Introduction to Two Dimensional Geometry

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 10
CHAPTER NAME : STRAIGHT LINES

## Introduction:

The system which helps us to locate the position of an object is called the coordinate system.

The two-dimensional coordinate system is introduced on a plane with the help of two straight lines namely X^' OX and Y'OY. These two lines are known as x-axis and y-axis respectively and the two lines taken together are called the coordinate axes or the axes of coordinates.

The x-axis is horizontal and is directed from left to right and the y-axis is vertical, which is directed from bottom to top. These two axes are mutually perpendicular to each other at a point O called the origin.
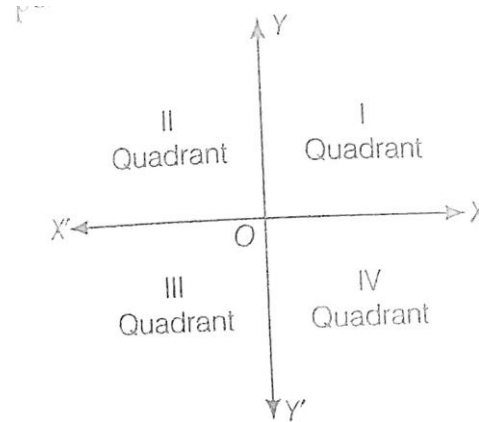
Coordinate Plane:

The coordinate axes divide the plane into four parts. These four parts are called quadrants, $\left(\frac{1}{4} \ th \ part\right)$ numbered I, II, III, IV anticlockwise from OX. Thus, the plane consists of the axes and four quadrants, is known as $XY - plane$ or Rectangular Cartesian coordinate plane or Euclidean plane and is denoted by $R \times R$ or R^2.

$$i.e. \ R \times R = \{(x, y) : x, y \in R \}$$

Coordinates of a Point in Cartesian Plane

Let P be any point in a plane. If the distance of P from $the\ y-axis$ is x and the distance of P from the $x-axis$ is y, the coordinates of a point P are (x, y). Here x is called the $x-$ coordinate or abscissa and y are called $y-$coordinate or ordinate.
Thus, for a given point, the abscissa and ordinate are the distances of the given point from $the\ y-axis$ and $x-axis$ respectively.
The coordinates of a point on the $x-axis$ are of the form $(x, 0)$ and a point on the $y-axis$ are of the form $(0, y)$. Thus, if the abscissa of a point is zero, it would lie somewhere on the $y-axis$ and if its ordinate is zero it would lie on $the\ x-axis$.
 The coordinates of origin O are $(0, 0)$.
There is one to one correspondence between the set of points and the set of ordered pairs of real numbers.
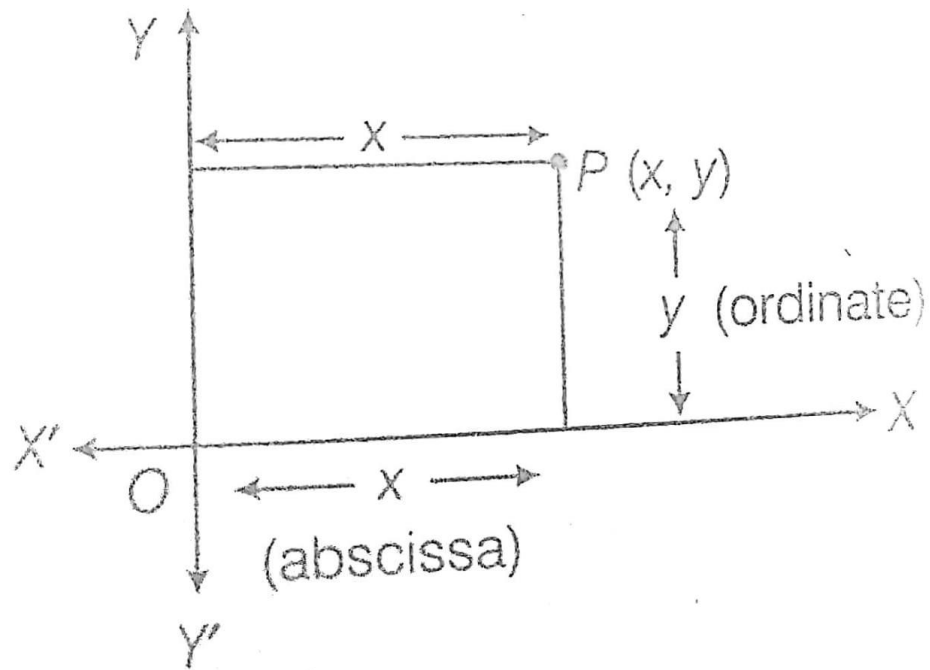
Sign Convention of Coordinates:

$0, y < 0$

II
QUADRANT
$(-, +)$

I
QUADRANT
$(+, +)$

X'

X

O

III
QUADRANT
$(-, -)$

IV
QUADRANT
$(+, -)$

Y

Y'
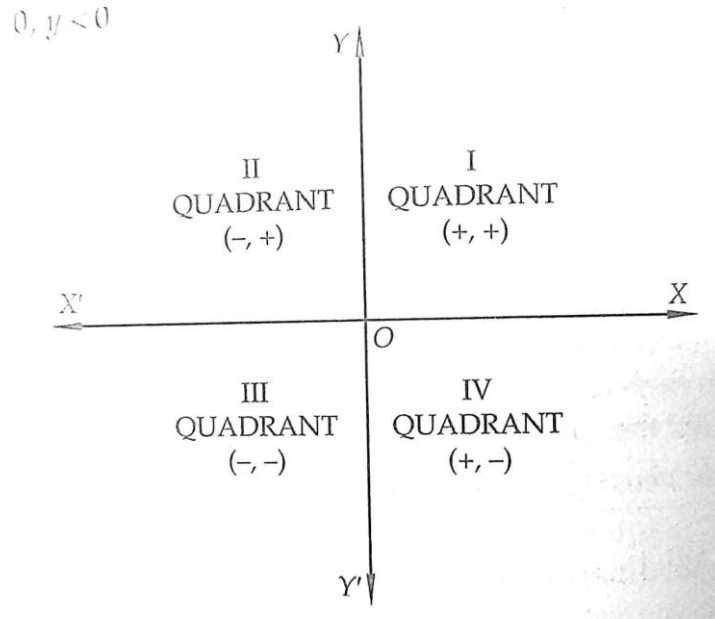
Distance between two Points

The distance between any two points in the plane is the length of the line segment joining them.

