Measuring of Angles in Radians and Degrees

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 03
CHAPTER NAME : TRIGONOMETRIC FUNCTIONS

Learning Objectives:

- Students will be able to learn the measurement of angles in different systems .
- Students will be able to learn different trigonometric functions and the relations between them.
- Students will be able to learn sign of trigonometric functions in different quadrants.
- Students will be able to learn values of trigonometric functions of allied angles .
- Students will be able to learn trigonometric functions of compound angles.
- Students will be able to learn trigonometric functions of multiple and sub – multiple angles.
- Students will be able to learn equations involving trigonometric functions.

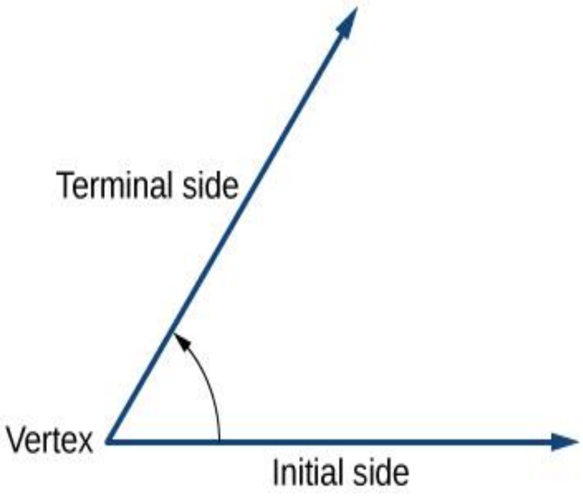
Introduction:

The word trigonometry is derived from the Greek words “**trigon**” and “**metron**” means the measurement of triangles.

It is the branch of mathematics that deals with the measurement of the sides and the angles of a triangle and the problems allied with angles.

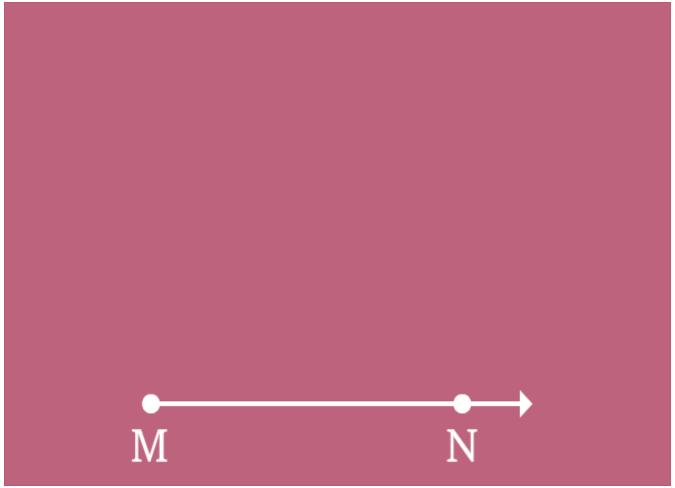
Angles:

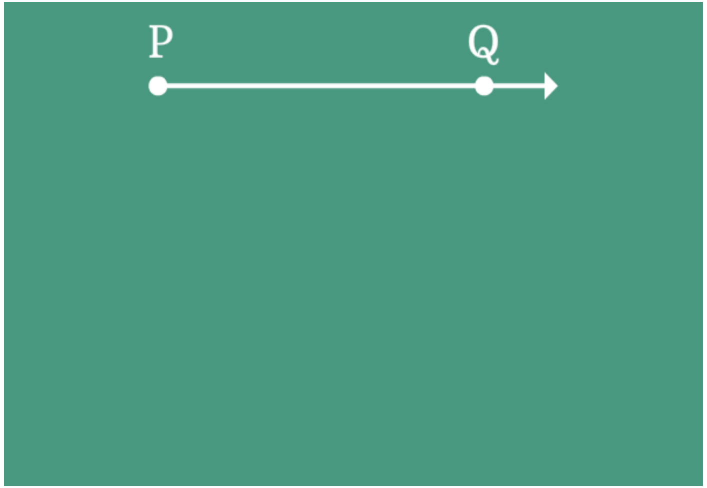
An angle is considered as the figure obtained by rotating a given ray about its endpoint.



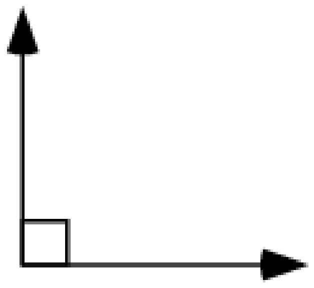
Measure of an Angle: The measure of an angle is the amount of rotation performed to get the initial side.

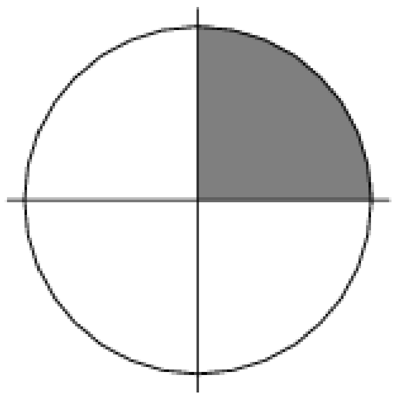
Sense of an Angle: The sense of an angle is said to be positive or negative according as the initial side rotates in anticlockwise or clockwise direction to get to the terminal side.





Right Angle: If the revolving ray starting from its initial position to final position describes one quarter of a circle, then we say that the measure of the angle formed is a right angle.





Systems of Measurement of Angles:

There are two systems for measuring angles

(i) **Sexagesimal or English System or Degree measure:**

In this system, the unit of measurement is degree.

