Introduction to Sets and their Representation

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 01
CHAPTER NAME : SETS

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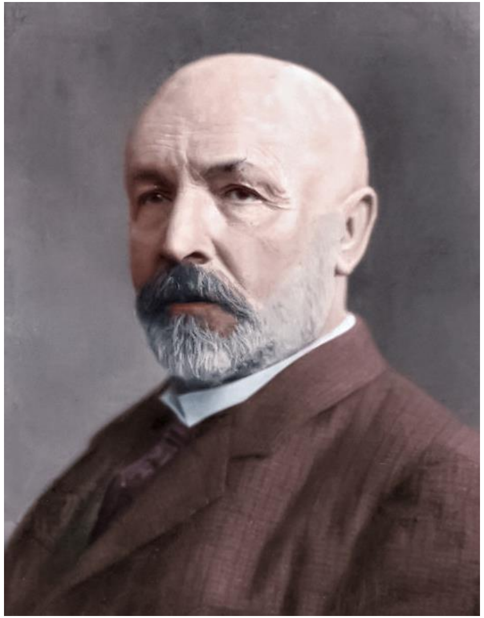
Learning Objectives:

- Students will be able to learn about sets and their representations .
- Students will be able to learn about different number sets.
- Students will be able to learn different types of sets.
- Students will be able to learn about subsets, power sets.
- Students will be able to learn the geometrical representations of sets.
- Students will be able to learn about different operations on sets .
- Students will be able to implement application oriented skills in their day to day life.

Introduction

The modern theory of sets was developed by German Mathematician ***George Cantor*** (1845 – 1918).



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Sets and their representations:

By **well – defined collections,** we mean that there is a rule, with the help of which it is possible to say, whether an object belongs to the given collection or not.

Example:

a) Even natural numbers less than 9, $i.e.$ 2, 4, 6, 8

b) The vowels in English alphabets, $i.e.\ a, e, i, o, u$

Further, examine the following collections:

a) The collection of five good doctors in our city.

b) The collection of five good novels of Premchand.

A doctor, which is good for one patient may not be the same as another patient. This collection is **not well - defined.**

Definition:

A **well – defined** collection of objects, is called a **set**.

Sets are usually denoted by the capital letters $A, B, C, \ldots$ etc. and all its members are represented by small letters $a, b, x, y, \ldots.$ etc.

The statement “x is an element of a set S” is written as $x \in S$ (read as x in S or x belongs to S).

Example: Which of the following are sets? Justify your answer.

- The collection of all the days in a week beginning with the letter S.
- The collection of famous dancers of India.

Representation of Sets:

a) Roster Form or Tabular Form :

- ***Example:*** The set of natural numbers less than 10 is represented in roster form as $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

b) Set – builder Form or Rule Method:

- ***Example:*** The collection of all peoples of India can be written as $S = \{x : x \text{ is an Indian}\}$.

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Example: Describe each of the following sets in roster form:

(i) $S = \{x : x$ is a positive integer and divisor of 21$\}$.

(ii) $S = \{x : x = n^2, 1 < n \leq 5, n \in N\}$.

(iii) $S = \{x : x$ is a positive integer and $x^2 < 40\}$.

Example: Describe the following sets in set-builder form:

(i) The set of reciprocals of all natural numbers.

$(ii)\ A = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$

(iii) B $= \{1, 2, 3, 4, 5, 6{,}7\}$

(iv) C $= \{ 1, 2\}$

Points to Remember:

- The repetition of any element in a set does not occur in any other element.
- The order in which the elements of a set are written is immaterial.

Example:

Describe the set of all letters of the word "COMMITTEE" in the roster form.

Assignments:

1. Describe each of the following sets in roster form:

(i) $S = \{x : x \in Z$ and $|x| \leq 2\}$

(ii) $S = \{x : x$ is a two-digit number such that the sum of its digits is 9$\}$

2. Describe the following sets in set-builder form:

(i) $A = \{1, 5, 25, 125, 625\}$

(ii) $\mathrm{B} = \{0\}$

3. List all the elements of the set $S = \{x : x\ is\ a\ letter\ of\ the\ word\ MISSISSIPPI\}$.

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Types of Sets

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Types of Sets:

Singleton Set:

A set consisting of a single element is called a singleton set.

Empty Set:

A set that does not contain any element, is called an empty set or null set or void set, denoted by $\emptyset$ (phi).

Example: The set $A = \{x : x^2 - 5 = 0$ and $x \in N\}$ is an empty set.

Finite Set: A set that is empty or consists of a definite number of elements, is called a finite set.

Cardinal Number: The number of distinct elements in a finite set A is called cardinal number or **cardinality** of A and it is denoted by $n(A)$ or $|A|$.

Points to Remember:

- The cardinal number of the empty set is 0.
- The cardinal number of a singleton set is 1.

Infinite Set: A set that consists of an infinite number of elements is called an infinite set.

Example: $S = \{1, 4, 9, 16, 25, \ldots\ldots\}$ is an infinite set.

Example: From the sets given below, select empty set, singleton set, finite set, an infinite set.

(i) $A = \{x: x < 1 \; and \; x > 3\}$

(ii) $B = \{x : x^3 - 1 = 0, x \in R\}$

(iii) $C = \{x : x \in N$ and x is a prime number$\}$

(iv) $D = \{2, 4, 6, 8, 10\}$

(v) $E =$ Set of odd natural numbers divisible by 2.

Example: Find the cardinal number of the following sets:

$(i)\ \{\{\emptyset\}, \{\{\emptyset\}\}\}$ $(ii)\ \{\{\emptyset\}\}$ $(iii)\ \{0, \{5\}\}$ $(iv)\{a, b, \{a, b\}\}$

Assignments:

1. Which of the following is a finite set?

i) The set of months of year.

ii) The set of positive integers greater than 100.

2. Is the following set $S = \{\, x : (2x-1)(2x-3)(2x-5) = 0 \; and \; x \in N\}$ an empty set?

3. Write down the cardinality of the set $\{a, \{a\}, \{\{a\}\}\}$

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Equal and Equivalent Sets

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Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Equivalent Sets: Two sets A and B are said to be equivalent or **similar** (denoted as $\boldsymbol{A \sim B}$) if they have the same number of elements.