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by
$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
The distance of any point $P(x, y)$ from the origin is $|OP| = \sqrt{x^2 + y^2}$
The distance of any point $P(x, y)$ from $the\ x - axis$ is $|y|$.
The distance of any point $P(x, y)$ from the $y - axis$ is $|x|$.
Example: Find the distance between the following points.

$(i)(2, -3)$ and $(-7, 0)$
Sol: The given points are $P(2, -3)$ and $Q(-7, 0)$.
So, $|PQ| = \sqrt{(-7 - 2)^2 + (0 + 3)^2} = 3\sqrt{10}\ units$

(ii) $P(acos\alpha, a\ sin\alpha)$ and $Q(a\ cos\beta, a\ sin\beta)$

Sol: $|PQ| = \sqrt{(a\ cos\beta - a\ cos\alpha)^2 + (a\ sin\beta - a\ sin\alpha)^2}$

$= \sqrt{a^2\ (cos\beta - cos\alpha)^2 + a^2(sin\beta - sin\alpha)^2}$

$= a\ \sqrt{cos^2\beta + cos^2\alpha - 2\cos\alpha\cos\beta + sin^2\beta + sin^2\alpha - 2\ sin\alpha\ sin\beta}$

$= a\ \sqrt{(cos^2\beta + sin^2\beta) + (cos^2\alpha + sin^2\alpha) - 2(\ cos\alpha\ cos\beta + sin\alpha\ sin\beta)}$

$= a\sqrt{1 + 1 - 2\cos(\alpha - \beta)}$

$= a\ \sqrt{2\{1 - \cos(\alpha - \beta)\}}$

$= a\ \sqrt{2 \times 2\ sin^2\left(\dfrac{\alpha - \beta}{2}\right)}$

$= 2a\sin\left(\dfrac{\alpha - \beta}{2}\right)$

Section or Division Formulae

Internal Division:
The coordinates of the point P which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are
$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$$

External Division:
The coordinates of the point P which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m : n$ are
$$\left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n} \right)$$

Example: Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio $2 : 3$ (i) internally (ii) externally.

Sol: Let the given points are $A(-1, 7)$ and $B(4, -3)$.

(i) Here, P divides AB internally in the ratio $2 : 3$. So, the coordinates of P are

$\left(\frac{2 \times 4 + 3 \times (-1)}{2 + 3}, \frac{2 \times (-3) + 3 \times 7}{2 + 3} \right) i.e., (1, 3)$

(ii) Here, P divides AB externally in the ratio $2 : 3$. So, the coordinates of P are

$\left(\frac{2 \times 4 - 3 \times (-1)}{2 - 3}, \frac{2 \times (-3) - 3 \times 7}{2 - 3} \right) i.e., (-11, 27)$

Mid –Point Formula

If P be the mid-point of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$, then the coordinates of P are

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example: Without using distance formula, show that the points $(-2, -1), (4, 0), (3, 3)$ and $(-3, 2)$ are the vertices of a parallelogram.

Sol: Let the given points are $A(-2, -1), B(4, 0), C(3, 3)$ and $D(-3, 2)$.

Now, mid-point of AC is $\left(\frac{-2+3}{2}, \frac{-1+3}{2}\right) = \left(\frac{1}{2}, 1\right)$

and mid-point of BD is $\left(\frac{4-3}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$

We get mid-point of AC = mid-point of BD. Thus mid-points of both diagonals are coinciding each other. Hence, the points A, B, C and D are vertices of a parallelogram.

Centroid of a Triangle

If G is the centroid of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) , then coordinates of G are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

The centroid divides each median in the ratio $2 : 1$.

Example: If the vertices of a triangle are $P(1, 3), Q(2, 5)$ and $R(3, -5)$, then find the centroid of a $\Delta\,PQR$.

Sol: The coordinates of the centroid G are $\left(\frac{1+2+3}{3}, \frac{3+5-5}{3}\right) = \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$

Area of a Triangle

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of an ΔABC, then the area of the triangle is

$$\frac{1}{2} \left| [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \right|$$

NOTE:
➤ The area of any triangle is always non –negative
➤ If the points A, B, C are collinear, then the area of the triangle is zero.

$$i.e. \, x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

➤ Using area formula, we can determine the area of any polygon of n sides.

Example: Find the area of a ΔABC, whose vertices are $A(6, 3), B(-3, 5)$ and $C(4, -2)$.

Sol: Area of the triangle $ABC = \frac{1}{2} \left| 6(5 + 2) + (-3)(-2 - 3) + 4(3 - 5) \right|$

$$= \frac{1}{2} \left| 6 \times 7 - 3(-5) + 4(-2) \right| = \frac{49}{2} \, sq. \, units$$

Example: For what value of k are points $(k, 2 - 2k), (-k + 1, 2k), (-4 - k, 6 - 2k)$ are collinear.

Sol: Since the points are collinear, so $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow k(2k - 6 + 2k) + (-k + 1)(6 - 2k - 2 + 2k) + (-4 - k)(2 - 2k - 2k) = 0$

$\Rightarrow 4k^2 - 6k - 4k + 4 + 4k^2 + 14k - 8 = 0$

$\Rightarrow 8k^2 + 4k - 4 = 0$

$\Rightarrow 2k^2 + k - 1 = 0$

$\Rightarrow (2k - 1)(k + 1) = 0$

$\Rightarrow k = \frac{1}{2}, -1$

Hence, the given points are collinear for $k = \frac{1}{2}$ or $k = -1$.

