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Introduction to Two Dimensional Geometry

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 10
CHAPTER NAME : STRAIGHT LINES

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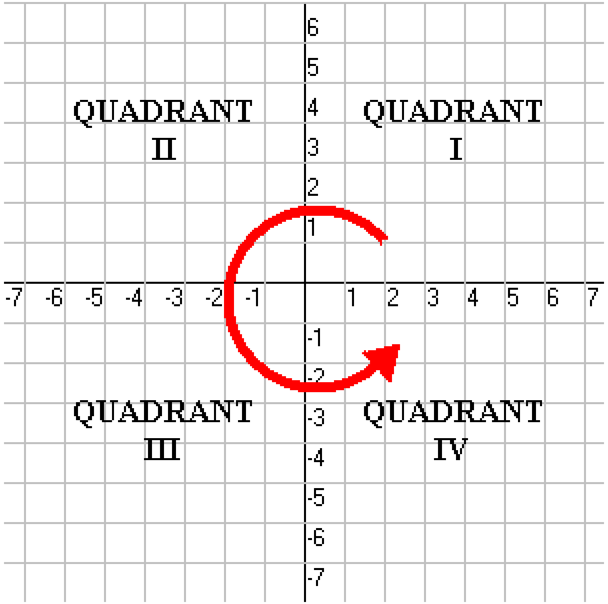
Learning Objectives:

- Students will be able to learn how to introduce two dimensional coordinate geometry on the plane.
- Students will be able to learn how to find the slope of a line segment .
- Students will be able to learn different forms equations of a line .
- Students will be able to learn about the parallelism and perpendicularity of two lines .
- Students will be able to learn about system of lines .
- Students will be able to learn about shifting of origin.
- Students will be able to implement application oriented skills in their day to day life.

Introduction:

The system which helps us to locate the position of an object is called the coordinate system.

The two-dimensional coordinate system is introduced on a plane with the help of two straight lines namely X' OX and Y'OY.



Coordinates of a Point in Cartesian Plane

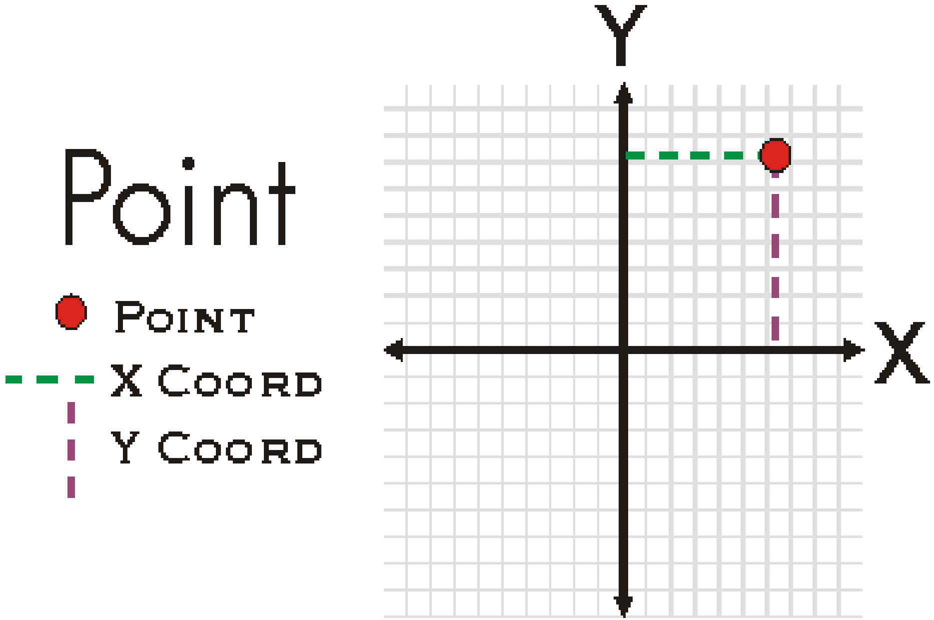
Let P be any point in a plane. If the distance of P from $the\ y - axis$ is x and the distance of P from the $x - axis$ is y, the coordinates of a point P are (x, y).

