to find the sum.
- Students will be able to learn the some special series.
- Students will be able to implement application oriented skills in their day to day life.

Sequences

The numbers are called terms of the sequence. We denote the terms of a sequence by $a_1, a_2, a_3, \ldots.., etc.$

a_1 is the term in the first position, a_2 is the term in the second position and so on.

In general, the term at nth position is denoted by a_n. It is called nth term of the sequence.

A sequence can be written as $\{a_n\}_{n=1}^{\infty}$ or $\{a_n\}$ or (a_n) .

Representation of a Sequence:

There are several ways of representing a real sequence.

One way to represent a real sequence is to list its first few terms till the rule for writing down other terms becomes clear. For example, $1, 3, 5, 7, \ldots.$ is a sequence whose nth term is $(2n-1)$.

Another way to represent a real sequence is to give a rule of writing the nth term of the sequence. For example, the sequence $1, 3, 5, 7, \ldots$ can be written as $a_n = 2n - 1$.

Sometimes we represent a real sequence by using a recursive relation. For example, the Fibonacci sequence is given by

$a_1 = 1, a_2 = 1$ and $a_{n+1} = a_n + a_{n-1}\ , n \geq 2.$

The terms of this sequence are $1, 1, 2, 3, 5, 8, \ldots$ .....

Finite and Infinite Sequences:

A sequence containing a finite number of terms is called a finite sequence. For example, $1, 3, 9, 27, 81$ is a finite sequence.

A sequence is called infinite, if it is not finite.

For example, $2, 5, 8, 11, \ldots.$ Is an infinite sequence.

Example: Write the first three terms of the sequence whose nth terms are:

$(i)\ a_n = n(n+1)$

$(ii)\ a_n = (-1)^{n-1} . 2^n$

Example: Find the 15th and 26th term of the sequence whose nth term is given by $a_n = \frac{n(n+2)}{n+4}$.

Example: Write the first five terms of the sequence (a_n) defined by $a_1 = 2, a_n = a_{n-1} + 4,$

for all $n > 1$.

Series

If $\{a_n\}_{n=1}^{\infty}$ be a sequence, then the series is the sum of the terms of the sequence.

Thus $a_1 + a_2 + a_3 + \cdots$ is the series corresponding to (a_n). Using the symbol $\sum(sigma)$, means summation, we can write $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$

A series is finite or infinite according to as the number of terms in the corresponding sequence is finite or infinite.

Progressions

The terms of a sequence don't need to always follow a certain pattern or they are described by some explicit formula for the nth term. Those sequences whose terms follow certain patterns are called progressions.

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Arithmetic Progression

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A sequence is called an arithmetic progression if the difference between a term and the previous term is always the same.

$i.e.\ a_{n+1} - a_n = constant\ (= d)$ for all $n \in N$.

The constant difference d is called the common difference.

Example: Show that the sequence defined by $a_n = 4n + 5$ is an A.P. Also, find its common difference.

Remember:

A sequence is an A.P. if its nth term is a linear expression in n and in such a case the common difference is equal to the coefficient of n.

General term of an A.P.

Let $'a'$ be the first term and $'d'$ be the common difference of an A.P.

Then first term $= \; a_1 = a = a + (1-1)d$

Second term $= \; a_2 = a + d = a + (2-1)d$

Third term $= \; a_3 = a + 2d = a + (3-1)d$

Hence general term $= nth$ term $= \; a_n = a + (n-1)d$

Example: Show that the sequence 9, 12, 15, 18, … is an A.P. Find its 16^{th} term and the general term.

Example: Which term of sequence 72, 70, 68, 66, …. Is 40?

Example: How many terms are there in the sequence 3, 6, 9, 12, …, 111?

Example: Is 184 a term of sequence 3, 7, 11, …?

Example: Which term of the sequence $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$, … is the first negative term?

Example: If the 5^{th} and 9^{th} term of an A.P. are respectively 12 and 16, then find the 10^{th} term.

Example: If m times the mth term of an A.P. is equal to n times its nth term, show that the $(m+n)th$ term of the A.P. is zero.

Example: If the pth term of an A.P. is q and the qth term is p, prove that its nth term is $(p+q-n)$.

Example: The sum of three numbers in A.P. is -3 and their product is 8. Find the numbers.