THANKING YOU

ODM EDUCATIONAL GROUP

Slope of a Line and
Angle between two Lines

SUBJECT : MATHEMATICS
**CHAPTER NUMBER: 10**
CHAPTER NAME : STRAIGHT LINES

Inclination of a Line

A straight line is a curve, such that all the points on the line segment joining any two points of it lies on it.

An angle θ made by the line with positive $x-axis$ in an anti-clockwise direction is called the angle of inclination of a line. A line in coordinate plane forms two angles with the $x-axis$, which are supplementary. Thus $0 \leq \theta \leq 180°$.
The inclination of the $x-axis$ or any line parallel to the $x-axis$ is 0°.
The inclination of the $y-axis$ or any line parallel to the $y-axis$ is 90°
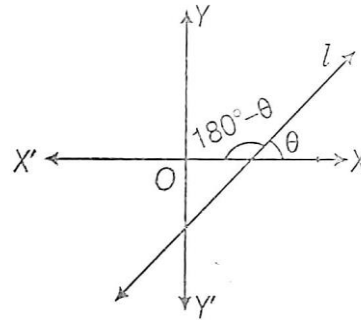
Slope or Gradient of a Line

If θ is the angle of inclination of a line l, then $tan\theta$ is called the slope or gradient of the line l and it is denoted by m.

$i.e. m = \tan\theta$

The slope of $x - axis$ is $m = tan0° = 0$

The slope of a line when $\theta = 90°$ is not defined $i.e.,$ the slope of the $y - axis$ is not defined.

The slope of a line is positive or negative according to the line leans to right or left.

Example: The slope of a line whose inclination is $150°$ is $tan150° - \dfrac{1}{\sqrt{3}}$

Example: Find the slope of the line making inclination of $60°$ with the positive direction of $the\ x - axis$.

Sol: Here inclination of the line is $\theta = 60°$. Therefore, the slope of the line is $m = tan60° = \sqrt{3}$

Slope of a Line Joining two Points

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given points. Then the slope of the line segment joining AB is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Sol: The slope of the line $= \frac{-4-2}{3-1} = -3$

Example: Find the angle between the $x - axis$ and the line joining the points $(3, -1)$ and $(4, -2)$.

Sol: The slope of the line $= \frac{-2+1}{4-3} = -1$.

If θ is the angle between $the\ x - axis$ and the given line, then $tan\theta = -1$

$\Rightarrow \theta = 135°$

Example: A ray of light coming from the point $(1,2)$ is reflected at a point A on the $x-axis$ and then passes through the point $(5,3)$. Find the coordinates of point A.

Sol: Let the coordinates of point A be $(x, 0)$. From the figure, the slope of the reflected ray is given by $tan\theta = \dfrac{3}{5-x} \ldots (i)$
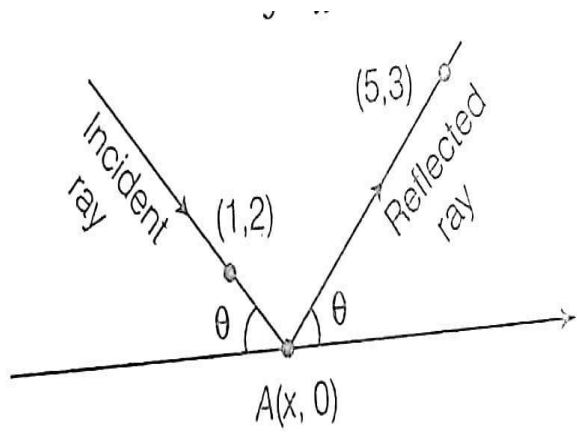
Again, the slope of the incident ray is given

by $\tan(\pi - \theta) = \dfrac{-2}{x-1}$

$\Rightarrow -tan\theta = -\dfrac{2}{x-1} \Rightarrow tan\theta = \dfrac{2}{x-1} \ldots (ii)$

From (i) and (ii), we get $\dfrac{3}{5-x} = \dfrac{2}{x-1}$

$\Rightarrow 3x - 3 = 10 - 2x \Rightarrow x = \dfrac{13}{5}$

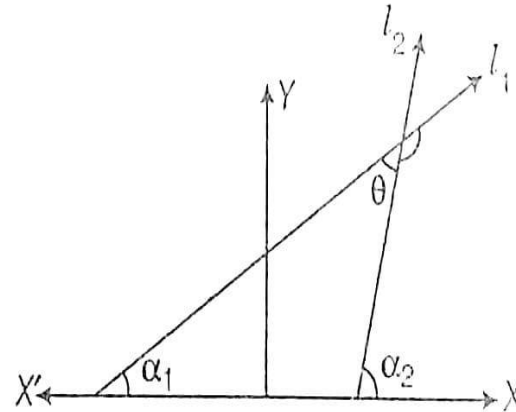Therefore, the required coordinates of the point A are $\left(\dfrac{13}{5}, 0\right)$.

ODM PUBLIC SCHOOL
SCHOOL OF HUMAN EXCELLENCE

Angle between Two Lines

Let l_1 and l_2 be two lines and their inclination are θ_1 and θ_2 respectively. Then, their slopes are $m_1 = tan\theta_1$ and $m_2 = \tan\theta_2$.

If θ is the angle between l_1 and l_2, then $tan\theta = \pm \frac{m_2 - m_1}{1 + m_1 m_2}$

For acute angle, we take $tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

For obtuse angle, we take $\varphi = \pi - \theta$

Example: Find the angle between the lines joining the points $(0, 0), (2, 3)$ and $(2, -2), (3, 5)$.