Here x is called the $x -$ coordinate or abscissa and y is called $y -$coordinate or ordinate.

The coordinates of a point on the $x - axis$ are of the form $(x, 0)$ and a point on the $y - axis$ are of the form $(0, y)$.

The coordinates of origin O are $(0, 0)$.

Point



Sign Convention of Coordinates:

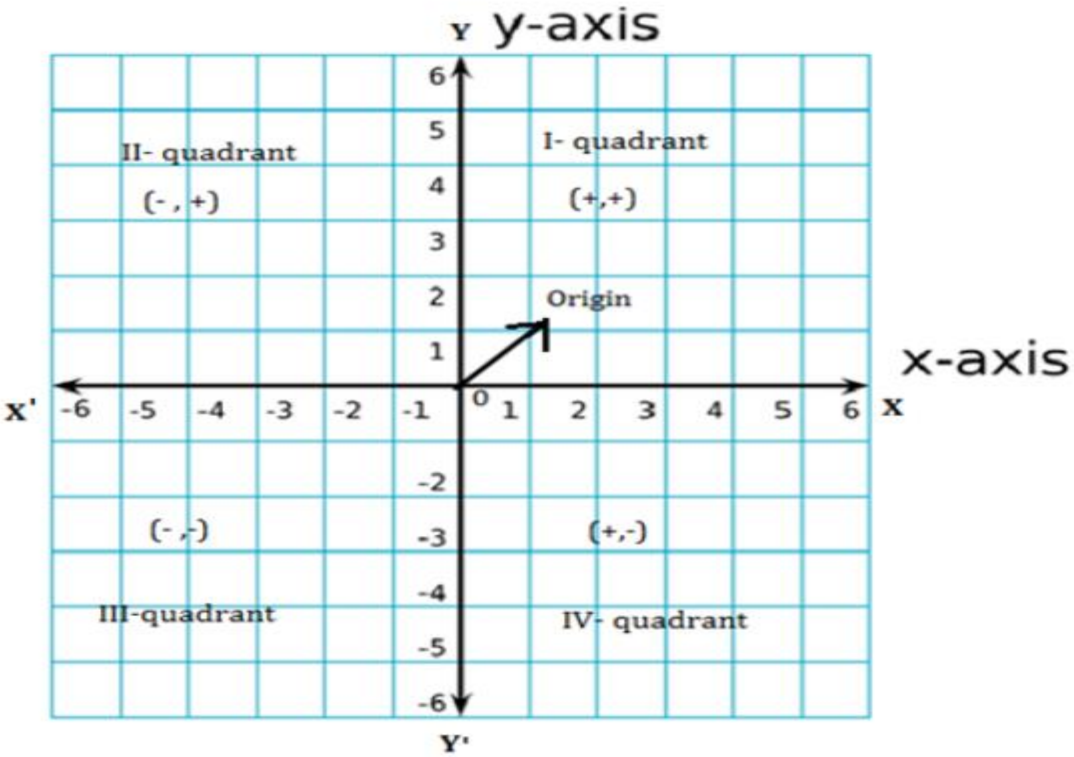


Figure 3. Coordinate system

Distance between two Points

The distance between any two points in the plane is the length of the line segment joining them.

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Find the distance between the points $(2, -3)$ and $(-7, 0)$.

Section or Division Formulae

Internal Division:

The coordinates of the point P which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

External Division:

The coordinates of the point P which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m : n$ are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Example: Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio $2 : 3$ (i) internally (ii) externally.

Mid –Point Formula

If P be the mid-point of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$, then the coordinates of P are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example: Without using distance formula, show that the points $(-2,-1),(4,0),(3,3)$ and $(-3,2)$ are the vertices of a parallelogram.

Centroid of a Triangle

If G is the centroid of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) , then coordinates of G are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Example: If the vertices of a triangle are $P(1,3), Q(2,5)$ and $R(3,-5)$, then find the centroid of a $\Delta\, PQR$.

