Distance of a Point from a Line

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 10
CHAPTER NAME : STRAIGHT LINES

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The distance of a Point from a Line

The distance of a point from a line is the length of the perpendicular drawn from the point to the line.

The distance of the point (x_1, y_1) from the line $Ax + By + C = 0$ is

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

The distance of origin from the line $Ax + By + C = 0$ is $d = \left|\frac{C}{\sqrt{A^2+B^2}}\right|$

Example: Find the distance of the point $(2,-3)$ from line $2x-3y+6=0$.

Sol: Required distance of the point from the line

$=$ The perpendicular distance from a point to the line$=\left|\frac{2\times2-3(-3)+6}{\sqrt{2^2+(-3)^2}}\right|=\frac{19}{\sqrt{13}}$

Example: Find the points on $the\ x - axis$ whose perpendicular distance from the line $4x + 3y = 12$ is 4.

Sol: Given points are on $the\ x - axis$. So, let its coordinates be $(\alpha, 0)$

Then, the length of the perpendicular from $(\alpha, 0)$ to the line $4x + 3y - 12 = 0$ is 4.

So, $\left|\frac{4\alpha+3\times0-12}{\sqrt{4^2+(-3)^2}}\right| = 4$

$\Rightarrow \left|\frac{4\alpha-12}{5}\right| = 4$

$\Rightarrow |4\alpha - 12| = 20$

$\Rightarrow 4\alpha - 12 = \pm 20$

$\Rightarrow 4\alpha = 12 \pm 20 = -8,\ 32$

$\Rightarrow \alpha = -2, 8$

Hence the required points are $(8, 0)$ and $(-2, 0)$

Equations of lines passing through a given point and making a given angle with a line

The equations of the straight lines which pass through a given point (x_1, y_1) and make a given angle α with the given straight line $y = mx + c$ are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

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Example: Find the equations of two straight lines through $(7, 9)$ and making an angle of $60°$ with the line $x - \sqrt{3}y - 2\sqrt{3} = 0$.

Sol: We know that the equations of two straight lines which pass through a point (x_1, y_1) and make a given angle α with $y = mx + c$ are

$$y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$$

Here, $x_1 = 7, y_1 = 9, \alpha = 60°$ and $m = \left(Slope\ of\ the\ line\ x - \sqrt{3}y - 2\sqrt{3} = 0\right) = \frac{1}{\sqrt{3}}$

So, equations of required lines are

$$y - 9 = \frac{\frac{1}{\sqrt{3}} + \tan 60°}{1 - \frac{1}{\sqrt{3}}\tan 60°}(x - 7) \text{ and } y - 9 = \frac{\frac{1}{\sqrt{3}} - \tan 60°}{1 + \frac{1}{\sqrt{3}}\tan 60°}(x - 7)$$

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or, $(y-9)\left(1 - \frac{1}{\sqrt{3}}\tan 60°\right) = \left(\frac{1}{\sqrt{3}} + \tan 60°\right)(x-7)$

and, $(y-9)\left(\frac{1}{\sqrt{3}} + \tan 60°\right) = \left(1 - \frac{1}{\sqrt{3}}\tan 60°\right)(x-7)$

or, $0 = \left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)(x-7)$ and $(y-9)(2) = \left(\frac{1}{\sqrt{3}} - \sqrt{3}\right)(x-7)$

or, $x - 7 = 0$ *and*, $x + \sqrt{3}y = 7 + 9\sqrt{3}$.

Hence, the required lines are $x = 7$ and $x + \sqrt{3}$y=7+9$\sqrt{3}$.

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Distance between Two Parallel Lines

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Distance between Two Parallel Lines

The distance between two parallel lines $y = mx + c_1$ and $y = mx + c_2$ is given by

$d = \left|\frac{c_1 - c_2}{\sqrt{1+m^2}}\right|.$

If the lines are given in general form

$i.e.$ Given lines are $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0,$

then distance between these lines is $d = \left|\frac{c_1 - c_2}{\sqrt{A^2+B^2}}\right|$

Example: Find the distance between the parallel lines $3x - 4y + 9 = 0$ and $6x - 8y - 15 = 0$.

Sol: Given lines are $3x - 4y + 9 = 0$ and

$6x - 8y - 15 = 0 \Rightarrow 3x - 4y - \frac{15}{2} = 0$

So, the required distance = $\left|\frac{c_1 - c_2}{\sqrt{A^2 + B^2}}\right| = \left|\frac{9 + \frac{15}{2}}{\sqrt{3^2 + (-4)^2}}\right| = \frac{33}{10}\ units$.

Example: Find the equations of lines parallel to $3x - 4y - 5 = 0$ at a unit distance from it.
Sol: Equation of any line parallel to $3x - 4y - 5 = 0$ is

$$3x - 4y + l = 0 \qquad \text{....(i)}$$

Putting $x = -1$ in $3x - 4y - 5 = 0$, we get $y = -2$. Therefore, $(-1, -2)$ is a point on $3x - 4y - 5 = 0$. Since the distance between the two lines is one unit. Therefore, the length of perpendicular from $(-1, -2)$ to $3x - 4y + l = 0$ is one unit.