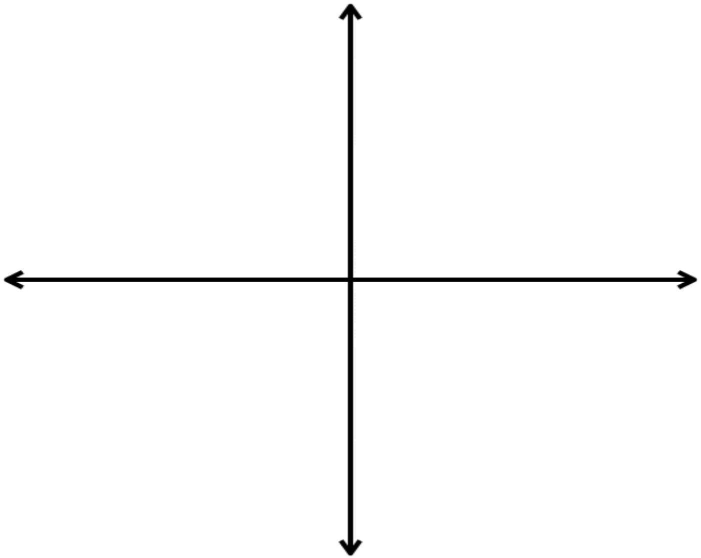
$1 \text{ right angle} = 90 \text{ degrees}(90^\circ)$