Area of a Triangle

If $A(x_{1,}y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a $\Delta\, ABC$, then the area of the triangle is

$$\frac{1}{2}\,|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\,|$$

Points to Remember:

- The area of any triangle is always non –negative
- If the points A, B, C are collinear, then the area of the triangle is zero.

  $$i.e. x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$
- Using area formula, we can determine the area of any polygon of n sides.

Example: Find the area of a ΔABC, whose vertices are $A(6,3), B(-3,5)$, and $C(4,-2)$.

Example: For what value of k are points $(k, 2-2k), (-k+1, 2k), (-4-k, 6-2k)$ are collinear.

Slope of a Line and Angle between two Lines

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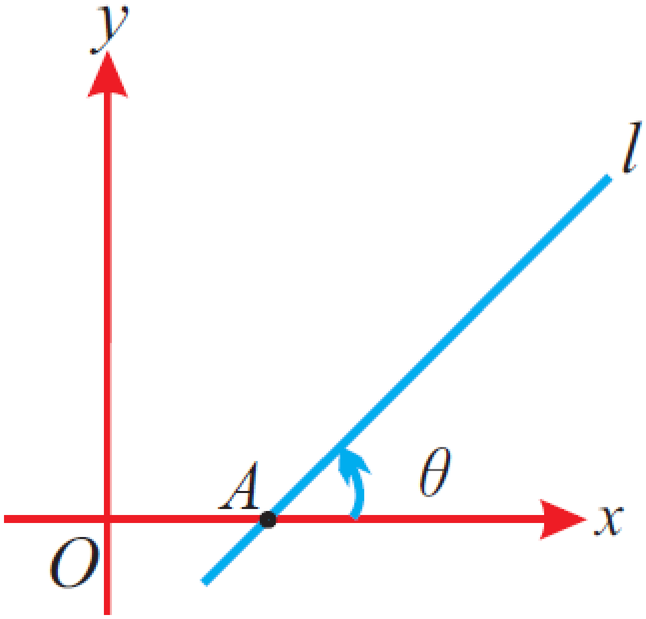
Inclination of a Line

An angle θ made by the line with positive $x - axis$ in an anti-clockwise direction is called the angle of inclination of a line.



Slope or Gradient of a Line

If θ is the angle of inclination of a line l, then $tan\theta$ is called the slope or gradient of the line l and it is denoted by m.

$i.e. m = \tan\theta$

Example: The slope of a line whose inclination is $150°$ is $tan150° = -\frac{1}{\sqrt{3}}$

Example: Find the slope of the line making inclination of $60°$ with the positive direction of $the\ x - axis.$

Slope of a Line Joining two Points

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points.

Then the slope of the line segment joining AB is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Example: Find the slope of a line joining the points $(2, 1)$ and $(-4, 3)$.

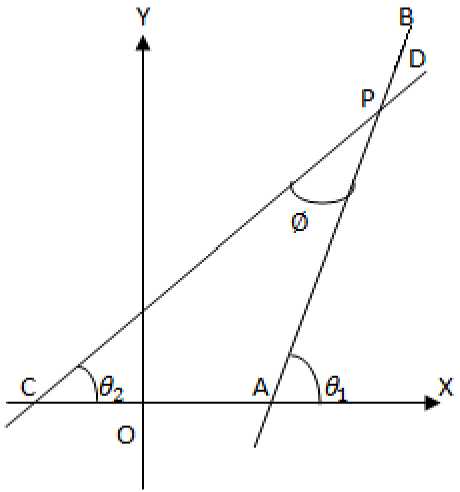
Example: A ray of light coming from the point $(1, 2)$ is reflected at a point A on the $x - axis$ and then passes through the point $(5, 3)$. Find the coordinates of point A.

Angle between Two Lines

Let l_1 and l_2 be two lines and their inclination are θ_1 and θ_2 respectively. Then, their slopes are $m_1 = tan\theta_1$ and $m_2 = \tan\theta_2$.