$i.e.\ \frac{|3\times -1 - 4\times -2+}{\sqrt{3^2+(-4)^2}} = 0$

$\frac{|5+}{\lambda|5} = 1 \Rightarrow$

$|5 + \lambda| = 5 \Rightarrow 5 + \lambda = \pm 5 \Rightarrow \lambda = 0\ or\ -10.$

Substituting the values of λ in (i), we get $3x - 4y = 0$ and, $3x - 4y - 10 = 0$ as the equations of the required lines.

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Family of Lines passing through the intersection of Two Lines

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Family of Lines passing through the intersection of Two Lines

The equation of the family of lines passing through the intersection of the lines

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is

$(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$, where λ is a parameter.

Example: Find the equation of the straight line which passes through the point $(2, -3)$ and the point of intersection of the lines $x + y + 4 = 0$ and $3x - y - 8 = 0$.

Sol: Any line through the intersection of the lines $x + y + 4 = 0$ and $3x - y - 8 = 0$ has the equation

$$(x + y + 4) + (3x - y - 8) = 0 \qquad \text{....}(1)$$

This will pass through $(2, -3)$, if

$$(2 - 3 + 4) + (6 + 3 - 8) = 0$$

$$3 + l = 0 \quad Þ \; l = -3.$$

Putting the value of λ in (1), the equation of the required line is $2x - y - 7 = 0$.

Example: Find the equation of the straight line which passes through the point of intersection of the straight lines $x + 2y = 5$ and $3x + 7y = 17$ and is perpendicular to the straight line $3x + 4y = 10$.

Sol: The equation of any line through the intersection of the lines

$x + 2y - 5 = 0$ and $3x + 7y - 17 = 0$ is

$$(x + 2y - 5) + (3x + 7y - 17) = 0$$

or, $x(3\lambda + 1) + y(7\lambda + 2) - (17\lambda + 5) = 0$ ....(1)

This is perpendicular to the line $3x + 4y = 10$.

$\therefore$ *Product of their slopes* $= -1$

Þ $-\left(\frac{3l+1}{7l+2}\right)\left(-\frac{3}{4}\right) = -1$ $\frac{11}{37}$

Putting this value of λ in (1), the equation of the required line is $4x - 3y + 2 = 0$.

Distance Form of a Line

The equation of the straight line passing through (x_1, y_1) and making an angle θ with the positive direction of the x-axis is $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$, where r is the distance of the point (x, y) on the line from the point (x_1, y_1).

NOTE 1 The equation of line is

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

$\Rightarrow x - x_1 = r\cos\theta$ and $y - y_1 = r\sin\theta$ $\Rightarrow x = x_1 + r\cos\theta$ and $y = y_1 + r\sin\theta$.

Thus, the coordinates of any point on the line at a distance r from the given point (x_1, y_1) are $(x_1 + r\cos\theta, y_1 + r\sin\theta)$. If P is on the right side of (x_1, y_1), then r is positive and if P is on the left side of (x_1, y_1), then r is negative.

Since different values of r determine different points on the line, therefore the above form of the line is also called parametric form or symmetric form of a line.

NOTE 2 In the above form one can determine the coordinates of any point on the line at a given distance from the given point through which it passes. At a given distance r from the point (x_1, y_1) on the line $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$ there are two points *viz*. $(x_1 + r\cos\theta, y_1 + r\sin\theta)$ and $(x_1 - r\cos\theta, y_1 - r\sin\theta)$.

Example: In what direction a line be drawn through the point $(1,2)$ that its point of intersection with the line $x + y = 4$ is at a distance $\frac{\sqrt{6}}{3}$ from the given point.

Sol: Let the line drawn through $A(1,2)$ makes an angle θ with the positive direction of the x-axis and intersects the line $x + y = 4$ at P such that $AP = \frac{\sqrt{6}}{3}$. Then, the coordinates of P are given by $\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \frac{\sqrt{6}}{3} \Rightarrow x = 1 + \sqrt{\frac{2}{3}}\cos\theta, y = 2 + \sqrt{\frac{2}{3}}\sin\theta$

So, the coordinates of P are $\left(1 + \sqrt{\frac{2}{3}}\cos\theta, 2 + \sqrt{\frac{2}{3}}\sin\theta\right)$.

Clearly, point P lies on the line $x + y = 4$.

$\therefore \quad 1 + \sqrt{\frac{2}{3}}\cos\theta + 2 + \sqrt{\frac{2}{3}}\sin\theta = 4$

$\Rightarrow \quad \cos\theta + \sin\theta = \sqrt{\frac{3}{2}}$

$$\Rightarrow \quad (\cos\theta + \sin\theta)^2 = \frac{3}{2}$$

$$\Rightarrow \quad 1 + \sin 2\theta = \frac{3}{2}$$

$$\Rightarrow \quad \sin 2\theta = \frac{1}{2}$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{6} \text{ or, } 2\theta = \frac{5\pi}{6}$$

$$\Rightarrow \quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}$$

Hence, the line drawn makes an angle whose measure is either $\frac{\pi}{12}$ or $\frac{5\pi}{12}$ with the x-axis.

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