Properties of Arithmetic Progressions

- If a constant is added to or subtracted from each term of an A.P., then the resulting sequence is also an A.P. with the same common difference.
- If each term of a given A.P. is multiplied or divided by a non -zero constant k, then the resulting sequence is also an A.P. with common difference kd or $\frac{d}{k}$, where d is the common difference of the given A.P.
- In a finite A.P., the sum of the terms equidistant from the beginning and end is always the same and is equal to the sum of the first and last term.
- A sequence is an A.P. iff its nth term is a linear expression in n $i.e. a_n = An + B$, where A, B are constants. In such a case the coefficients of n in a_n is the common difference of the A.P.

Example: If a, b, c are in A.P., prove that the following are also in A.P.

$(i)\ \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$

$(ii)\ b + c, c + a, a + b$

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Arithmetic Mean and its Related Problems

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Sum to n terms of an A.P.

The sum of n terms of an A.P. with first term $'a'$ and common difference $'d'$ is

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Or, $S_n = \frac{n}{2}(a + l)$, where $l =$ the last term $= a + (n-1)d$

Remember:

If the sum S_n of n terms of a sequence is given, then nth term a_n of the sequence can be determined by using the formula: $a_n = S_n - S_{n-1}$.

Example: Find the sum of 20 terms of A.P. 1, 4, 7, 10, …..

Example: If the sum of n terms of an A.P. is $nP + \frac{1}{2}n\,(n-1)Q$, where P and Q are constants, find the common difference.

Example: The sum of n terms of two arithmetic progressions are in the ratio $(3n+8):(7n+15)$. Find the ratio of their $12th$ terms.

Arithmetic Mean

If between two given quantities a and b we have to insert n quantities $A_1, A_2, \ldots, A_n$ such that $a, A_1, A_2, \ldots, A_n, b$ form an A.P., then we say that $A_1, A_2, \ldots, A_n$ are arithmetic means between a and b.

If a, A, b are in A.P., we say that A is the arithmetic mean of a and b.

So, $A - a = b - A$

$\Rightarrow 2A = a + b \;\Rightarrow A = \frac{a+b}{2}$

Insertion of Arithmetic Means

Let $A_1, A_2, \dots, A_n$ be n arithmetic means between two quantities a and b . Let d be the common difference of this A.P. Clearly, it contains $(n + 2)$ terms.

$\therefore\ b = (n + 2)th$ term $\Rightarrow b = a + (n + 1)d \ \Rightarrow d = \frac{b-a}{n+1}$

So, $A_n = a + nd = a + \frac{n(b-a)}{n+1}$

Example: Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.

Example: If eleven A.M.'s is inserted between 10 and 28, then find the number of integral A.M.'s.

Applications of A.P.

Example: The income of a person is ₹ 3,00,000 in the first year and he receives an increase of ₹ 10,000 to his income per year for the next 19 years. Find the total amount, he receives in 20 years.

Example: The digits of a positive integer, having three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

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Geometric Progression

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A sequence (a_n) of non – zero numbers is said to be a geometric progression (G.P.) or geometric sequence, if there exists a constant r such that $\frac{a_{n+1}}{a_n} = r$, $n \in N$ and the series $\sum_{n=1}^{\infty} a_n$ is called a geometric series. The above ratio is called the common ratio.

A geometric series is finite or infinite according to as the corresponding G.P. consists of a finite or infinite number of terms.

For example, sequence 5, 25, 125, … is a G.P. with first term 5 and common ratio 5.

Example: Show that the sequence given by $a_n = \frac{2}{3^n}$, $n \in N$ is a G.P.

General Term of a G.P.

Let $'a'$ be the first term and $'r'$ be the common ratio of a G.P.

Then the nth term or general term of the G.P. is $a_n = a\, r^{n-1}$

Thus the G.P. can be written as $a, ar, ar^2, ar^3, \ldots$

Example: Find the $9th$ term and the general term of the progression: $\frac{1}{4}, -\frac{1}{2}, 1, -2, \ldots$

Example: Which term of the G.P., 2, 8, 32, ... up to n terms is 131072?

Example: In a G.P., the $3rd$ term is 24 and the $6\ th$ terms are 192. Find the $10th$ term.