Sol: We have $m_1 = $ slope of the line joining $(0, 0)$ and $(2, 3) = \frac{3-0}{2-0} = \frac{3}{2}$

$m_2 = $ slope of the line joining $(2, -2)$ and $(3, 5) = \frac{5+2}{3-2} = 7$

If θ is the angle between then, $tan\theta = \pm \frac{m_2 - m_1}{1 + m_1 m_2} = \pm \left(\frac{7 - \frac{3}{2}}{1 + 7 \times \frac{3}{2}} \right) = \pm \left(\frac{11}{23} \right)$

Example: If the angle between two lines is $\frac{\pi}{4}$ and slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Sol: We know that the acute angle θ between two lines with slopes m_1 and m_2 is given by

$tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

Let $m_1 = \frac{1}{2}$, $m_2 = m$ and $\theta = \frac{\pi}{4}$

Now, putting these values, we get

$\tan\frac{\pi}{4} = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right| \Rightarrow 1 = \left|\frac{2m-1}{2+m}\right| \Rightarrow \pm 1 = \frac{2m-1}{2+m} \Rightarrow \frac{2m-1}{2+m} = 1$ or $\frac{2m-1}{2+m} = -1$

$\Rightarrow m = 3$ or $m = -\frac{1}{3}$

Condition of Parallelism of Lines

If two lines of slopes m_1 and m_2 are parallel, then the angle θ between them is $0°$.

$\therefore \tan \theta = \tan 0° = 0$

$\Rightarrow \frac{m_2 - m_1}{1 + m_1 m_2} = 0 \Rightarrow m_1 = m_2$

Thus, two lines are parallel if and only if their slopes are equal.

Example: What is the value of y so that the line through $(3, y)$ and $(2, 7)$ is parallel to the line through $(-1, 4)$ and $(0, 6)$?

Sol: Let $A(3, y), B(2, 7), C(-1, 4)$ and $D(0, 6)$ be the given points.

Then $m_1 = $ slope of the line $AB = \frac{7 - y}{2 - 3} = y - 7$

and $m_2 = $ slope of the line $CD = \frac{6 - 4}{0 - (-1)} = 2$

Since, AB and CD are parallel, so, $m_1 = m_2 \Rightarrow y - 7 = 2 \Rightarrow y = 9$.

Condition of Perpendicularity of Two Lines

If two lines of slopes m_1 and m_2 are perpendicular, then the angle θ between them is $90°$.

$\therefore \tan\theta = \tan 90° = \dfrac{1}{0}$

$\Rightarrow \dfrac{m_2 - m_1}{1 + m_1 m_2} = \dfrac{1}{0} \Rightarrow m_1 m_2 = -1$

Thus, two lines are perpendicular if and only if product of their slopes is -1.

Example: Line through the points $(-2, 6)$ and $(4, 8)$ is perpendicular to the line through $(8, 12)$ and $(x, 24)$. Find the value of x.

Sol: Slope of the line through the points $(-2, 6)$ and $(4, 8)$ is $m_1 = \dfrac{8-6}{4-(-2)} = \dfrac{1}{3}$

Slope of the line through the points $(8, 12)$ and $(x, 24)$ is $m_2 = \dfrac{24-12}{x-8} = \dfrac{12}{x-8}$

Since, the two lines are perpendicular, so $m_1 m_2 = -1$

$\Rightarrow \dfrac{1}{3} \times \dfrac{12}{x-8} = -1 \Rightarrow x = 4$

Collinearity of Three Points

If A, B and C are three points in $XY-$plane, then they will be collinear $i.e.$ will lie on the same line if and only if $slope\ of\ AB = slope\ of\ BC$.

Example: Prove that the points $A(1, 4), B(3, -2)$ and $C(4, -5)$ are collinear.

Sol: Now, slope of $AB = \frac{-2-4}{3-1} = -3$

Also, slope of $BC = \frac{-5+2}{4-3} = -3$

Since, slope of $AB =$ slope of BC,so points A, B and C are collinear.

THANKING YOU
ODM EDUCATIONAL GROUP

Various Forms of Equation of a Line

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 10
CHAPTER NAME : STRAIGHT LINES

Locus and Equation to a Locus

Locus: The curve described by a moving point under given geometrical condition(s), is called locus of that point.

Equation of the Locus of a Point:
The equation of the locus of a point is the relationship that is satisfied by the coordinates of every point on the locus of the point.

A straight line is a curve, such that all the points on the line segment joining any two points of it lies on it.

The Straight Lines

Various Forms of Equation of Line

The equation of a straight line is the relation between x(the abscissa) and y (the ordinate) which is satisfied by the coordinates of each and every point on the line and is not satisfied by the coordinates of any point which does not lie on the line.

Equation of Line Parallel to $x - axis$ or Equation of a horizontal line

Let L be a straight line parallel to the $x - axis$ at a distance a from it, then the equation of the line L is $y = a$ or $y = -a$.

The choice of the sign will depend upon the position of the line according to the line is on the positive or negative side of the $-axis$.

The equation of the $x - axis$ is $y = 0$.
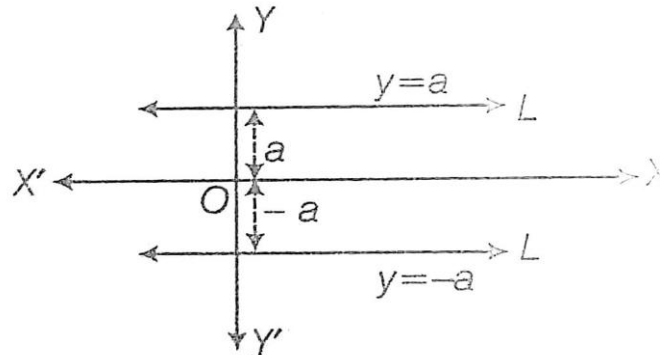
Example: Write down the equation of a line parallel to the $x-axis$ and at a distance of 6 units above the $x-axis$.
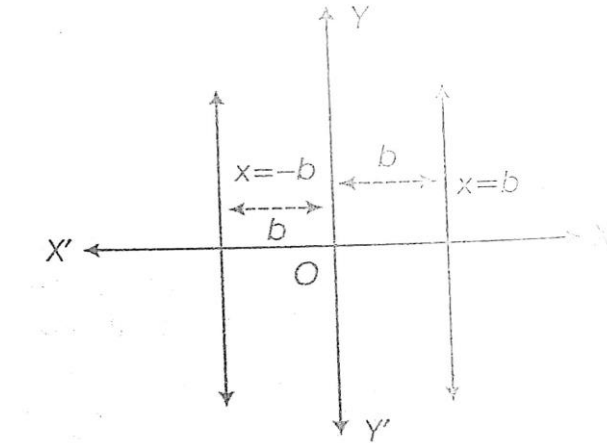
Sol: The equation of a line parallel to the $x-axis$ and at a distance of 6 units above the $x-axis$ is $y=6$.