If θ is the angle between l_1 and l_2,

then $tan\theta = \pm \frac{m_2 - m_1}{1 + m_1 m_2}$



Example: Find the angle between the lines joining the points $(0, 0), (2, 3)$ and $(2, -2), (3, 5)$.

Example: If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Condition of Parallelism of Lines

If two lines of slopes m_1 and m_2 are parallel, then the angle θ between them is $0°$.

$\Rightarrow m_1 = m_2$

Example: What is the value of y so that the line through $(3, y)$ and $(2, 7)$ is parallel to the line through $(-1, 4)$ and $(0, 6)$?

Condition of Perpendicularity of Two Lines

If two lines of slopes m_1 and m_2 are perpendicular, then the angle θ between them is $90°$.

$\Rightarrow m_1 m_2 = -1$

Example: Line through the points $(-2, 6)$ and $(4, 8)$ is perpendicular to the line through $(8, 12)$ and $(x, 24)$. Find the value of x.

Collinearity of Three Points

If A, B and C are three points in XY $-$plane, then they will be collinear $i.e.$ will lie on the same line if and only if $slope\ of\ AB = slope\ of\ BC$.

Example: Prove that the points $A(1,4), B(3,-2)$ and $C(4,-5)$ are collinear.

Various Forms of Equation of a Line

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Locus and Equation to a Locus

Locus: The curve described by a moving point under given geometrical condition(s), is called locus of that point.

Equation of the Locus of a Point:

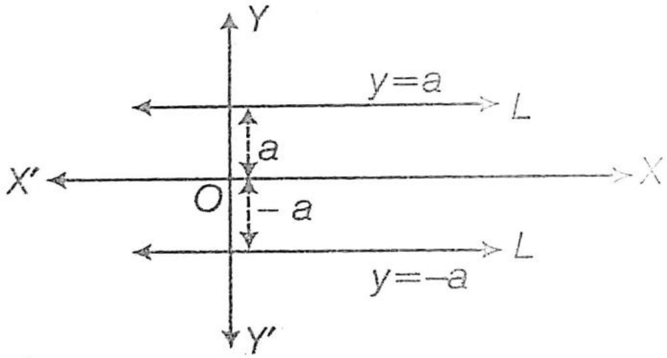
The equation of the locus of a point is the relationship that is satisfied by the coordinates of every point on the locus of the point.

The Straight Lines

Equation of Line Parallel to $x - axis$ or Equation of a horizontal line

Let L be a straight line parallel to the $x - axis$ at a distance a from it, then the equation of the line L is $y = a$ or $y = -a$.

The equation of the $x - axis$ is $y = 0$.

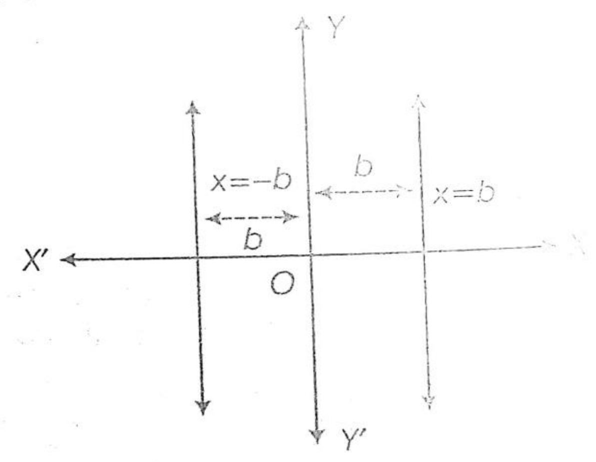


Example: Write down the equation of a line parallel to the $x - axis$ and at a distance of 6 units above the $x - axis$.

Equation of Line Parallel to $y - axis$ or Equation of vertical Line

Let L be a straight line parallel to $the\ y - axis$ at a distance b from it, then the equation of the line L is $x = b$ or $x = -b$.

The equation of the $y - axis$ is $x = 0$.