Example: The sum of the first three terms of a G.P. is $\frac{13}{12}$ and their product is -1. Find the common ratio and the terms.

Example: The product of the first three numbers of a G.P. is 1000. If 6 is added to its second term and 7 added to its third term, the terms become in A.P. Find the G.P.

Properties of Geometric Progression

- If all the terms of a G.P. are multiplied or divided by the same non – zero constant, then it remains a G.P. with the same common ratio.
- The reciprocals of the terms of a given G.P. form a G.P.
- If each term of a G.P. is raised to the same power, the resulting sequence also forms a G.P.
- In a finite G.P., the product of the terms equidistant from the beginning and the end is always the same and is equal to the product of the first and the last term.
- Three non – zero numbers a, b, c are in G.P. iff $b^2 = ac$.

Example: If a, b, c, d are in G.P., prove that $a+b, b+c, c+d$ are in G.P.

Example: If a, b, c are in G.P. and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, prove that x, y, z are in A.P.

Sum of the terms of a G.P.

The sum of n terms of a G.P. with first term $'a'$ and common ratio $'r'$ is given by

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right), r \neq 1.$$

If l is the last term of the G.P., then $S_n = \dfrac{lr - a}{r - 1}, r \neq 1.$

Example: Find the sum of first n terms and the sum of first 5 terms of the geometric series $1+\frac{2}{3}+\frac{4}{9}+\cdots$

Example: How many terms of the G.P. $3, \frac{3}{2}, \frac{3}{4}, \ldots.$ are needed to give the sum $\frac{3069}{512}$?

Example: Find the sum of the following sequences:

(i) $7, 77, 777, 7777, \ldots$ to n terms.

(ii) 0.5 , $0.55, 0.555, 0.5555, \ldots$ to n terms.

Example: A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.

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Geometric Mean
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Geometric Mean

Let a and b be two given numbers. If n numbers $G_1, G_2, \ldots, G_n$ are inserted between a and b such that the sequence $a, G_1, G_2, \ldots, G_n, b$ is a G.P. , then the numbers $G_1, G_2, \ldots, G_n$ are known as n geometric means between a and b.

The sequence $a, G_1, G_2, \ldots, G_n, b$ is a G.P. consisting of $(n+2)$ terms. Let r be the common ratio of this G.P.

Then, $b = (n+2)th$ term $= ar^{n+1} \Rightarrow r^{n+1} = \frac{b}{a} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$\therefore\ G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, \ldots, G_n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

If a, G, b are in G.P., then G is the G.M. of a and b.

Then $G^2 = ab \Leftrightarrow G = \sqrt{ab}$

If a and b are two numbers of opposite signs, then geometric mean between them does not exist.

Example: Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

Relationship Between A.M. and G.M.

- If A and G are respectively arithmetic and geometric means between two positive numbers a and b, then $A \geq G$ $i.e. \frac{a+b}{2} \geq \sqrt{ab}$.
- If A and G are respectively arithmetic and geometric means between two positive quantities a and b, then the quadratic equation having a, b as its roots is $x^2 - 2Ax + G^2 = 0$.
- If A and G be the A.M. and G.M. between two positive numbers, then the numbers are $A \pm \sqrt{A^2 - G^2}$.

Example: If $A.M.$ and $G.M.$ of two positive numbers a and b are 10 and 8 respectively, find the numbers.

Example: If $x \in R$, find the minimum value of the expression $3^x + 3^{1-x}$.

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Infinite G.P. and its sum

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The sum of an infinite G.P. with first term a and common ratio r $(-1 < r < 1\ i.e., |r| < 1)$ is

$$S = \frac{a}{1-r}$$

Note: If $r \geq 1$, then the sum of an infinite G.P. tends to infinity.

Example: Find the sum to infinity of the G.P. $-\frac{5}{4}, \frac{5}{16}, -\frac{5}{64}, \ldots$

Example: The sum of an infinite G.P. is 8, its second term is 2, find the first term.

Example: Prove that: $6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times \cdots = 6$.

Example: If $x = 1 + a + a^2 + \cdots$ and $y = 1 + b + b^2 + \cdots$ where $|a| < 1$ and $|b| < 1$, prove that $1 + ab + a^2b^2 + \cdots = \frac{xy}{x+y-1}$

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