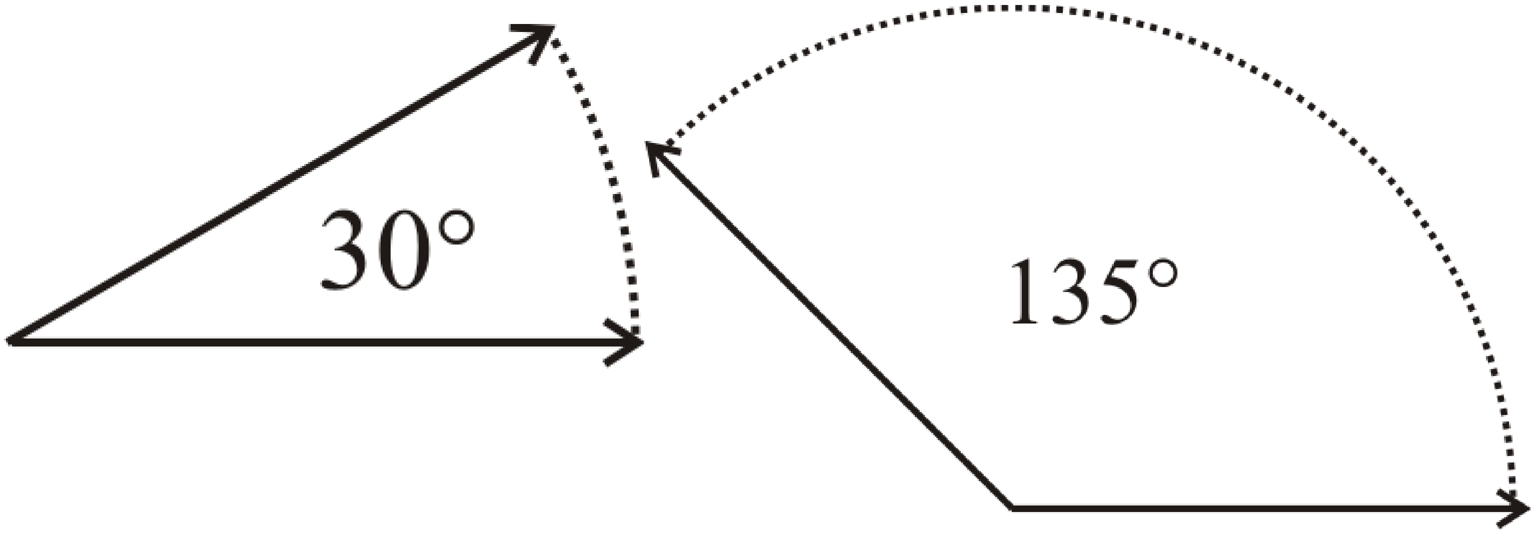
$1^\circ = 60 \text{ minutes}(60')$

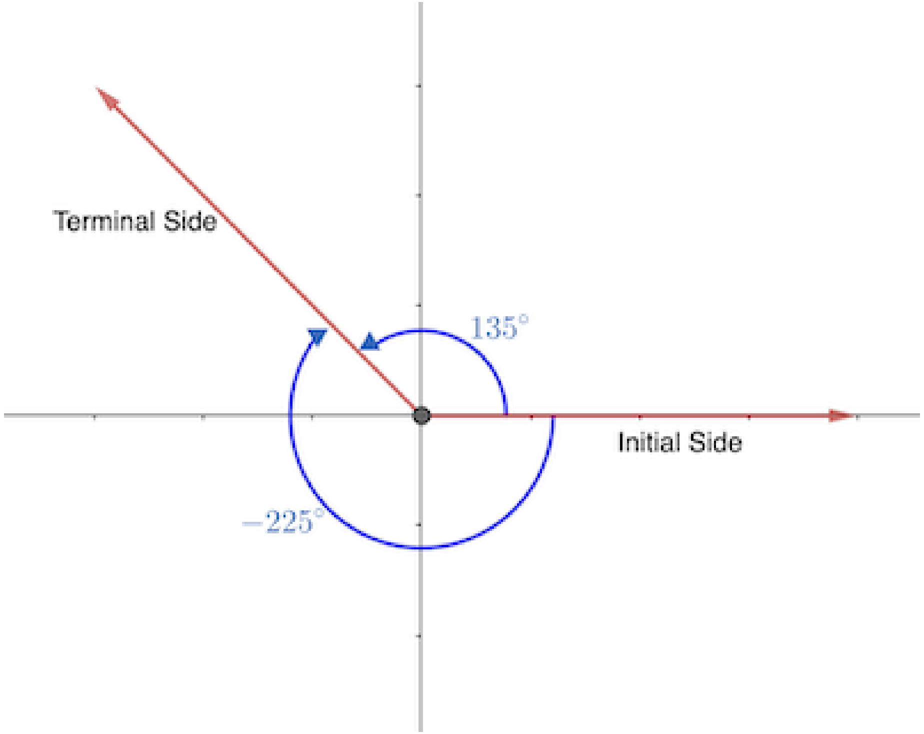
$1' = 60 \text{ seconds}(60'')$

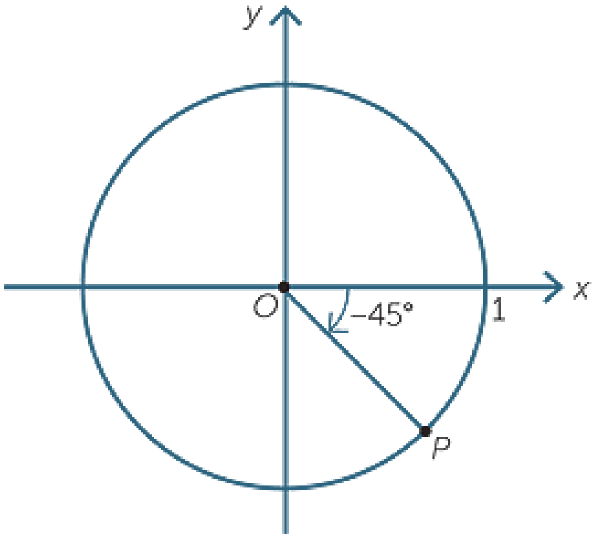

0°
A
B


0°
R
S



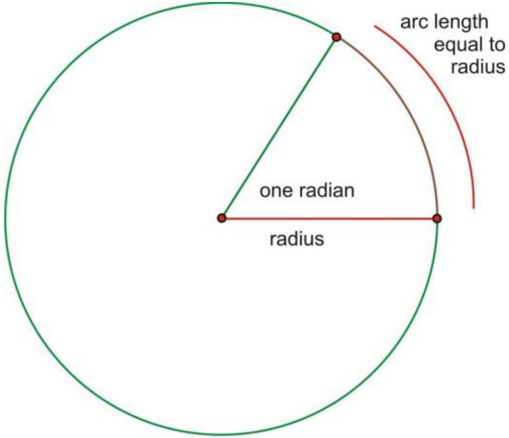

30°
135°


Terminal Side
135°
−225°
Initial Side


y
x
O
−45°
1
P

(ii) **Circular System or Radian Measure:**

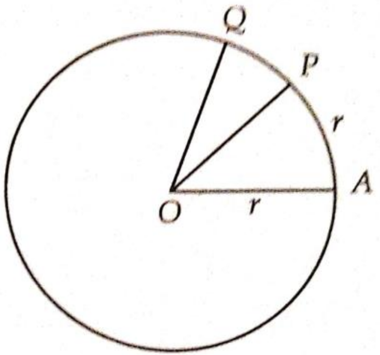
One radian, written as 1^c, is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.



Points to Remember:

- Radian is a constant angle.
- The number of radians in angle subtended by an arc of a circle at the centre is equal to $\frac{arc}{radius}$.

$i.e., \theta = \frac{l}{r}\ radians$



Relations between Degrees and Radians:

$$one\ radian = \frac{180°}{\pi} \Rightarrow \boldsymbol{\pi\ radians = 180°}.$$

$$\text{Radian measure} = \frac{\pi}{180} \times \text{Degree measure}$$

$$\text{Degree measure} = \frac{180}{\pi} \times \text{Radian measure}$$

The relation between degree measure and radian measure of some common angles are tabulated below.

Degree	$\mathbf{30°}$	$\mathbf{45°}$	$\mathbf{60°}$	$\mathbf{90°}$	$\mathbf{180°}$	$\mathbf{270°}$	$\mathbf{360°}$
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Some Points to Remember:

- All integral multiples of $\frac{\pi}{2}$ are called quadrant angles.
- When an angle is expressed in radians, the word radian is generally omitted.
- We have $1\ radian = 57^\circ\ 16'22''(approx)$ and $1^\circ = 0.01746\ radians$.
- The angle between two consecutive digits of a clock is $30^\circ (= \frac{\pi}{6} radians)$.
- The hour hand rotates through an angle of 30° in one hour $i.e.\ \left(\frac{1}{2}\right)^\circ$ in one minute.
- The minute hand rotates through an angle of 6° in one minute.

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Problems on Measurement of Angles

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Some Examples:

Example: Find the radian measure corresponding to the following degree measures:

$(i)\ 340°$ $(ii)\ 40°\ 20'$ $(iii)\ 5°37'30''$

Example: Find the degree measure corresponding to the following radian measures:

$(i)\ \left(\frac{2\pi}{15}\right)^c$ $(ii)\ \left(\frac{\pi}{8}\right)^c$ $(iii)\ 6^c$

Example: Find the length of an of a circle of radius 5 cm subtending a central angle measuring $15°$.

Example: The angles of a triangle are in the ratio $3 : 4 : 5$, find the smallest angle in degrees and the greatest angle in radians.

Example: The minute hand of a watch is 35 cm long. How far does its tip move in 18 minutes? (Use $\pi = \frac{22}{7}$)

Example: Find the angle between the minute hand of a clock and the hour hand when the time is $7 : 20\ AM$.

Example: A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 88 meters when it has traced out $72°$ at the center, find the length of the rope.

Example: The moon's distance from the earth is 360,000 kms and its diameter subtends an angle of $31'$ at the eye of the observer. Find the diameter of the moon.

Example: The wheel of a railway carriage is 40 cm in diameter and makes 6 revolutions in a second; how fast is the train going?