$i.e. n(A) = n(B)$

Example: Let $A = \{a, b, c, d\}$ and $= \{1, 29, 53, 107\}$.

Here $n(A) = 4$ and $n(B) = 4$. Therefore A and B are equivalent sets.

Equal Sets: Two sets A and B are said to be equal if they contain the same elements and we write it as $A = B$.

Example: Let $A = \{1, 2\}$ and $B = \{x : x^2 - 3x + 2 = 0\} = \{1, 2\}$.

So $A = B$.

Mathematically, $A = B$

if $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$

$i.e. x \in A \Leftrightarrow x \in B$

Points to Remember:

- Two equal sets are equivalent but not conversely.

Example: In the following, state whether $A = B$ or not:

(i) $A = \{x : x$ is a letter in the word 'area'$\}$

$B = \{y : y$ is a letter in the word 'ear'$\}$

(ii) $A = \{4^n - 3n - 1 : n \in N\}$

$B = \{9(n - 1) : n \in N\}$

Example: Which of the following sets are equal?

$A = \{1, 3, 5\}, B = \{x \in N : (x - 1)(x - 3)(x - 5) = 0\}$

$C = \{1, 3, 3, 3, 5, 5, 5, 5\}, D = \{x: x \text{ is an odd natural number less than } 6\}$

Example: From the sets given below, select equal sets and equivalent sets.

$A = \{0, a\}, \quad B = \{1, 2,3, 4\}, C = \{1, -1\}\ D = \{3, 1,2, 4\}$

Assignments:

1. State true or false: The sets $A = \{e\}$ and $B = \{\{e\}\}$ are equal.

2. State which two of the following sets are similar:

$$A = \{1, 2, 3, 4, \dots\}$$

$$B = \{2, 4, 6, 8, \dots\}$$

$$C = \{x : (x - 1)(x - 2)(x - 3)(x - 4) = 0\}$$

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Subsets and Super Sets

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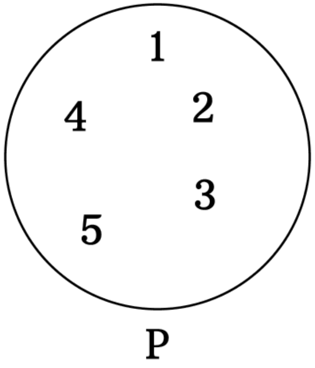
Subset: If every element of a set A is an element of a set B, then A is called a subset of B.

We write $A \subseteq B$. Here B is called a superset of A.

Mathematically, $A \subseteq B$ if $x \in A \Rightarrow x \in B$.

Example: Let $Q = \{2, 3\}$ and $P = \{1, 2, 3, 4, 5\}$. So $Q \subseteq P$.



Proper Subset: If $A \subseteq B$ and $A \neq B$ then A is called a proper subset of B and written as $A \subset$ B.

Example: Let $A = \{2, 5,7\}$ and $B = \{2, 3, 5, 7\}$.

So, A is a proper subset of B.

Points to Remember:

- If $A \subseteq B$, then $n(A) \leq n(B)$
- If $A \subset B$, then $n(A) < n(B)$

Some Important Results:

- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
- Every set is a subset of itself, $i.e. A \subseteq A$.
- Every set is not a proper subset of itself.
- The set A is called a proper subset of B, if there must be an element $x \in B$ such that $x \notin A$.
- The empty set $\emptyset$ is a subset of every set, $i.e. \emptyset \subseteq A$.
- $N \subset Z \subset Q \subset R \subset C$.

Points to Remember:

- The total number of subsets of a finite set A containing n elements is 2^n.
- The total number of proper subsets of a finite set A containing n elements is $2^n - 1$.
- The total number of non – empty proper subsets of a finite set A containing n elements is $2^n - 2$.

Example: Consider the following sets:

$\emptyset, A = \{a, e\}, B = \{e, i\},\ C = \{a, e, i, o, u\}$. Insert the correct symbol $\subset$ or $\not\subset$ between each of the following pairs of sets.

$(i)\ \emptyset\ \ldots.\ B$ $\quad (ii)\ A\ \ldots.B$ $\quad (iii)\ A \ldots C$ $\quad (iv)\ B \ldots C$

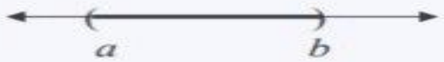
Example: Write down all the non – empty proper subsets of $\{3, 4, 5\}$.

Example: Prove that $A \subseteq \emptyset$ implies $A = \emptyset$.

Example: Every set is a subset of itself.

Example: Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. Find the values of m and n.

Intervals as Subsets of R

Inequality	Interval notation	Graph
$a < x < b$	(a, b)	
$a \leq x \leq b$	$[a, b]$	
$a \leq x < b$	$[a, b)$	
$a < x \leq b$	$(a, b]$	
$a < x$	(a, ∞)	
$a \leq x$	$[a, \infty)$	
$x < b$	$(-\infty, b)$	
$x \leq b$	$(-\infty, \infty)$	

The set of real numbers R is given in interval form as $(-\infty, \infty)$.

Example: Write the following intervals in set – builder form.

$(i)\ (-3, 0)$ $(ii)\ (6, 12]$ $(iii)[-23, 5)$ $(iv)\ \left[\frac{1}{3}, 7\right]$

Example: Write the following as intervals and also represent on the number line.

$(i)\{x : x \in R, -5 < x \leq 6\}$

$(ii)\{x : x \in R, -11 < x < -9\}$

$(iii)\{x : x \in R, 2 \leq x < 8\}$

$(iv)\{x : x \in R, 5 \leq x \leq 6\}$

Assignments:

1. Make correct statement by using these symbols $\subseteq$ and $\not\subset$ in blank spaces.

i) $\{2, 3, 4\}$ __$\{1, 2, 3, 4, 5\}$

ii) $\{a, b, c\}$____$\{b, c, d\}$

2. Is the given statement true? $\{a, b\} \not\subset \{b, c, a\}$

3. Is the given statement true? Statement: If $x \in A$ and $A \subseteq B$ then $x \in B$.

4. State true or false : $\emptyset \subseteq \{1, 2, \emptyset\}$.

5. Write the given set in the roaster form: $A = \{x : x \text{ is an integer and } -3 \leq x < 7\}$.

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Power Sets, Venn Diagrams

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Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Power Set:

The collection of all subsets of a given set S is called the power set and it is denoted by $P(S)$.