Equation of Line Parallel to $y-axis$ or Equation of vertical Line

Let L be a straight line parallel to $the\ y-axis$ at a distance b from it, then the equation of the line L is $x=b$ or $x=-b$.
The choice of the sign will depend upon the position of the line according to the line is on the positive or negative side of the $-axis$.
The equation of the $y-axis$ is $x=0$.

Example: Find the equation of the lines parallel to the axes and passing through the point $(-3, 5)$.

Sol: The equation of a line parallel to the $x-axis$ and passing through $(-3, 5)$ is $y = 5$.

The equation of a line parallel to the $y-axis$ and passing through $(-3, 5)$ is $x = -3$.

Example: Find the equation of a line that is equidistant from the lines $x = -4$ and $x = 8$.
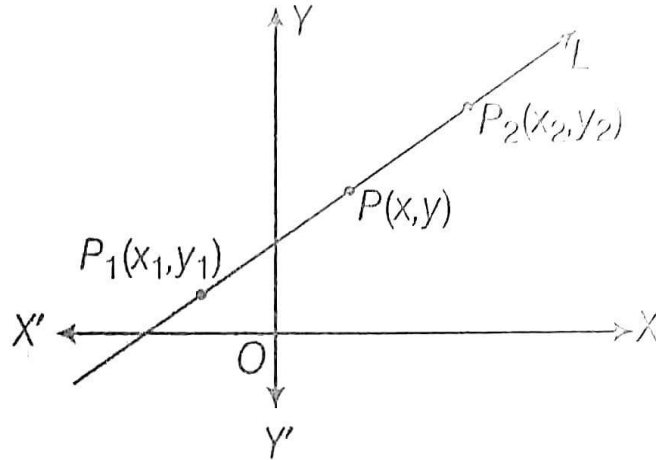
Sol: Since the given lines are both parallel to $the \ y-axis$ and the required line is equidistant from these lines, so it is also parallel to $the \ y-axis$ and its distance from the $y-axis$ is $\frac{1}{2}(-4 + 8) = 2 \ units$.

Hence, its equation is $x = 2$.

Two – point Form of a Line

The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Example: Find the equation of the line joining the points $(-1, 3)$ and $(4, -2)$.

Sol: By two –point form, the equation of the line is $y - 3 = \frac{-2-3}{4-(-1)}\left(x - (-1)\right)$

$\Rightarrow\ y - 3 = -(x + 1)$
$\Rightarrow\ y - 3 = -x - 1$
$\Rightarrow\ x + y - 2 = 0$

Example: If $A(2, 1), B(-2, 3)$ and $C(4, 5)$ are the vertices of a ΔABC, then find the equation of the median through the vertex C.

Sol: Let F be the midpoint of side AB and CF is a median through C.

Now, coordinates of F are $\left(\frac{2+(-2)}{2}, \frac{1+3}{2}\right)\ i.e.\ (0, 2)$.

So, the equation of the median CF is given by $y - 5 = \frac{2-5}{0-4}(x - 4)$

$\Rightarrow 3x - 4y + 8 = 0$.

Point Slope Form

The equation of the straight line having slope m and passes through the point (x_0, y_0) is
$$y - y_0 = m(x - x_0)$$
Example: Find the equation of the line passing through $(-4, 3)$ and having slope $\frac{1}{2}$.

Sol: Given, $m =$ slope of the line $= \frac{1}{2}$ and $x_0 = -4$ and $y_0 = 3$.

Therefore, the equation of the line is $y - 3 = \frac{1}{2}\left(x - (-4)\right)$

$\Rightarrow 2y - 6 = x + 4 \Rightarrow x - 2y + 10 = 0$.

Example: If the line joining two points $A(2, 0)$ and $B(3, 1)$ is rotated about A in an anti-clockwise direction through an angle of $15°$. Find the equation of the line in a new position.

Sol: The slope of line AB is given by $m = \dfrac{1-0}{3-2}$

$\Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$

After rotation of the line AB about A is anticlockwise direction through an angle of $15°$, the slope of the line AC in a new position is given by $m_1 = \tan(45° + 15°) = \tan 60° = \sqrt{3}$.

Therefore the equation of the new line AC is $y - 0 = \sqrt{3}(x - 2)$

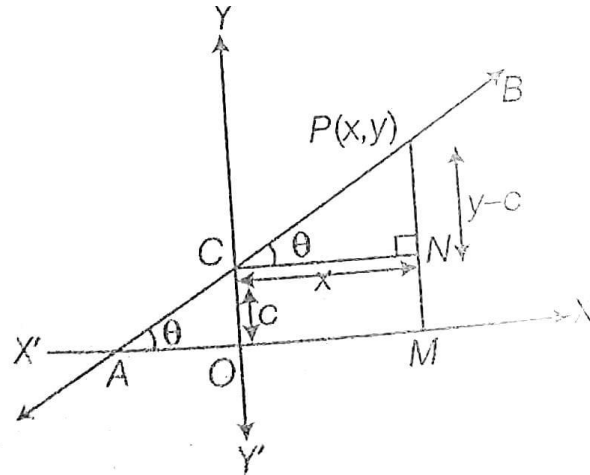$\Rightarrow \sqrt{3}x - y - 2\sqrt{3} = 0$.

Slope Intercept Form

Suppose a line L with slope m cuts the $y-axis$ at a distance c from the origin. (distance c is called the $y-$intercept of the line).
Then the equation of the line is $y = mx + c$.
If a line L with slope m cuts the $x-axis$ at a distance d from the origin $i.e.$ Makes $x-$intercept d, then the equation of line L is $y = m(x - d)$.