Example: A circular wire of radius 3 cm is cut and bent to lie along the circumference of a hoop whose radius is 48 cm. Find the angle in degrees which is subtended at the centre of the hoop.

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Trigonometric Functions

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Introduction:

There are six possible ratios among three sides of a triangle.

These six ratios are called trigonometrical ratios and defined as follows:

$$\sin\theta = \frac{PM}{OP} = \frac{y}{r}$$

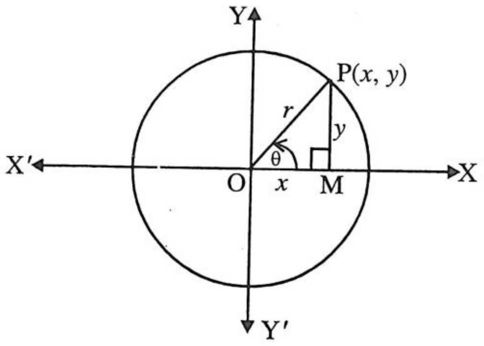
$$cos\theta = \frac{OM}{OP} = \frac{x}{r}$$

$$tan\theta = \frac{PM}{OM} = \frac{y}{x}$$

$$cot\theta = \frac{OM}{PM} = \frac{x}{y}$$

$$sec\theta = \frac{OP}{PM} = \frac{r}{x}$$

$$cosec\ \theta = \frac{OP}{PM} = \frac{r}{y}$$



The functions $sin\theta, cos\theta, tan\theta, cot\theta, sec\theta$, and $cosec\theta$ are called trigonometric functions.

We define $cosec\theta = \frac{1}{sin\theta}, sec\theta = \frac{1}{cos\theta},\ cot\theta = \frac{1}{tan\theta}$

Also, $tan\theta = \frac{sin\theta}{cos\theta}$ and $cot\theta = \frac{cos\theta}{sin\theta}$

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Trigonometric Table

θ	0°	30°	45°	60°	90°	180°	270°	360°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	0
cosec θ	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
sec θ	1	$\frac{1}{2}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1
cot θ	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined

Trigonometric Identities:

An equation involving trigonometric functions which is true for all these values of the variable for which the function is defined is called a trigonometric identity.

We have the following three identities among trigonometrical ratios:

$(i)\ sin^2\theta + cos^2\theta = 1$

$(ii)\ sec^2\theta - tan^2\theta = 1$

$(iii)\ cosec^2\theta - cot^2\theta = 1$

These are called “Pythagorean Identities”.

Signs of the Trigonometric Ratios / Functions:

In the first quadrant: We have $x > 0$ and $y > 0$.

Thus, in the first quadrant, all trigonometric ratios are positive.

In the second quadrant: We have $x < 0$ and $y > 0$.

Thus, in the second quadrant sine and cosecant functions are positive and all others are negative.

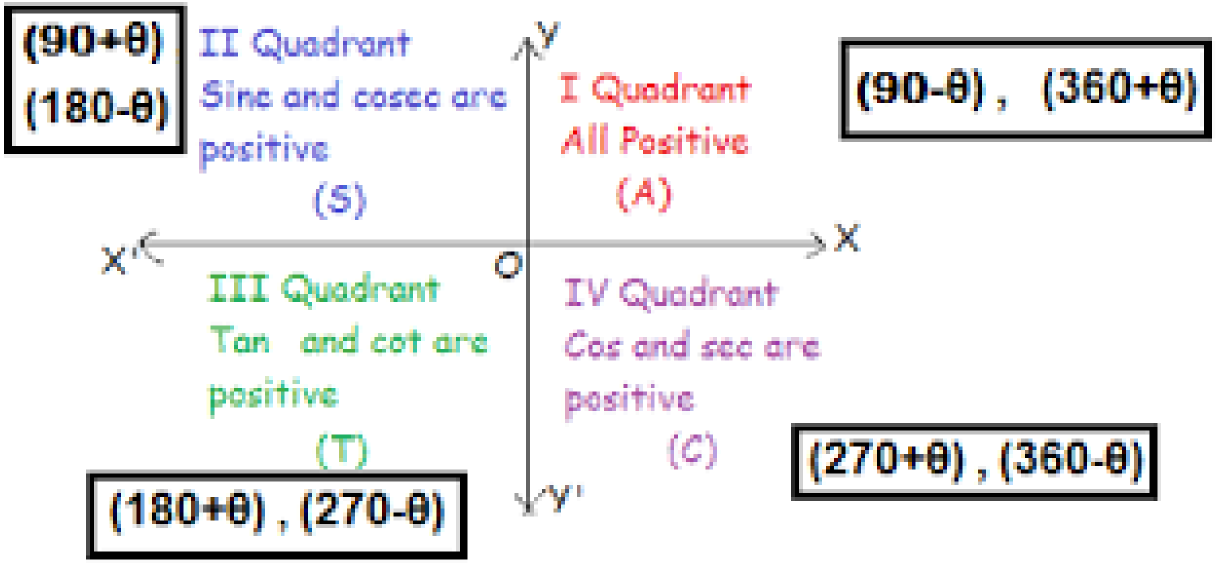
In the third quadrant: We have $x < 0$ and $y < 0$.

Thus, in the third quadrant, all trigonometric functions are negative except tangent and cotangent.

In the fourth quadrant: We have $x > 0$ and $y < 0$.

Thus, in the fourth quadrant, all trigonometric functions are negative except cosine and secant.

The above rule is known as $ASTC$ Rule.