Points to Remember:

- The number of subsets $=$ Number of elements of power set.
- If a set S has n elements, then the total number of elements in its power set is 2^n.
- The power set of a given set is always non – void.
- If $a \in S$ then $\{ a \} \in P(S)$. Also $\{ a \} \subset$ S

$$\mathcal{P}(\{1,2,3\}) =$$

$$\left\{ \begin{array}{ccc} & \{\ \} & \\ \{1\} & \{2\} & \{3\} \\ \{1,2\} & \{1,3\} & \{2,3\} \\ & \{1,2,3\} & \end{array} \right\}$$

Example: If $S = \{x\}$, then find the power set of S.

Example: If $S = \{x, y\}$, then find the power set of S.

Example: If a set A has k elements, then how many elements has $P\big(P(A)\big)$?

Universal Set:

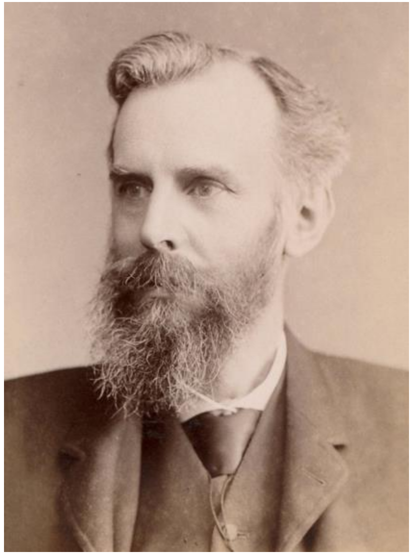
If there are some sets under consideration, then a set can be chosen arbitrarily which is a superset of each one of the given sets. Such a set is known as the universal set and it is denoted by U.

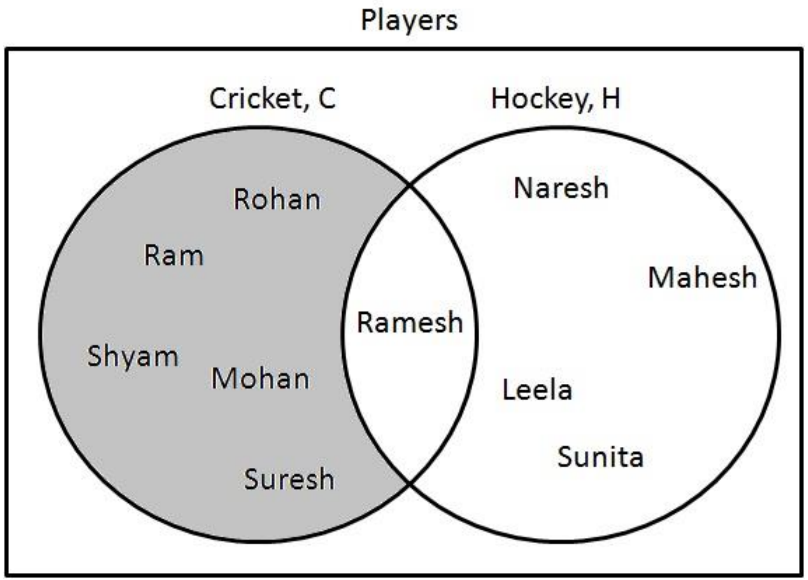
$e.g.$ Let $\mathrm{A} = \{2, 4, 6\}, B = \{1, 3, 5\}, C = \{0, 7\}$. Then $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$ is a universal set.

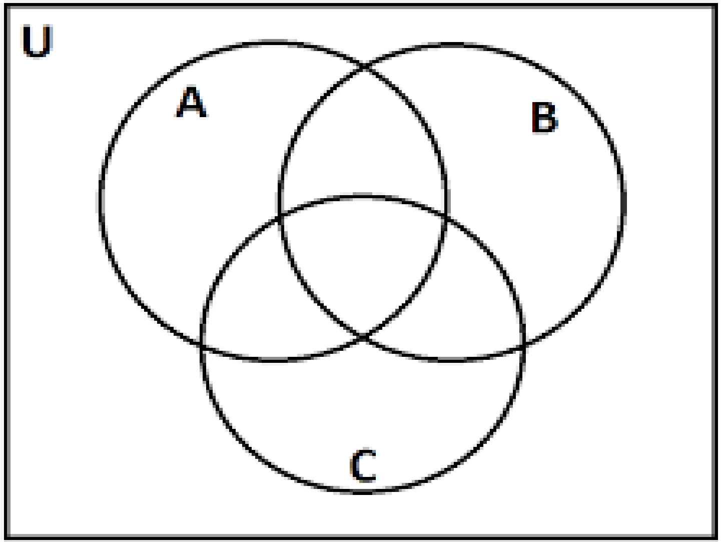
Venn Diagrams:

Venn diagrams are named after the English Mathematician John Venn (1834–1883).

These diagrams represent most of the relationships between sets.




Players
Cricket, C
Hockey, H
Rohan
Ram
Shyam
Mohan
Suresh
Ramesh
Naresh
Mahesh
Leela
Sunita


U
A
B
C

Example: Draw a Venn diagram to represents the sets,

$U = \{1, 2, 3, \dots., 10\}, A = \{2, 3, 4\}, B = \{1, 4, 5\}, C = \{6, 7, 8\}$

Assignments:

1. How many elements has $P(A), if\ A = \ \phi$.

2. Write down all possible subsets of the set $\{\ 1, \{1\}\}$.

3. Write the power set of $A = \{\ a, b, c\ \}$

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Operation on Sets

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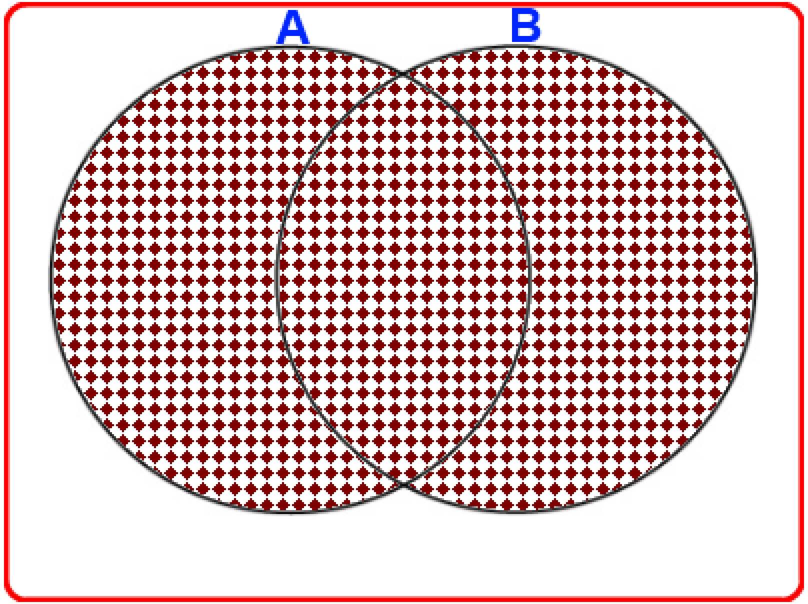
Union of Sets:

$A \cup B$, is the set all elements which belong either to A or to B or both A and B.

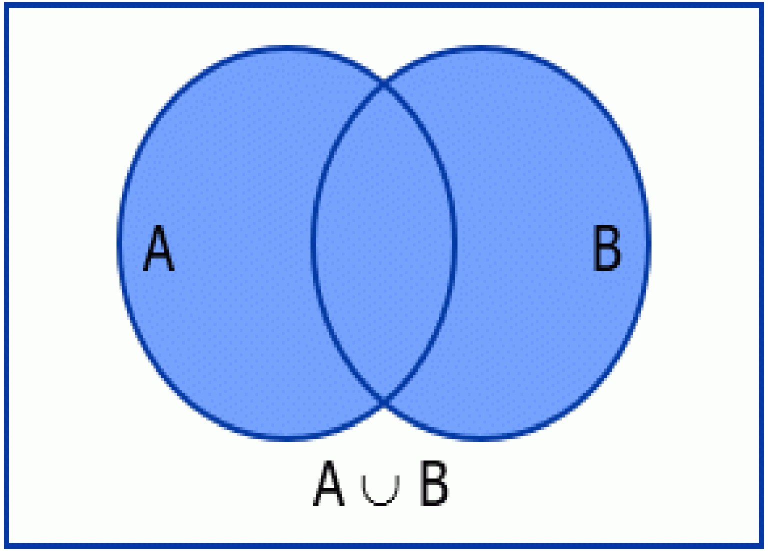
Mathematically, $A \cup B = \{x : x \in A \; or \; x \in B\}$

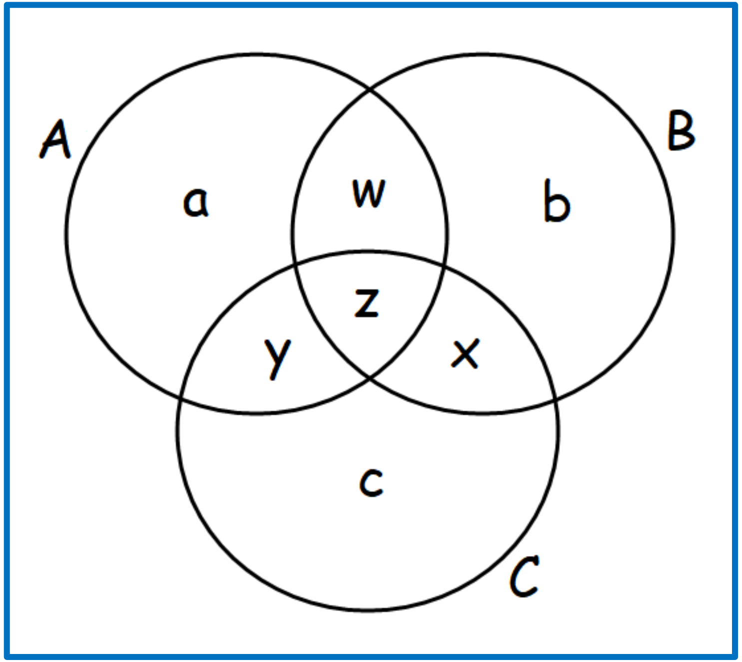
Example: Let $\mathrm{A} = \{a, e, i\}$, $B = \{i, o, u\}$. Then $A \cup B = \{a, e, i, o, u\}$

Venn Diagram:



Venn Diagrams:





Laws of Union of Sets:

- Commutative Law $i.e. A \cup B = B \cup A$
- Associative Law $i.e. A \cup (B \cup C) = (A \cup B) \cup C$
- Idempotent Law $i.e. A \cup A = A$
- Identity Law $i.e. A \cup \emptyset = A$
- Universal Law $i.e. A \cup U = U$
- $A \subseteq A \cup B$ and $B \subseteq A \cup B$
- If $A \subseteq B$, then $A \cup B = B$

Example: If $A = \{x : x \leq 10, x \in N\}$ and $B = \{y : 6 < y < 20, y \in N\}$, find $A \cup B$.

Example: If $A = \{4, 5, 7, 8, 10\}$, $B = \{4, 5, 9\}$, and $C = \{1, 4, 6, 9\}$, then verify that

$(A \cup B) \cup C = A \cup (B \cup C)$.