NOTE: If the line passing through the origin then $c = 0$.

Therefore, the equation of the line passing through the origin is $y = mx$, where m is the slope of the line.
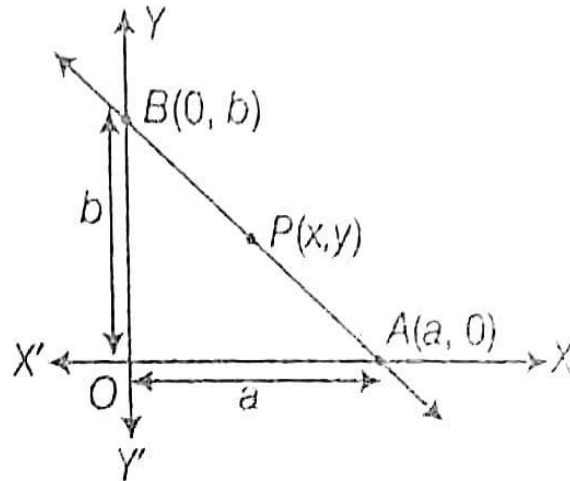
Example: Find the equation of the line which has slope $\frac{1}{2}$ and cuts $-$ off an intercept -5 on $the\ y - axis$.

Sol: Given, $m =$ slope of the line $= \frac{1}{2}$ and $c = -5$.

Hence, the required equation of the line is $y = \frac{1}{2}x - 5 \Rightarrow x - 2y - 10 = 0$.

Intercept Form

The equation of a line which cuts – off intercepts a and b on the $x - axis$ and $y - axis$ respectively, is $\frac{x}{a} + \frac{y}{b} = 1$.

Example: Find the equation of the lines which cuts –off intercepts on the axes whose sum and product are 1 and -6 respectively.

Sol: Let a and b be the intercepts of the line $x - axis$ and $-axis$, respectively.

Then the equation of the line is $\frac{x}{a} + \frac{y}{b} = 1 \ldots (i)$

Given, sum of intercepts $a + b = 1 \ldots (ii)$ and product of intercepts, $ab = -6 \ldots (iii)$

On putting the value of b from (ii) in (iii), we get $a(1 - a) = -6$

$\Rightarrow a^2 - a - 6 = 0 \Rightarrow a = 3$ or -2.

From (iii), when $a = 3, b = -2$ and when $a = -2, b = 3$. Hence the required lines are $\frac{x}{3} + \frac{y}{-2} = 1$ and $\frac{x}{-2} + \frac{y}{3} = 1 \Rightarrow 2x - 3y - 6 = 0$ and $3x - 2y + 6 = 0$.

Example: Find the equation of the line which passes through the point $(-4, 3)$, the portion of the line intercepted between the axes is divided internally in the ratio $5 : 3$ by this point.

Sol: Let the line intersects $x - axis$ and $y - axis$ respectively at $A(x, 0)$ and $B(0, y)$

Then $-4 = \frac{5 \times 0 + 3x}{5 + 3} \Rightarrow \frac{3x}{8} = -4 \Rightarrow x = -\frac{32}{3}$

and $3 = \frac{5 \times y + 3 \times 0}{5 + 3} \Rightarrow \frac{5y}{8} = 3 \Rightarrow y = \frac{24}{5}$

Thus the intercepts on the coordinate axes are $-\frac{32}{3}$ and $\frac{24}{5}$ respectively.

Hence, the required equation of the line is $\frac{x}{-\frac{32}{3}} + \frac{y}{\frac{24}{5}} = 1$

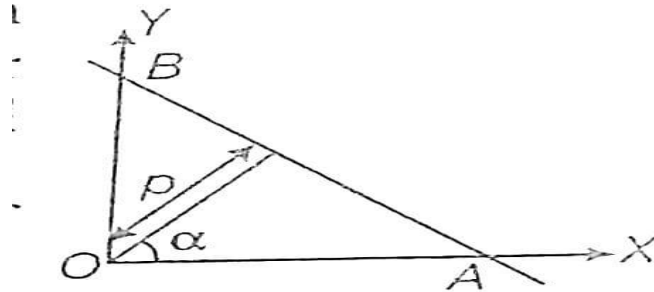$\Rightarrow 9x - 20y + 96 = 0$

Normal Form or Perpendicular Form of a Line

The equation of a straight line upon which the length of the perpendicular $i.e.$ Normal from the origin is p and this perpendicular makes an angle α with the positive direction of $the\ x - axis$ is
$$x\ cos\alpha + y\ sin\alpha = p$$

Here slope of line $= -\dfrac{1}{tan\alpha} = -\dfrac{cos\alpha}{sin\alpha}$

Example: The perpendicular distance of a line from the origin is 7 cm and its slope is -1. Find the equation of the line.

Sol: Here slope$(m) = -\dfrac{1}{tan\alpha} = -1 \Rightarrow tan\alpha = 1 \Rightarrow \alpha = 45°$

Thus the required equation of the line is $x\ cos\alpha + y\ sin\alpha = p$

$\Rightarrow\ x\cos 45° + y\ sin45° = 7$

$\Rightarrow\ x\dfrac{.1}{\sqrt{2}} + y\,.\dfrac{1}{\sqrt{2}} = 7 \Rightarrow x + y - 7\sqrt{2} = 0.$

THANKING YOU

ODM EDUCATIONAL GROUP

General Equation of a Line

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 10
CHAPTER NAME : STRAIGHT LINES

General Equation of a Line

An equation of the form $Ax + By + C = 0$, where $A, B, C \in R, \sqrt{A^2 + B^2} \neq 0$ is called general linear equation or general equation of the line.

Different Forms of $Ax + By + C = 0$

The general equation of a line can be reduced into various forms of the equation of the line, which are given below.