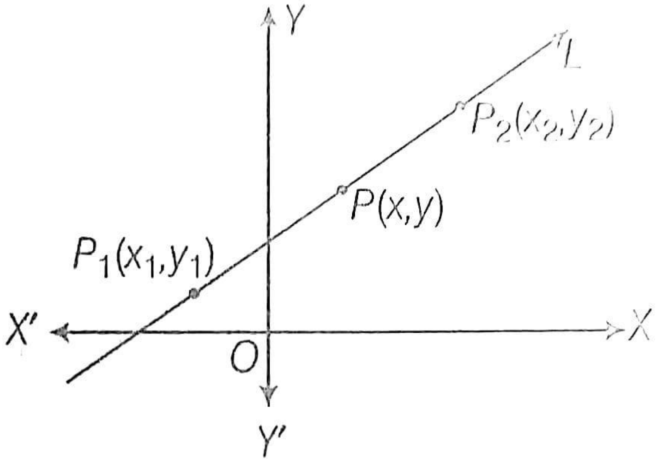
Example: Find the equation of the lines parallel to the axes and passing through the point $(-3, 5)$.

Example: Find the equation of a line that is equidistant from the lines $x = -4$ and $x = 8$.

Two – point Form of a Line

The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



Example: Find the equation of the line joining the points $(-1, 3)$ and $(4, -2)$.

Example: If $A(2, 1), B(-2, 3)$ and $C(4, 5)$ are the vertices of a ΔABC, then find the equation of the median through the vertex C.

Point Slope Form

The equation of the straight line having slope m and passes through the point (x_0, y_0) is

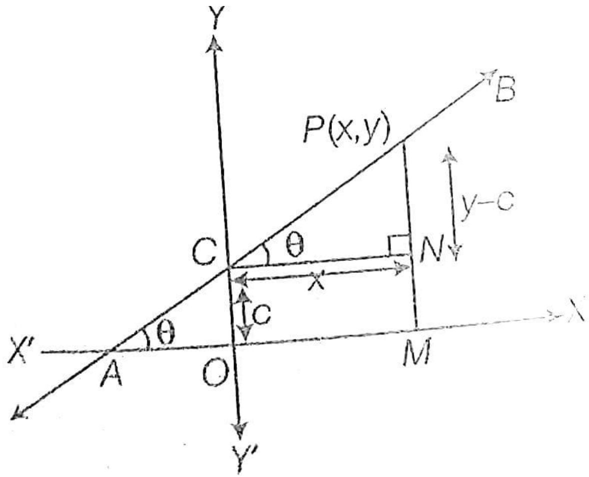
$$y - y_0 = m(x - x_0)$$

Example: Find the equation of the line passing through $(-4, 3)$ and having slope $\frac{1}{2}$.

Slope Intercept Form

Suppose a line L with slope m cuts the $y-axis$ at a distance c from the origin. (distance c is called the y –intercept of the line).

Then the equation of the line is $y = mx + c$.



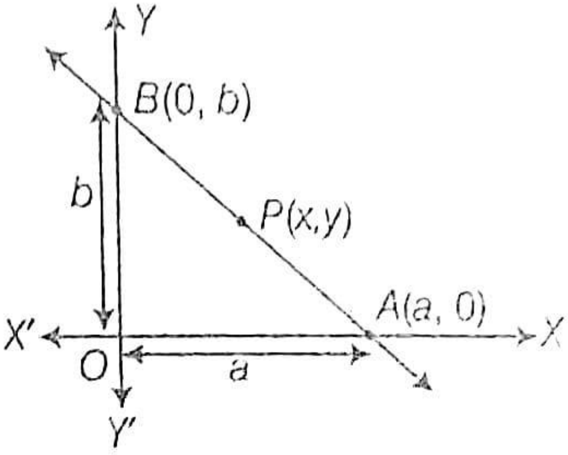
Remember:

The equation of the line passing through the origin is $y = mx$, where m is the slope of the line.

Example: Find the equation of the line which has slope $\frac{1}{2}$ and cuts – off an intercept -5 on the $y - axis$.