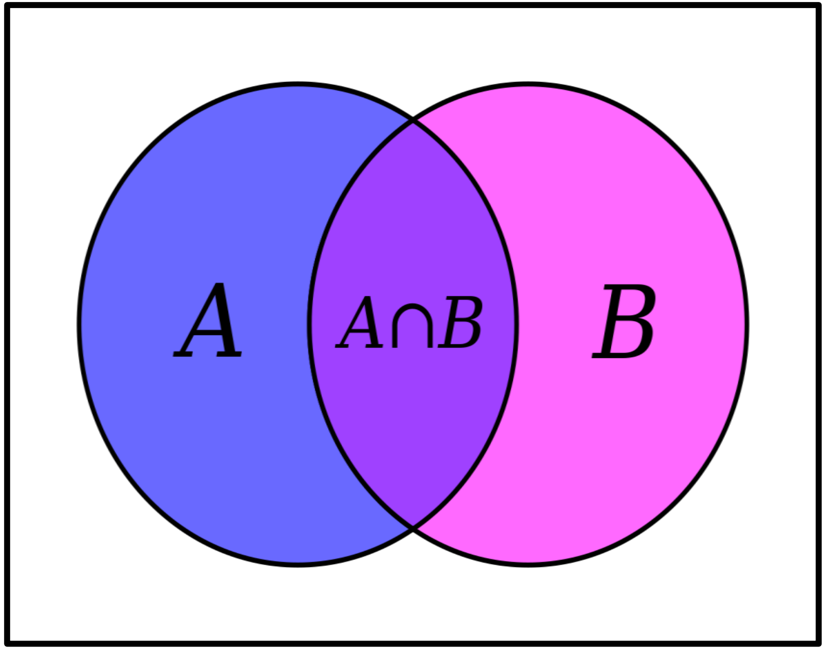
Intersection of two Sets:

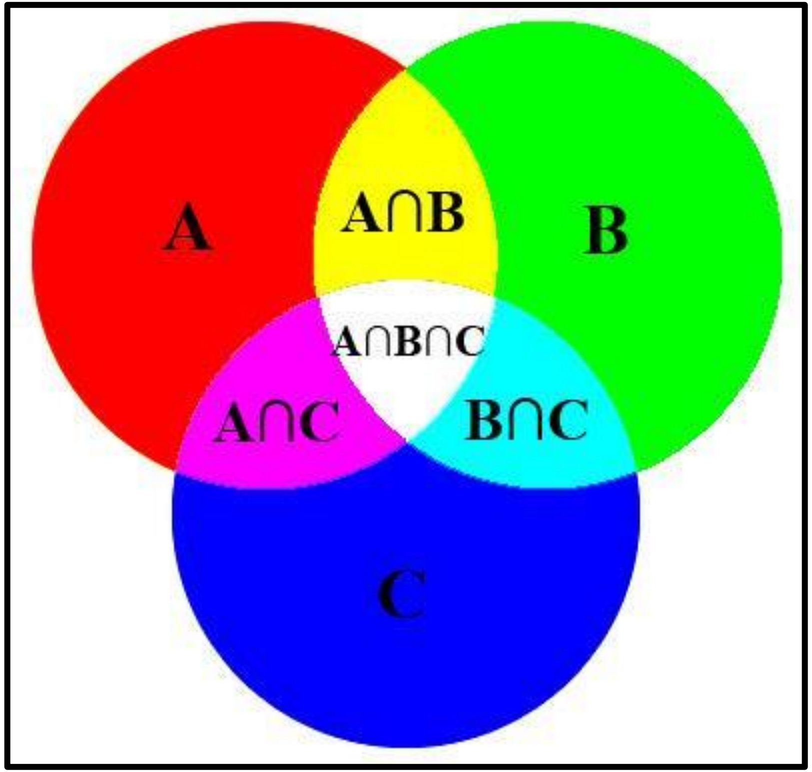
$A \cap B$ is the set of those elements which belong to both A and B.

Mathematically, $A \cap B = \{x : x \in A \; and \; x \in B\}$

Example: Let $A = \{2, 3, 4, 5\}$ and $B = \{1, 3, 5, 6\}$. Then, $A \cap B = \{3, 5\}$

Venn Diagrams:





Laws of Intersection of Sets:

- Commutative Law $i.e. A \cap B = B \cap A$
- Associative Law $i.e\ A \cap (B \cap C) = (A \cap B) \cap C$
- Idempotent Law $i.e.\ A \cap A = A$
- Identity Law $i.e. A \cap \emptyset = \emptyset$
- Universal Law $i.e. U \cap \emptyset = A$
- $A \cap B \subseteq A$ and $A \cap B \subseteq B$
- If $A \subseteq C$ and $B \subseteq C$ then $A \cap B \subseteq C$

Example: Let $P = \left\{\frac{1}{x} : x \in N , x < 7\right\}$ and Q $= \left\{\frac{1}{2x} : x \in N , x \leq 7\right\}. Find\ P \cap Q$

Example: If $A = \{3, 5, 7, 9, 11\}$, B= $\{7, 9, 11, 13\}$, $\mathrm{C} = \{11, 13, 15\}$,then find

$(i)\ A \cap B \cap C \qquad (ii)\ A \cap (B \cup C)$

Example: If $a \in N$ such that $aN = \{an : n \in N\}$. Describe the set $3N \cap 7N$.

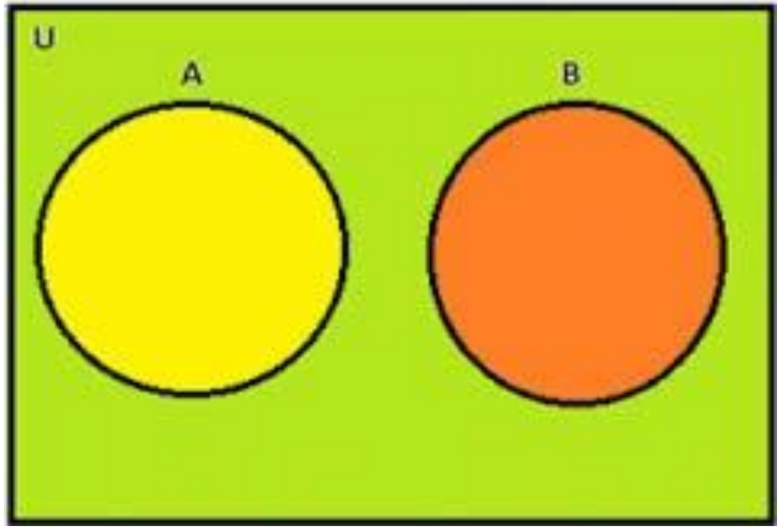
Disjoint Sets:

Two sets A and B are said to be disjoint or non-overlapping if they have no common elements.

$i.e. A \cap B = \emptyset$.

Let A $= \{1, 2, 3, 4, 5, 6\}$ and $B = \{7, 8, 9, 10\}$. Then $A \cap B = \emptyset$.