Slope Intercept Form

If $B \neq 0$, then $Ax + By + C = 0$ can be written as $y = -\frac{A}{B} x + \left(-\frac{C}{B}\right)$

where $m = -\frac{A}{B}$ and $c = -\frac{C}{B}$

Intercept Form

If $C \neq 0$, then $Ax + By + C = 0$ can be written as $\frac{x}{-C/A} + \frac{y}{-C/B} = 1$, where $a = -\frac{C}{A}$, $b = -\frac{C}{B}$

Normal Form

Let $x \cos\omega + y \sin\omega = p$ be the normal form of the line represented by the equation $Ax + By + C = 0$

or $Ax + By = -C$

Thus, both the equations are same and therefore $\dfrac{A}{\cos\omega} = \dfrac{B}{\sin\omega} = -\dfrac{C}{p}$

$\Rightarrow \cos\omega = -\dfrac{Ap}{C}$ and $\sin\omega = -\dfrac{Bp}{C}$

Since $\sin^2\omega + \cos^2\omega = 1$, so, $\left(-\dfrac{Ap}{C}\right)^2 + \left(-\dfrac{Bp}{C}\right)^2 = 1$

$\Rightarrow \dfrac{A^2 p^2}{C^2} + \dfrac{B^2 p^2}{C^2} = 1 \Rightarrow p^2 = \dfrac{C^2}{A^2 + B^2} \Rightarrow p = \pm\dfrac{C}{\sqrt{A^2 + B^2}}$

Therefore, $\cos\omega = \pm\dfrac{A}{\sqrt{A^2 + B^2}}$ and $\sin\omega = \pm\dfrac{B}{\sqrt{A^2 + B^2}}$

Thus the normal form of the equation $Ax + By + C = 0$ is $x \cos\omega + y \sin\omega = p$, where

$\cos\omega = \pm\dfrac{A}{\sqrt{A^2 + B^2}}$, $\sin\omega = \pm\dfrac{B}{\sqrt{A^2 + B^2}}$ and $p = \pm\dfrac{C}{\sqrt{A^2 + B^2}}$ (p should be positive)

To transform the general equation of a line to the normal form, we use the following steps.

(i) Shift the constant term to the RHS and make it positive.

(ii) Divide both sides by $\sqrt{(coefficient\ of\ x)^2 + (coefficient\ of\ y)^2}$

The equation so obtained is in the normal form.

Example: Transform the equation of the line $3x + 2y - 7 = 0$ to

(i) slope intercept form and also find the slope and $y-$intercept.

(ii) intercept form and the intercepts on the coordinate axes.

(iii) normal form and also find the inclination of the perpendicular segment from the origin on the line with the axis and its length.

Sol: (i) Given equation is $3x + 2y - 7 = 0$

It can be written as $2y = -3x + 7 \Rightarrow y = -\dfrac{3}{2}x + \dfrac{7}{2}$

which is the required slope intercept form where slope $m = -\frac{3}{2}$ and $y-$intercept $c = \frac{7}{2}$.

(ii) Given equation can be rewritten as $3x + 2y = 7$

$\Rightarrow \frac{x}{7/3} + \frac{y}{7/2} = 1$, which is the required intercept form of the given line.

Here $x-$intercept $= \frac{7}{3}$ and $y-$intercept $= \frac{7}{2}$

(iii) Given equation is $3x + 2y - 7 = 0 \Rightarrow 3x + 2y = 7$

On dividing both sides by $\sqrt{3^2 + 2^2} = \sqrt{13}$, we get

$\frac{3}{\sqrt{13}} x + \frac{2}{\sqrt{13}} y = \frac{7}{\sqrt{13}}$, which is the required normal form of the given line.

Here $cos\omega = \frac{3}{\sqrt{13}}$, $sin\omega = \frac{2}{\sqrt{13}}$ and $p = \frac{7}{\sqrt{13}}$. Since $cos\omega$ and $sin\omega$ both are positive, so ω is in

the first quadrant and is obtained from $\tan \omega = \frac{2}{3}$.

Example: Equation of a line is $3x - 4y + 10 = 0$. Find its (i) slope $(ii) x -$ and $(iii) y -$ intercepts.

Sol: (i) Given equation $3x - 4y + 10 = 0$ can be written as $y = \frac{3}{4}x + \frac{5}{2} \dots (1)$

Comparing (1) with $y = mx + c$, we have slope of the given line is $m = \frac{3}{4}$.

(ii) Equation $3x - 4y + 10 = 0$ can be written as $3x - 4y = -10$

$\Rightarrow \frac{x}{-10/3} + \frac{y}{5/2} = 1 \dots (2)$

Comparing (2) with $\frac{x}{a} + \frac{y}{b} = 1$, we have $x - intercept$ as $a = -\frac{10}{3}$ and $y - intecept$ as $b = \frac{5}{2}$

The Angle between Two Lines, having General Equations

Let, general equations of lines be $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$.

Then slope of given lines are $m_1 = -\dfrac{A_1}{B_1}$ and $m_2 = -\dfrac{A_2}{B_2}$

Let θ be the angle between two lines, then $tan\theta = \pm \dfrac{m_2 - m_1}{1 + m_1 m_2} = \pm \left(\dfrac{-\frac{A_1}{B_1} + \frac{A_2}{B_2}}{1 + \frac{A_1}{B_1}\frac{A_2}{B_2}} \right)$

Example: Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}\, y - x + 6 = 0$.

Sol: Given lines are $y = \sqrt{3}x + 5$ … (1) and $y = \dfrac{1}{\sqrt{3}}\, x - 2\sqrt{3}$ … (2)

The slope of a line (1) is $m_1 = \sqrt{3}$ and slope of the line (2) is $m_2 = \dfrac{1}{\sqrt{3}}$

The acute angle θ between two lines is given by $tan\theta = \left| \dfrac{m_2 - m_1}{1 + m_1 m_2} \right|$

Putting the value of m_1 and m_2, we get $tan\theta = \left|\dfrac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}\right| = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \ \theta = 30°$

Hence, angle between two lines is either $30°$ or $180° - 30° = 150°$.