Intercept Form

The equation of a line which cuts – off intercepts a and b on the $x - axis$ and $y - axis$ respectively, is $\frac{x}{a} + \frac{y}{b} = 1.$



Example: Find the equation of the lines which cuts –off intercepts on the axes whose sum and product are 1 and -6 respectively.

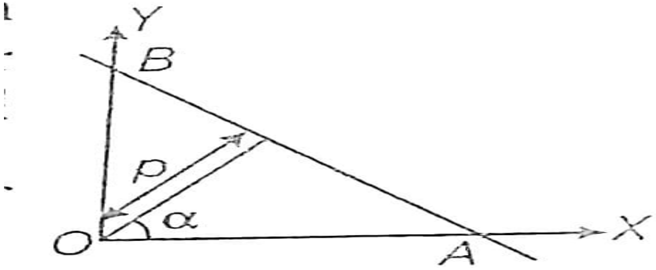
Example: Find the equation of the line which passes through the point $(-4, 3)$, the portion of the line intercepted between the axes is divided internally in the ratio $5 : 3$ by this point.

Normal Form or Perpendicular Form of a Line

The equation of a straight line upon which the length of the perpendicular $i.e.$ normal from the origin is p and this perpendicular makes an angle α with the positive direction of $the\ x - axis$ is

$$x\ cos\alpha + y\ sin\alpha = p$$

Here slope of line $= -\frac{1}{tan\alpha} = -\frac{cos\alpha}{sin\alpha}$



Example: The perpendicular distance of a line from the origin is 7 cm and its slope is -1. Find the equation of the line.

General Equation of a Line

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An equation of the form $Ax + By + C = 0$, where $A, B, C \in R$, $\sqrt{A^2 + B^2} \neq 0$ is called general linear equation or general equation of the line.

Different Forms of $\boldsymbol{Ax + By + C = 0}$

Slope Intercept Form

If $B \neq 0$, then $Ax + By + C = 0$ can be written as $y = -\frac{A}{B}x + \left(-\frac{C}{B}\right)$

where $m = -\frac{A}{B}$ and $c = -\frac{C}{B}$

Intercept Form

If $C \neq 0$, then $Ax + By + C = 0$ can be written as $\frac{x}{-C/A} + \frac{y}{-C/B} = 1$, where $a = -\frac{C}{A}$, $b = -\frac{C}{B}$

Normal Form

The normal form of the equation $Ax + By + C = 0$ is $x\, cos\omega + y \sin\omega = p$, where

$cos\omega = \pm \frac{A}{\sqrt{A^2+B^2}}$, $sin\omega = \pm \frac{B}{\sqrt{A^2+B^2}}$ and $p = \pm \frac{C}{\sqrt{A^2+B^2}}$ (p should be positive)

Example: Transform the equation of the line $3x + 2y - 7 = 0$ to

(i) slope intercept form and also find the slope and y –intercept.

(ii) intercept form and the intercepts on the coordinate axes.

(iii) normal form and also find the inclination of the perpendicular segment from the origin on the line with the axis and its length.

Example: Equation of a line is $3x - 4y + 10 = 0$. Find its

(i) slope $(ii) x -$ intercept and (iii) $y -$ intercept.

The Angle between Two Lines, having General Equations

Let, general equations of lines be $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$.

Then slope of given lines are $m_1 = -\frac{A_1}{B_1}$ and $m_2 = -\frac{A_2}{B_2}$

Let θ be the angle between two lines, then $tan\theta = \pm \frac{m_2 - m_1}{1 + m_1 m_2} = \pm \left(\frac{-\frac{A_1}{B_1} + \frac{A_2}{B_2}}{1 + \frac{A_1}{B_1}\frac{A_2}{B_2}} \right)$

Example: Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}\, y - x + 6 = 0$.