Venn Diagram:



Example: Which of the following pairs of sets are disjoint?

$(i)\ A = \{1, 2, 3, 4, 5, 6\}$

$B = \{x : x \text{ is a natural number and } 4 \leq x \leq 8\}$

$(ii)\ A = \{x : x \text{ is the boys in your school}\}$

$B = \{x : x \text{ is the girls in your school}\}$

Difference of Sets:

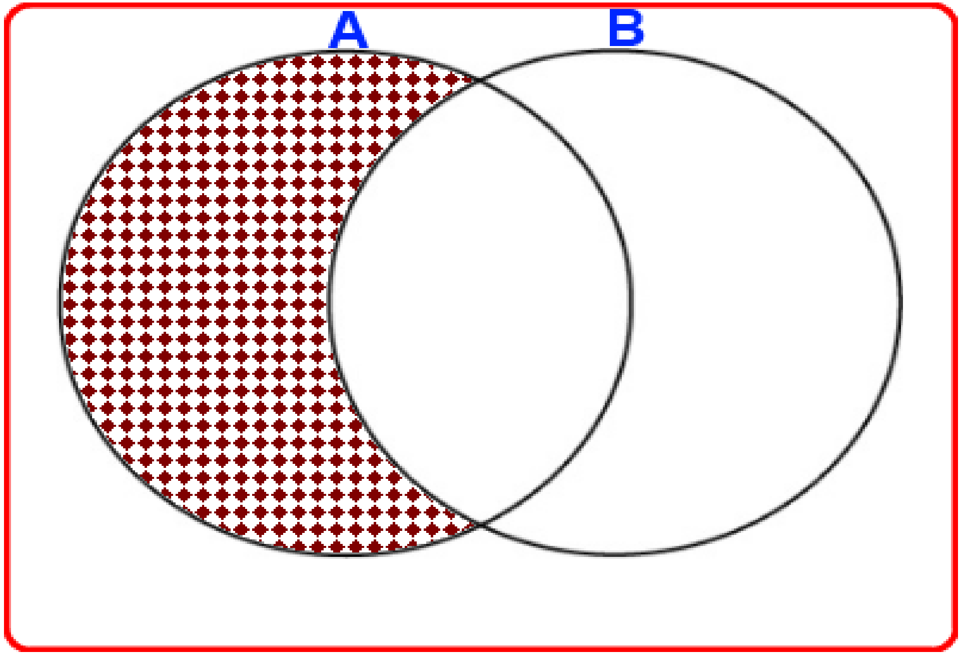
The difference of sets A and B in this order is the set of all those elements of A which do not belong to B.

It is denoted by $A - B$ or $A \setminus B$.

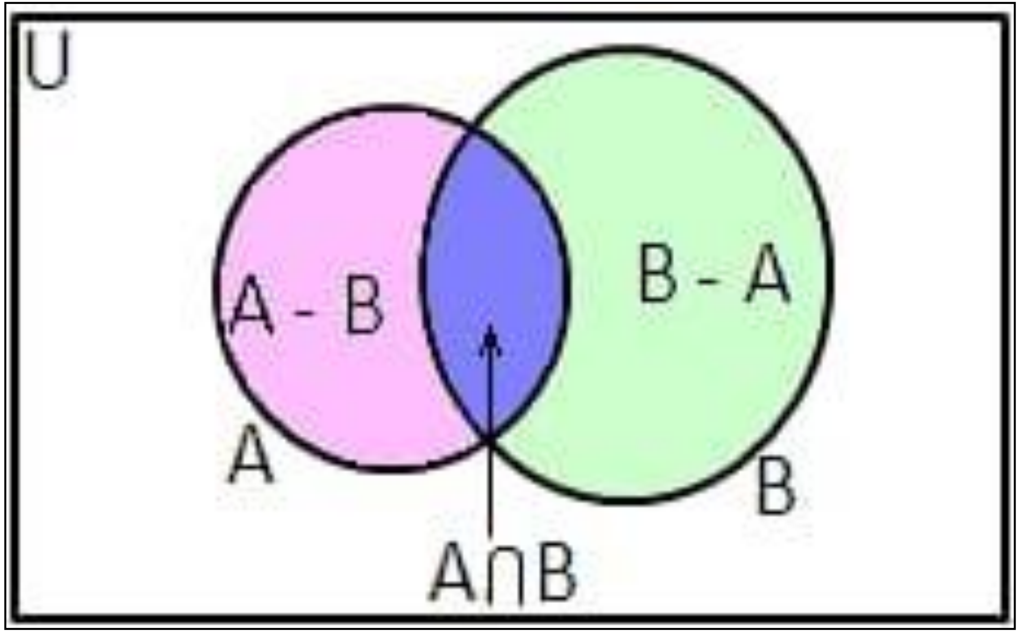
Mathematically, $A - B = \{x : x \in A \text{ and } x \notin B\}$

Example: Let $A = \{2, 3, 4, 5\}$ and $B = \{1, 3, 5, 6\}$. Then, $A - B = \{2, 4\}$.

Venn Diagram:



Venn Diagram:



Laws of Difference of Sets:

- The difference is not commutative $i.e. A - B \neq B - A$
- $A - A = \emptyset$
- $A - \emptyset = A$
- $A - B = A - (A \cap B)$
- $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
- $(A - B) \cup B = A \cup B \text{ and } (A - B) \cap B = \emptyset$

Example: If A = $\{1,2,3,4\}$, B = $\{3,4,5,6\}$, and C= $\{1,4,6,7,8\}$, then verify the following.