(90+θ)
(180-θ)
II Quadrant
Sine and cosec are positive
(S)
Y
I Quadrant
All Positive
(A)
(90-θ) , (360+θ)
X'
X
O
III Quadrant
Tan and cot are positive
(T)
IV Quadrant
Cos and sec are positive
(C)
(270+θ) , (360-θ)
(180+θ) , (270-θ)
Y'


CAST / ASTC RULE
Math Dot Com
Quadrant II
Sin
All
Quadrant I
Quadrant III
Tan
Cos
Quadrant IV

Points to Remember:

Since $sin^2\theta + cos^2\theta = 1$, so, $|sin\theta| \leq 1$ and $|cos\theta| \leq 1$

$\Rightarrow$ $-1 \leq sin\theta \leq 1$ and $-1 \leq cos\theta \leq 1$

Also, $0 \leq sin^2\theta \leq 1,\ 0 \leq cos^2\theta \leq 1$

Since, $cosec\theta = \frac{1}{sin\theta}$, therefore $cosec\theta \geq 1$ or $cosec\theta \leq -1$

Since, $c\theta = \frac{1}{cos\theta}$, therefore $sec\theta \geq 1$ or $sec\theta \leq -1$

Domain and Range of Trigonometric Functions:

$sin: R \to [-1, 1]$

$cos: R \to [-1, 1]$

$tan: R - \left\{(2n+1)\frac{\pi}{2}, n \in Z\right\} \to R$

$cot: R - \{n\pi, n \in Z\} \to R$

$sec: \ R - \left\{(2n+1)\frac{\pi}{2}, n \in Z\right\} \to (-\infty, -1] \cup [1, \infty)$

$cosec: \ R - \{n\pi, n \in Z\} \to (-\infty, -1] \cup [1, \infty)$

Example: Find $sinx$ and $tanx$, if $cosx = -\frac{12}{13}$ and x lies in the third quadrant.

Example: Find all other trigonometric ratios, if $sinx = -\frac{2\sqrt{6}}{5}$ and x lies in quadrant III.

Example: If $secx = \sqrt{2}$ and $\frac{3\pi}{2} < x < 2\pi$, find the value of $\frac{1+tanx+cosecx}{1+cotx-cosecx}$.

Example: If x is any non – zero real number, show that $cos\theta$ and $sin\theta$ can never be equal to $x + \frac{1}{x}$.

Example: Prove the following. $sec^2\theta + cosec^2\theta \geq 4$.

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Trigonometric Functions of Complementary and Supplementary Angles

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Values of Trigonometric Functions at Allied Angles:

Two angles are said to be allied when their sum or difference is either zero or multiples of $\frac{\pi}{2}$.

The angles allied to θ are $-\theta, \frac{\pi}{2} \pm \theta, \pi \pm \theta, \frac{3\pi}{2} \pm \theta, 2\pi \pm \theta$, etc.

Trigonometrical Ratios of $(-\theta)$

$\sin(-\theta) = -sin\theta$

$\cos(-\theta) = cos\theta$,

$\tan(-\theta) = -tan\theta$

Similarly, $cosec(-\theta) = -cosec\theta$, $\sec(-\theta) = sec\theta$ and $\cot(-\theta) = -cot\theta$

Trigonometrical Ratios of $\frac{\pi}{2} - \theta$

$$\sin\left(\frac{\pi}{2} - \theta\right) = cos\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = cot\theta$$

Trigonometrical Ratios of $\frac{\pi}{2} + \theta$

$$\sin\left(\frac{\pi}{2} + \theta\right) = \sin\left(\frac{\pi}{2} - (-\theta)\right) = \cos(-\theta) = cos\theta$$

$$cos\left(\frac{\pi}{2} + \theta\right) = \cos\left(\frac{\pi}{2} - (-\theta)\right) = \sin(-\theta) = -sin\theta$$

$$\tan\left(\frac{\pi}{2} + \ \theta\right) = \tan\left(\frac{\pi}{2} - (-\theta)\right) = \cot(-\theta) = -cot\theta$$

Trigonometrical Ratios of $\pi \pm \theta$

$$\sin(\pi - \theta) = \sin\left(\frac{\pi}{2} + \overline{\frac{\pi}{2} - \theta}\right) = \cos\left(\frac{\pi}{2} - \theta\right) = sin\theta$$

Similarly, $\cos(\pi - \theta) = -cos\theta$ and $\tan(\pi - \theta) = -tan\theta$

Also, $\sin(\pi + \theta) = \sin\left(\frac{\pi}{2} + \overline{\frac{\pi}{2} + \theta}\right) = \cos\left(\frac{\pi}{2} + \theta\right) = -sin\theta$

Similarly, $\cos(\pi + \theta) = -cos\theta$ and $\tan(\pi + \theta) = tan\theta$

Trigonometrical Ratios of $\frac{3\pi}{2} \pm \theta$

$$\sin\left(\frac{3\pi}{2} - \theta\right) = \sin\left(\pi + \overline{\frac{\pi}{2} - \theta}\right) = -\sin\left(\frac{\pi}{2} - \theta\right) = -cos\theta$$

Similarly, $\cos\left(\frac{3\pi}{2} - \theta\right) = -sin\theta$ and $\tan\left(\frac{3\pi}{2} - \theta\right) = cot\theta$

Also, $\sin\left(\frac{3\pi}{2} + \theta\right) = \sin\left(\pi + \overline{\frac{\pi}{2} + \theta}\right) = -\sin\left(\frac{\pi}{2} + \theta\right) = -cos\theta$

Similarly, $\cos\left(\frac{3\pi}{2} + \theta\right) = sin\theta$ and $\tan\left(\frac{3\pi}{2} + \theta\right) = -cot\theta$

Trigonometrical Ratios of $2\pi \pm \theta$

$\sin(2\pi - \theta) = \sin(\pi + \overline{\pi - \theta}) = -\sin(\pi - \theta) = -sin\theta$

Similarly, $\cos(2\pi - \theta) = cos\theta$ and $\tan(2\pi - \theta) = tan\theta$

Again, $\sin(2\pi + \theta) = \sin(2\pi - (-\theta)) = -\sin(-\theta) = sin\theta$