Condition for two lines to be Parallel

If the lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ are parallel, then their slopes are equal

$i.e. m_1 = m_2 \Rightarrow -\frac{A_1}{B_1} = -\frac{A_2}{B_2} \Rightarrow \frac{A_1}{B_1} = \frac{A_2}{B_2}$

Condition for two lines to be Perpendicular

If the lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ are perpendicular, then the product of their slopes is -1. $i.e. m_1 \times m_2 = -1 \Rightarrow \left(-\frac{A_1}{B_1}\right) \times \left(-\frac{A_2}{B_2}\right) - 1 \Rightarrow A_1A_2 + B_1B_2 = 0.$

Example: A line passing through the points $(a, 2a)$ and $(-2, 3)$ is perpendicular to the line $4x + 3y + 5 = 0$. Find the value of a.

Point of Intersection of Two Lines

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Lines Parallel and Perpendicular to a given Line

Line Parallel to a given Line

The equation of a line parallel to a given line A$x + By + C = 0$ is $Ax + By + k = 0$, where k is a constant.

Example: Find the equation of the line which is parallel to $3x - 2y + 5 = 0$ and passes through the point $(5, -6)$.

Line Perpendicular to a given Line

The equation of a line perpendicular to a given $Ax + By + C = 0$ is $Bx - Ay + k = 0$, where k is a constant.

Example: Find the equation of the straight line that passes through the point $(3, 4)$ and perpendicular to the line $3x + 2y + 5 = 0$.

Example: Find the equation of the line perpendicular to $x - 7y + 5 = 0$ and having x −intercept 3.

Point of Intersection of Two Lines

Let the equations of two lines be $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$.

The coordinates of the point of intersection of the given lines are

$$\left(\frac{B_1C_2 - B_2C_1}{A_1B_2 - A_2B_1}, \frac{C_1A_2 - C_2A_1}{A_1B_2 - A_2B_1}\right).$$

Concurrent Lines

Three lines are said to be concurrent if they pass through a common point $i.e.$ They meet at a point.

Thus, if three lines are concurrent, the point of intersection of two lines lies on the third line.

Let $A_1x + B_1y + C_1 = 0$, $A_2x + B_2y + C_2 = 0$ and $A_3x + B_3y + C_3 = 0$ be three lines.

Then the condition for which these three lines are concurrent is

$A_1(B_2C_3 - B_3C_2) + B_1(C_2A_3 - C_3A_2) + C_1(A_2B_3 - A_3B_2) = 0.$

Example: Show that the lines $x - y - 6 = 0$, $4x - 3y - 20 = 0$ and $6x + 5y + 8 = 0$ are concurrent. Also, find their common point of intersection.

Distance of a Point from a Line

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The distance of the point (x_1, y_1) from the line $Ax + By + C = 0$ is

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|.$$

The distance of origin from the line $Ax + By + C = 0$ is $d = \left|\frac{C}{\sqrt{A^2+B^2}}\right|$.

Example: Find the distance of the point $(2, -3)$ from line $2x - 3y + 6 = 0$.

Example: Find the points on $the\ x - axis$ whose perpendicular distance from the line $4x + 3y = 12$ is 4.

Distance between Two Parallel Lines

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The distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \left|\frac{C_1 - C_2}{\sqrt{A^2 + B^2}}\right|.$$

Example: Find the distance between the parallel lines $3x - 4y + 9 = 0$ and $6x - 8y - 15 = 0$.

Example: Find the equations of lines parallel to $3x - 4y - 5 = 0$ and at a unit distance from it.

Family of Lines passing through the intersection of Two Lines

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The equation of the family of lines passing through the intersection of the lines

$A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ is

$(A_1x + B_1y + C_1) + \lambda(A_2x + B_2y + C_2) = 0$, where λ is a parameter.

Example: Find the equation of the straight line which passes through the point $(2, -3)$ and the point of intersection of the lines $x + y + 4 = 0$ and $3x - y - 8 = 0$.

Example: Find the equation of the straight line which passes through the point of intersection of the straight lines $x + 2y = 5$ and $3x + 7y = 17$ and is perpendicular to the straight line $3x + 4y = 10$.

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