$(i)\ A \cap (B - C) = (A \cap B) - (A \cap C)$

$(ii)\ A - (B \cup C) = (A - B) \cap (A - C)$

$(iii)\ (A \cap B) - C = (A - C) \cap (B - C)$

Assignments:

1. Fill in the blank: If A and B are two sets, then $A \cap (A \cup B) =$ ___.

2. Let $A = \{a, e, i, o, u\}$ and $B = \{a, i, u\}$, find $A \cup B$.

3. Is the given pair of sets disjoint: $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

4. What can you say about A and B if $A \cup B = \emptyset$?

5. If $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$ find $A - B$.

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Complement of a Set and its Properties

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The Complement of a Set:

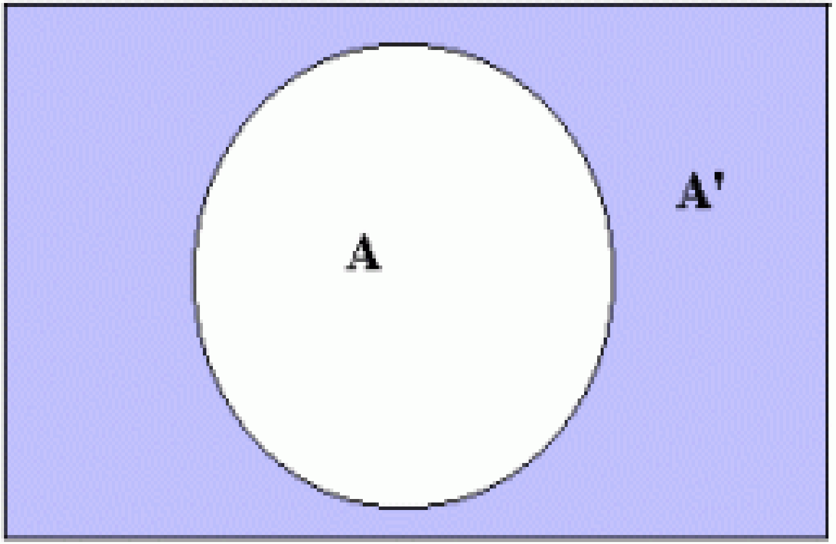
The complement of A concerning U is the set of all those elements of U which are not in A.

It denoted by A'.

Mathematically, $A' = \{x : x \in U \; and \; x \notin A\} = U - A$.

Example: Let $U = \{1, 2, 3, 4, 5\}$ and $= \{2, 4\}$, Then $A' = \{1, 3, 5\}$.

Venn Diagram:



Laws of Complement of a Set:

- **Involution Law :** $(A')' = A$
- **Complement Laws:**

  (i) $A \cup A' = U$ (ii) $A \cap A' = \emptyset$

- **Law of empty set and universal set:**

  (i) $\emptyset' = U$ (ii) $U' = \emptyset$

- **De – Morgan's Laws:**

  (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$.

Example: Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}, B = \{2, 4, 6, 8\}, C = \{3, 4, 5, 6\}$. Find

$(i)\ B'$ $(ii)\ (A \cup B)'$ $(iii)(A \cap C)'$ $(iv)(B - C)'$

Example: If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}, A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, then verify that

$(i)\ (A \cap B)' = A' \cup B'$ $(ii)(A \cup B)' = A' \cap B'$

Law of Algebra of Sets

1. **Idempotent Laws:** For any set A

$(i)\ A \cup A = A \qquad (ii)\ A \cap A = A$

2. **Identity Laws:** For any set A,

$(i)\ A \cup \emptyset = A \qquad (ii)\ A \cap U = A$

3. **Commutative Laws:**

$(i)\ A \cup B = B \cup A \qquad (ii)\ A \cap B = B \cap A$

Law of Algebra of Sets

4. Associative Laws:

$(i)\ (A \cup B) \cup C = A \cup (B \cup C)$ $(ii)\ (A \cap B) \cap C = A \cap (B \cap C)$

5. Distributive Laws:

$(i)\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $(ii)\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. De – Morgan's Laws:

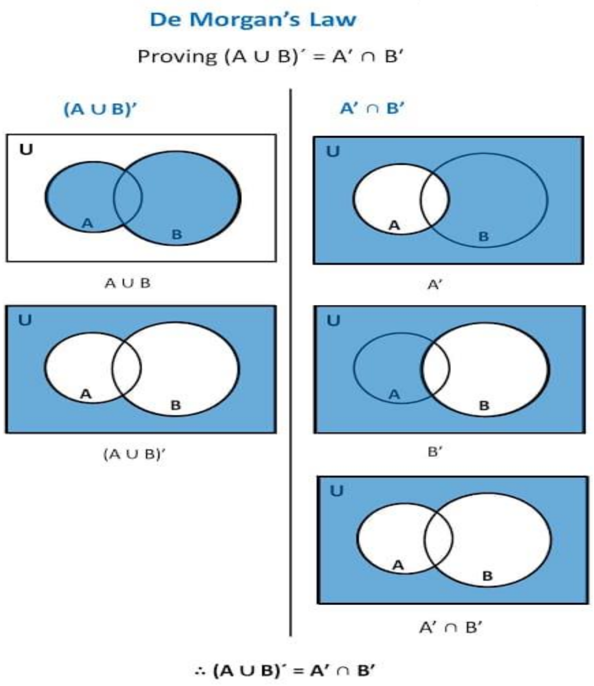
(i) The complement of the union is equal to the intersection of complements.

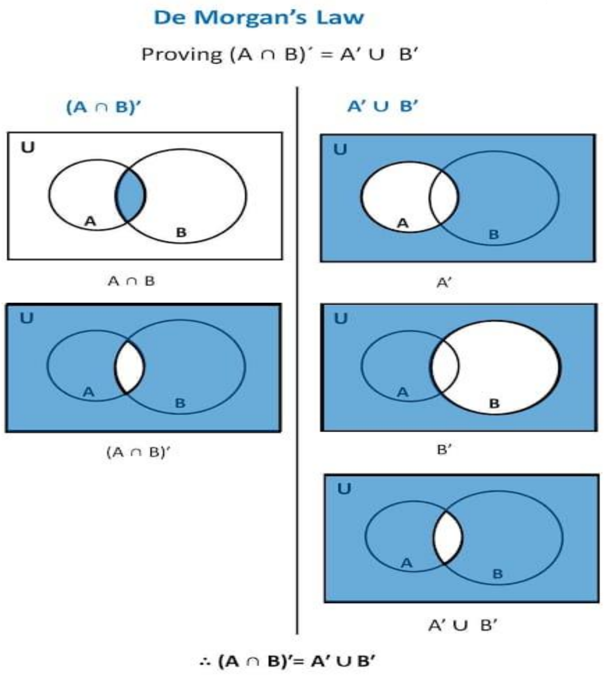
$i.e., (A \cup B)' = A' \cap B'$

(ii) The complement of the intersection is equal to the union of complements.