Similarly, $\cos(2\pi + \theta) = cos\theta$ and $\tan(2\pi + \theta) = tan\theta$

Points to remember

$$(i)\ \sin\left(n\frac{\pi}{2}+\theta\right)=\begin{cases}(-1)^{\frac{n-1}{2}}\ cos\theta, if\ n\ be\ an\ odd\ integer\\ (-1)^{\frac{n}{2}}\ sin\theta\ , if\ n\ be\ an\ even\ integer\end{cases}$$

$$(ii)\ \cos\left(n\frac{\pi}{2}+\theta\right)=\begin{cases}(-1)^{\frac{n+1}{2}}\ sin\theta, if\ n\ be\ an\ odd\ integer\\ (-1)^{\frac{n}{2}}\ cos\theta\ , if\ n\ be\ an\ even\ integer\end{cases}$$

$$(iii)\ \tan\left(n\frac{\pi}{2}+\theta\right)=\begin{cases}-\ cot\theta, if\ n\ be\ an\ odd\ integer\\ tan\theta, if\ n\ be\ an\ even\ integer\end{cases}$$

Periodicity of Trigonometric Functions

(i) $sinx, cosx, secx, cosecx$ are all periodic with period 2π.

(ii) $tanx$ and $cotx$ are periodic with period π.

Even and Odd Functions

$sine$ and $tangent$ are odd functions, where $cosine$ is an even function.

Example: Evaluate the following:

$(i)\ \sin\frac{31\pi}{3}$

$(ii)\ \cos\frac{7\pi}{6}$

$(iii)\ \sin\left(-\frac{25\pi}{4}\right)$

$(iv)\ \tan\left(\frac{19\pi}{3}\right)$

Example: Evaluate the following:

$(i)\ \tan 480°$

$(ii)\ \cos(-1710°)$

Example: Prove that $cos510° \ cos330° + sin390° \ cos120° = -1$

Example: Prove that $\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\ \cos\left(\frac{\pi}{2}+x\right)} = \ cot^2x$

Example: Prove that $\frac{\cos(2\pi+x)cosec(2\pi+x)\ \tan\left(\frac{\pi}{2}+x\right)}{\sec\left(\frac{\pi}{2}+x\right)cosx\ \cot(\pi+x)} = 1.$

Example: Find the value of $\frac{\cot 54°}{\tan 36°} + \frac{\tan 20°}{\cot 70°}$.

Example: State the sign of $sin201° + cos201°$.

Example: Find the value of $sin1° \, . \, sin2° . \, sin3° \; .... \sin 200°$.

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Trigonometric Functions of Compound Angles

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Trigonometric Ratios of Compound Angles:

The algebraic sums of two or more angles are generally called compound angles and the angles are known as the constituent angles.

If A, B, C are three angles then $A \pm B, A + B + C, A - B + C$, etc. are compound angles.

Trigonometric Ratios of Sum and Difference of two angles

$$\sin(A+B) = sinA\ cosB + cosA\ sinB \text{ .... } (1)$$

$$\cos(A+B) = cosA\ cosB\ - sinA\ sinB \text{ .... } (2)$$

$$\tan(A+B) = \frac{\tan A + tanB}{1 - tanA.tanB} \text{ .... } (3)$$

Replacing B by $(-B)$ in (1), (2) and (3) we have

$$\sin(A-B) = sinA\cos(-B) + cosA\sin(-B) = sinA\ cosB - cosA\sin B$$

$$\cos(A-B) = cosA\cos(-B) - sinA\sin(-B) = cosA\ cosB + sinAsinB$$

$$\tan(A-B) = \frac{tanA + \tan(-B)}{1 - tanA\tan(-B)} = \frac{tanA\ - tanB}{1 + tanA\ tanB}$$

Similarly, $\cot(A+B) = \frac{cotA\ cotB\ -1}{cotB + cotA}$ and $\cot(A-B) = \frac{cotA\ cotB+1}{cotB - cotA}$

More Useful Result:

$(i)\sin(A+B)\sin(A-B) = sin^2A - sin^2B = cos^2B - cos^2A$

$(ii)\cos(A+B)\cos(A-B) = cos^2A - sin^2B = cos^2B - sin^2A$

$(iii)\sin(A+B+C) = sinA\ cosB\ cosC + cosA\ sinB\ cosC + cosA\ cosB\ sinC - sinAsinBsinC$

$(iv)\cos(A+B+C) = cosA\ cosBcosC - cosA\ sinB\ sinC - sinA\ cosBsinC - sinAsinBcosC$

$$(v)\tan(A+B+C) = \frac{tanA + tanB + tanC - tanA\ tanB\ tanC}{1 - tanA\ tanB - tanB\ tanC - tanC\ tanA}$$

Example: Find the value of $\sin 15°$.

Example: Prove that $\frac{\sin(x+y)}{\sin(x-y)} = \frac{tanx+tany}{tanx-tany}$

Example: Show that $tan3x\ tan2x\ tanx = tan3x - tan2x - tanx$

Example: Prove that $\tan 75° + \cot 75° = 4$.

Example: Prove that $\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2}\ cosx$

Example: If $tanA = \frac{1}{2}$ and $tanB = \frac{1}{3}$, show that $cos2A = \sin 2B$.

Example: If $tanA - tanB = x$ and $cotB - \cot A = y$, prove that $\cot(A - B) = \frac{1}{x} + \frac{1}{y}$.

Example: If $A + B = \frac{\pi}{4}$, prove that $(1 + tanA)(1 + tanB) = 2$.

Example: If angle θ is divided into two parts such that the tangent of one part is k times the tangent of other and φ is their difference, then show that $sin\theta = \frac{k+1}{k-1}\ sin\varphi$.