Condition for two lines to be Parallel

If the lines $A_1 x + B_1 y + C_1 = 0$ and $A_2 x + B_2 y + C_2 = 0$ are parallel, then their slopes are equal

$i.e. m_1 = m_2 \Rightarrow -\dfrac{A_1}{B_1} = -\dfrac{A_2}{B_2} \Rightarrow \dfrac{A_1}{B_1} = \dfrac{A_2}{B_2}$

Condition for two lines to be Perpendicular

If the lines $A_1 x + B_1 y + C_1 = 0$ and $A_2 x + B_2 y + C_2 = 0$ are perpendicular, then the product of their slopes is -1. $i.e. m_1 \times m_2 = -1 \Rightarrow \left(-\dfrac{A_1}{B_1}\right) \times \left(-\dfrac{A_2}{B_2}\right) - 1 \Rightarrow A_1 A_2 + B_1 B_2 = 0$.

Example: A line passing through the points $(a, 2a)$ and $(-2, 3)$ is perpendicular to the line $4x + 3y + 5 = 0$. Find the value of a.

Sol: Let m_1 be the slope of the line joining $A(a, 2a)$ and $B(-2, 3)$.

Then $m_1 = \frac{3-2a}{a+2}$

Let m_2 be the slope of line $4x + 3y + 5 = 0$. Then $m_2 = -\frac{4}{3}$.

Since, the given lines perpendicular so, $m_1 . m_2 = -1$

$\Rightarrow \frac{3-2a}{a+2} \times \left(-\frac{4}{3}\right) = -1$

$\Rightarrow 8a - 12 = 3a + 6$

$\Rightarrow a = \frac{18}{5}$

THANKING YOU
ODM EDUCATIONAL GROUP

Point of Intersection of Two Lines

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 10
CHAPTER NAME : STRAIGHT LINES

Lines Parallel and Perpendicular to a given Line

Line Parallel to a given Line

The equation of a line parallel to a given line $Ax + By + C = 0$ is $Ax + By + k = 0$, where k is a constant.

Example: Find the equation of the line which is parallel to $3x - 2y + 5 = 0$ and passes through the point $(5, -6)$.

Sol: The equation of any line parallel to the line $3x - 2y + 5 = 0$ $\quad 3x - 2y + k = 0$ ... (1)

This passes through $(5, -6)$, so $3(5) - 2(-6) + k = 0$

$\Rightarrow \; k = -27$.

Putting $k = -27$ in (1), we obtain $3x - 2y - 27 = 0$ as the required equation.

Line Perpendicular to a given Line

The equation of a line perpendicular to a given $Ax + By + C = 0$ is $Bx - Ay + k = 0$, where k is a constant.

Example: Find the equation of the straight line that passes through the point $(3, 4)$ and perpendicular to the line $3x + 2y + 5 = 0$.

Sol: The equation of a line perpendicular to $3x + 2y + 5 = 0$ is $2x - 3y + k = 0$ … (1)

This passes through the point $(3, 4)$

So, $3 \times 2 - 3 \times 4 + k = 0 \Rightarrow k = 6$

Putting $k = 6$ in (1), we obtain $2x - 3y + 6 = 0$ as the required equation.

Example: Find the equation of the line perpendicular to $x - 7y + 5 = 0$ and having $x-$intercept 3.

Sol: The equation of a line perpendicular to $x - 7y + 5 = 0$ is $7x + y + k = 0$ ... (1)

Its $x-$intercept is 3. So it passes through the point $(3, 0)$ on the $x - axis$

Thus $7 \times 3 + 0 + k = 0 \Rightarrow k = -21$

Putting $k = -21$ in (1) we obtain $7x + y - 21 = 0$ as the equation of the required line.

Point of Intersection of Two Lines

Point of Intersection of Two Lines

Let the equations of two lines be $A_1 x + B_1 y + C_1 = 0$ and $A_2 x + B_2 y + C_2 = 0$.

Suppose these two lines intersect at a point $P(x_1, y_1)$. Then (x_1, y_1) satisfies each of the given equations.

So $A_1 x_1 + B_1 y_1 + C_1 = 0$ and $A_2 x_2 + B_2 y_2 + C_2 = 0$.

By cross $-$ multiplication we get $x_1 = \dfrac{B_1 C_2 - B_2 C_1}{A_1 B_2 - A_2 B_1}$, $y_1 = \dfrac{C_1 A_2 - C_2 A_1}{A_1 B_2 - A_2 B_1}$

Hence, the coordinates of the point of intersection of the given lines are

$\left(\dfrac{B_1 C_2 - B_2 C_1}{A_1 B_2 - A_2 B_1} , \dfrac{C_1 A_2 - C_2 A_1}{A_1 B_2 - A_2 B_1} \right)$

Concurrent Lines

Three lines are said to be concurrent if they pass through a common point $i.e.$ They meet at a point.

Thus, if three lines are concurrent, the point of intersection of two lines lies on the third line.

Let $A_1x + B_1y + C_1 = 0$, $A_2x + B_2y + C_2 = 0$ and $A_3x + B_3y + C_3 = 0$ be three lines.

Then the condition for which these three lines are concurrent is

$$A_1(B_2C_3 - B_3C_2) + B_1(C_2A_3 - C_3A_2) + C_1(A_2B_3 - A_3B_2) = 0.$$

Example: Show that the lines $x - y - 6 = 0$, $4x - 3y - 20 = 0$ and $6x + 5y + 8 = 0$ are concurrent. Also, find their common point of intersection.

Sol: The given lines are $x - y - 6 = 0$ … (1)

$4x - 3y - 20 = 0$ … (2) and $6x + 5y + 8 = 0$ … (3)

Solving (1) and (2) by cross – multiplication, we get

$\frac{x}{20-18} = \frac{y}{-24+20} = \frac{1}{-3+4} \Rightarrow x = 2$ and $y = -4$

Thus, the first two lines intersect at the point $(2, -4)$.

Putting $x = 2$ and $y = -4$ in (3), we get $6 \times 2 + 5 \times (-4) + 8 = 0$

Thus the point $(2, -4)$ lies on the line (3).

Hence, the given lines are concurrent and their common point of intersection is $(2, -4)$.

THANKING YOU
ODM EDUCATIONAL GROUP