$i.e., (A \cap B)' = A' \cup B'$

Proof of De – Morgan’s Law by using Venn - Diagram





Example: If $n(A) = p, n(B) = q$ and $p < q$, then find the maximum and the minimum number of elements present in $A \cup B$ and $A \cap B$.

Example: For any two sets A and B, prove that $A \cup B = A \cap B \Leftrightarrow A = B$.

Example: For any two sets A and B, prove that $P(A \cap B) = P(A) \cap P(B)$.

Assignments:

1. State true or false: $A = A' \Rightarrow U \neq \emptyset$

2. If $U = \{ x : x \leq 10, x \in N\}$, $A = \{x : x \in N, x \text{ is prime}\}$, $B = \{x : x \in N, x \text{ is even}\}$, write $A \cap B'$ in roaster form.

3. Verify De-Morgan's laws for the following sets $A = \{ 1, 2, 3, 4, 5, 6\}$, $B = \{ 1, 2, 3\}$, $E = \{ 1, 2, 3, 4, 5, 6\}$, where E is the universal set.

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Practical Problems based on Operation on Sets

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 01
CHAPTER NAME : SETS

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: 1800 120 2316
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Some Important Results on Number of Elements in Sets

If A, B, and C are finite sets and U be the finite universal set. Then

$(i)\ n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint sets

$(ii)\ n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$(iii)\ n(A - B) = n(A) - n(A \cap B)$

$(iv)\ n(A' \cup B') = n\big((A \cap B)'\big) = n(U) - n(A \cap B)$

$(v)\ n(A' \cap B') = n\big((A \cup B)'\big) = n(U) - n(A \cup B)$

$(vi)\ n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(vii) Number of elements present exactly in A is

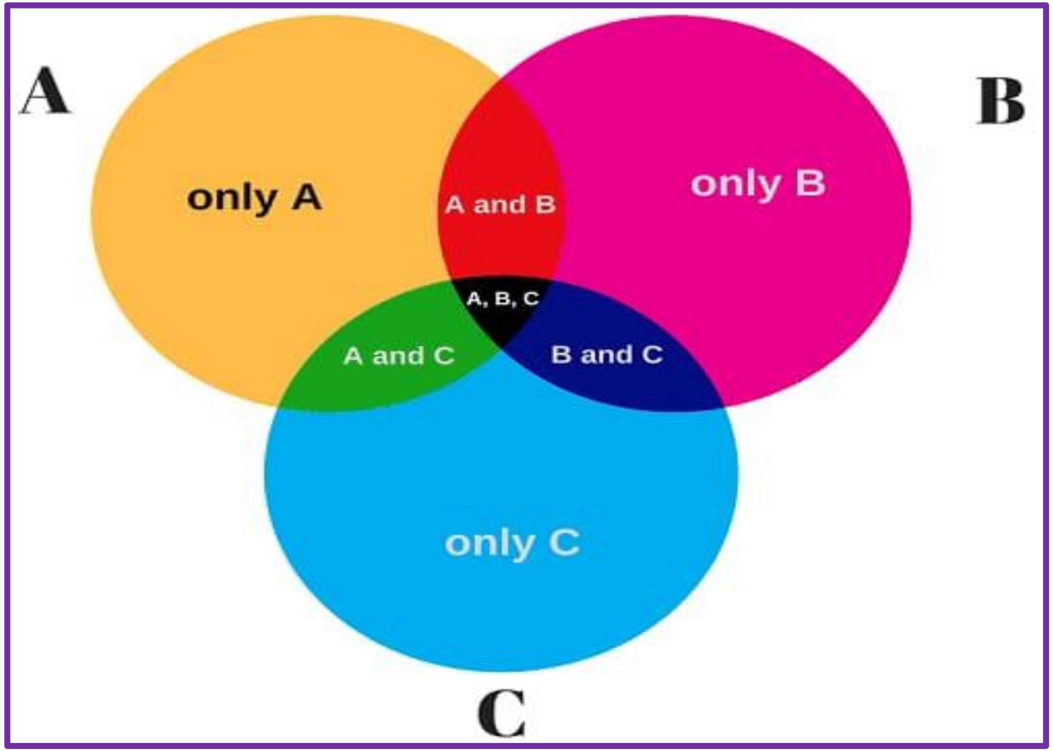
$$n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$$

$(viii)$ Number of elements in exactly one of the sets A, B, C is

$$n(A) + n(B) + n(C) - 2\,n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3\,n(A \cap B \cap C)$$

(ix) Number of elements in exactly two of the sets A, B, C is

$$n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$$


A
B
only A
A and B
only B
A, B, C
A and C
B and C
only C
C

Different Application Based Problems:

Example: If X and Y are any two sets such that $n(X) = 45, n(Y) = 43$ and $n(X \cup Y) = 76$, find $n(X \cap Y)$.

Example: If A and B are two sets such that $n(A) = 35, n(A \cap B) = 11$ and $n\big((A \cup B)'\big) = 17$. If $n(U) = 57$, find $n(B - A)$.

Example: In a school, 20 teachers teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics?

Example: In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?

Example: In a group of 50 people, 35 speak Hindi, 25 speak both English and Hindi and all people speak at least one of two languages. How many people speak only English and not Hindi? How many people speak English?

Example: In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking the orange juice, and 75 were listed as taking both apples as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

Example: In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had taken Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had taken

(i) only Mathematics.

(ii) Physics and Chemistry but not Mathematics.

(iii) only one of the subjects.

(iv) at least one of the three subjects.

(v) none of the subjects.

Assignments:

1. Find the value of $n(S) + n(T)$, where $S = \{x : x$ is a multiple of 3 less than 100$\}$ and $T = \{ x : x$ is a prime number less than 20$\}$

2. Consider three movies A, B and C released on the same date, 50% of the population like movie A, 20% like B and 20% C: 10% of the population like both A and B, 15% like both B and C, 10% like both C and A and 7% like all the three. Find the percentage of the population who likes

(i) at least two movies (ii) no movie at all.

3. Out of 100 students, 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many students passed in

(a) English and Mathematics but not in science? (b) Mathematics and Science but not in English?

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