Maximum and Minimum Values of Trigonometrical Expressions:

The maximum and minimum values of $a\ sin\theta + b\ cos\theta$ are $\sqrt{a^2 + b^2}$ and $-\sqrt{a^2 + b^2}$

Example: Find the maximum and minimum values of $7\ cos\theta + 24\ sin\theta$.

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Transformation Formulae

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From compound angle formulae we have

$$\sin(A+B) = sinA\,cosB + cosA\,sinB \ldots (i)$$

$$\sin(A-B) = sinA\,cosB - cosA\,sinB \ldots (ii)$$

$$\cos(A+B) = cosA\,cosB - sinA\,sinB \ldots (iii)$$

$$\cos(A-B) = cosA\,cosB + sinA\,sinB \ldots (iv)$$

Adding (i) and (ii), we obtain

$\sin(A+B)+\sin(A-B)=2\ sinA\ cosB$

Subtracting (ii) from (i), we obtain

$\sin(A+B)-\sin(A-B)=2\ cosA\ sinB$

Adding (iii) and (iv), we obtain

$\cos(A+B)+\cos(A-B)=2\ cosA\ .\ cosB$

Subtracting (iii) from (iv), we obtain

$\cos(A-B)-\cos(A+B)=2\ sinA\ sinB$

In above formulae $A>B$.

Formulae to transform the sum or difference into products:

Let $A + B = C$ and $A - B = D$. Then, $A = \frac{C+D}{2}$ and $B = \frac{C-D}{2}$

Substituting the values of A, B, C and D in the above formulae, we get

$$\sin C + sinD = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$sinC - sinD = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

Formulae to transform the sum or difference into products:

$$cosC + cosD = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$cosD - cosC = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

Or, $\cos C - cosD = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

Or, $cosC - cosD = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

Example: Convert each of the following products into the sum or difference of sines and cosines.

$(i) 2\sin 5\theta\ cos\theta$

$(ii)\ cos75° \cos 15°$

Example: Express each of the following as a product.

$(i)\ \sin 6\theta\ - \sin 2\theta$

$(ii)\ \cos 6\theta\ - \cos 8\theta$

Example: Prove that $\frac{\cos 7x+\cos 5x}{sin7x -\sin 5x} = cotx$

Example: Prove that $\frac{\sin 5x-2 \sin 3x+sinx}{\cos 5x -\cos x} = tanx$

Example: Prove that $\frac{\sin 5A-sin7A+sin8A -sin4A}{cos4A+cos7A -cos5A-cos8A} = \cot 6A$

Example: Prove that $cos18° - sin18° = \sqrt{2}\, sin27°$.

Example: Prove that $sin10°\, sin30°\, sin50°\, sin70° = \frac{1}{16}$.

***Example**:* Prove that $\sin x + \sin\left(x + \frac{2\pi}{3}\right) + \sin\left(x + \frac{4\pi}{3}\right) = 0$.

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Problems on Compound Angles and Transformation Formulae

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 03
CHAPTER NAME : TRIGONOMETRIC FUNCTIONS

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Different Examples:

Example: Prove that $cos3A + \cos 5A + cos7A + cos15A = 4\, cos4A\, cos5A \cos 6A$.

Example: If $a\ sinx = b\ \sin\left(x + \frac{2\pi}{3}\right) = c \sin\left(x + \frac{4\pi}{3}\right)$, prove that $ab + bc + ca = 0$.

Example: Prove that $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \tan 54°$.

Example: Prove that $\frac{sinx}{cos3x}+\frac{sin3x}{cos9x}+\frac{sin9x}{cos27x}=\frac{1}{2}(\,tan27x-tanx)$.

Example: Prove that $cos2\alpha\cos 2\beta+\ sin^2(\alpha-\beta)-\ sin^2(\alpha+\beta)=\cos 2(\alpha+\beta)$.

Example: If α and β are the solutions of the equation $a\ tanx+b\ secx=c$, show that

$\tan(\alpha+\beta)=\frac{2ac}{a^2-c^2}$.

Example: Prove that : $\cos A \cos\left(\frac{\pi}{3} - A\right)\cos\left(\frac{\pi}{3} + A\right) = \frac{1}{4}\cos 3A.$

Note :

Similarly sinA $\sin\left(\frac{\pi}{3} - A\right)\sin\left(\frac{\pi}{3} + A\right) = \frac{1}{4}\sin 3A$

and $\tan A \tan\left(\frac{\pi}{3} - A\right)\tan\left(\frac{\pi}{3} + A\right) = \tan 3A$

Example: If three angles A, B, and C are in $A.P.$, prove that $\cot B = \frac{sinA - sinC}{\cos C - \cos A}$.

Example: If A, B, and C are the angles of a triangle, then prove that

$sin^2 A + sin^2 B + sin^2 C - 2\, cosA \cos B \cos C = 2.$

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Trigonometric Functions of Multiple and Sub multiple Angles

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 03
CHAPTER NAME : TRIGONOMETRIC FUNCTIONS

Multiple and Submultiple Angles:

If A is any angle, then angles $2A, 3A, 4A, \ldots$ etc are called multiple angles and the angles $\frac{A}{2}, \frac{A}{3}, \frac{A}{4}, \ldots$ etc are called sub - multiple angles of A.

Trigonometric Ratios of angle $2A$ in terms of that of angle A:

(i) $\sin 2A = \sin(A + A) = sinA\ cosA + cosA\ sinA = 2\ sinA\ cosA$

(ii) We have $sin2A = 2\ sinA\ cosA = \dfrac{2\ sinA\ cos^2A}{cosA} = \dfrac{2\ tanA}{sec^2A} = \dfrac{2\ tanA}{1+\ tan^2A}$

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$(iii)\ \cos 2A = \cos(A + A) = cosA\, cosA - sinA\, sinA = cos^2 A - sin^2 A$

$(iv)\ \cos 2A = cos^2 A - sin^2 A = cos^2 A - (1 - cos^2 A) = 2cos^2 A - 1$

$(v)\ \cos 2A = cos^2 A - sin^2 A = 1 - sin^2 A - sin^2 A = 1 - 2sin^2 A$

$$(vi)\ cos2A = cos^2 A - sin^2 A = \frac{cos^2 A - sin^2 A}{cos^2 A + sin^2 A} = \frac{1 - tan^2 A}{1 + tan^2 A}$$

(vii) We have $\cos 2A = 1 - 2sin^2A \Rightarrow \; sin^2A = \frac{1-cos2A}{2}$

$(viii)$ We have $cos2A = 2cos^2A - 1 \Rightarrow cos^2A = \frac{1+\cos 2A}{2}$

(ix) $\tan 2A = \tan(A + A) = \dfrac{\tan A + tanA}{1 - tanA\, tanA} = \dfrac{2\, tanA}{1 - \; tan^2A}$

(x) $tan^2\, A = \frac{1 - \cos 2A}{1 + \cos 2A}$

Trigonometric Ratios of the Angle A in terms of Angle $\frac{A}{2}$:

Replacing A by $\frac{A}{2}$ in the relations of multiple angles of $2A$, we obtain

$(i)\ sinA = 2\sin\frac{A}{2}\cos\frac{A}{2}$

$(ii)\ sinA = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

$(iii)\ cosA = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$

$(iv)\ cosA = 2\cos^2\frac{A}{2} - 1$

$(v)\ cosA = 1 - 2\sin^2\frac{A}{2}$

$(vi)\ cosA = \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

$$(vii)\ sin^2\frac{A}{2} = \frac{1-cosA}{2}$$

$$(viii)\ cos^2\frac{A}{2} = \frac{1+cosA}{2}$$

$$(ix)\ tanA = \frac{2\tan\frac{A}{2}}{1-tan^2\frac{A}{2}}$$

$$(x)\ tan^2\frac{A}{2} = \frac{1-\cos A}{1+\cos A}$$

Trigonometric Ratios of Angle $3A$ in terms of Angle A:

(i) $\sin 3A = 3\ sinA - 4sin^3 A$

$(ii)\ cos3A = 4cos^3 A\ - 3\ cosA$

$(iii)\ tan3A = \frac{3\ tanA - tan^3 A}{1-3tan^2 A}$

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Notes:

1. $$\sin\frac{A}{2}=\begin{cases}\sqrt{\frac{1-\cos A}{2}} & if\ \frac{A}{2}\ lies\ in\ quadrant\ I\ or\ II\\ -\sqrt{\frac{1-\cos A}{2}} & if\ \frac{A}{2}\ lies\ in\ quadrant\ III\ or\ IV\end{cases}$$

2. $$\cos\frac{A}{2}=\begin{cases}\sqrt{\frac{1+\cos A}{2}} & if\ \frac{A}{2}\ lies\ in\ quadrant\ \mathrm{I\ or}\ IV\\ -\sqrt{\frac{1+\cos A}{2}} & if\ \frac{A}{2}\ lies\ in\ quadrant\ \mathrm{II\ or\ III}\end{cases}$$

3. $$\tan\frac{A}{2}=\begin{cases}\sqrt{\frac{1-\cos A}{1+\cos A}} & if\ \frac{A}{2}\ lies\ in\ quadrant\ \mathrm{I\ or\ III}\\ -\sqrt{\frac{1-\cos A}{1+\cos A}} & if\ \frac{A}{2}\ lies\ in\ quadrant\ \mathrm{II\ or}\ IV\end{cases}$$

Different Examples:

Example: If $sinA = \frac{3}{5}$, where $0° < A < 90°$, find the values of $sin2A, cos2A, tan2A$.

Example: Prove that $\frac{1+sin2\theta+cos2\theta}{1+sin2\theta\ -cos2\theta} = cot\theta$.

Example: Prove that $\frac{\sin 5x\ -2\ sin3x+sinx}{\cos 5x-cosx} = tanx$.

Example: Prove that $\left(1 + \cos\frac{\pi}{8}\right)\left(\ 1 + \cos\frac{3\pi}{8}\right)(1 + \cos\frac{5\pi}{8})\ (\ 1 + \cos\frac{7\pi}{8}\ ) = \frac{1}{8}$.

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Problems on Multiple and Sub multiple Angles

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 03
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Different Examples:

Example: Prove that $cos^2x + \ cos^2\left(x + \frac{\pi}{3}\right) + \ cos^2 \left(x - \frac{\pi}{3}\right) = \frac{3}{2}$.

Example: Find the value of $\tan\frac{\pi}{8}$.

Example: If $tanx = \frac{3}{4}$ $, \pi < x < \frac{3\pi}{2}$, find the values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

Example: If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, prove that $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta)$ $= 1 - 2a^2 - 2b^2$.

Example: If $cos\theta = \frac{\cos\alpha - \cos\beta}{1 - cos\alpha \cos\beta}$, prove that $\tan\frac{\theta}{2} = \pm\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$.

Example: Prove that $sin3x\ sin^3x + \cos 3x\ cos^3x = cos^3 2x$.

Example: Show that $2\,sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$.

Example: Prove that $\sqrt{3}\,cosec\,20° - \sec 20° = 4$.

Example: Prove that $\cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$.

Example: Prove that $\frac{\tan 5x+\tan 3x}{\tan 5x-\tan 3x} = 4\cos 2x \cos 4x$.

Example: Prove that $\frac{\cos 5x+\cos 4x}{1-2\cos 3x} = -(\cos 2x + cosx)$.

Example: If $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\alpha}{2}$, prove that $\cos\alpha = \frac{\cos\theta-e}{1-e\cos\theta}$.

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