Chapter- 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1. Evaluate $1 + i^{10} + i^{100} - i^{1000}$
2. Simplify $1 + i + i^2 + i^3 + i^4 + i^5 + i^6 + i^7$
3. Find $\frac{-\sqrt{-81}}{\sqrt{-9}}$
4. Show that if n is an odd natural number then $i^n + i^{2n} + i^{3n} + i^{4n} = 0$
5. Find $(1 + i)^{1000}$.
6. Find the real values of x and y if $(3x - 2iy)(2 + i)^2 = 10(1 + i)$
7. Find the real values of x and y if $\frac{x-1}{3+i} + \frac{y-1}{3-i} = i$
8. Find the real values of x and y if $(1 + i)y^2 + (6 + i) = (2 + i)x$
9. Find the value of x and y if $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$
10. Prove that $(1 + i)^2 \left(1 + \frac{1}{i}\right)^2 = 4$
11. If $z_1 = 1 + i,\ z_2 = 2 + 3i,\ z_3 = 3 - 2i$, verify that $(z_1 - z_2)z_3 = z_1z_3 - z_2z_3$
12. Find the additive and multiplicative inverse of $-3 + \sqrt{7}i$
13. Multiply $(i + i)$ by $(a - bi)$.
14. If $z_1 = 1 + i,\ z_2 = 2 - 3i,\ z_3 = -4 - 5i$ then verify that $z_1(z_2 + z_3) = z_1z_2 + z_1z_3$.
15. Prove that the complex number $\left(\frac{2+3i}{3+4i}\right)\left(\frac{2-3i}{3-4i}\right)$ is purely real.
16. Express in the form of $x + iy$: $\frac{3+2i}{5-3i}$
17. Express in the form of $x + iy$: $\left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$
18. Find the conjugate of $(\sqrt{7} + 5i)(\sqrt{7} - 5i)^2$
19. Show that the moduli of the complex numbers $3 + 4i,\ 2\sqrt{6} - i, -\sqrt{15} - 10i$ are equal.
20. Prove that $|z| = |-z| = |\bar{z}|$
21. Find the modulus and argument of $-\sqrt{3} + i$
22. Find the polar representation of $-1 + \sqrt{3}i$
23. Find x if $(1 - i)^x = 2^x$
24. Find the polar representation of $2 + 2i$
25. If $z = x + iy$ and $|2z - 1| = |z + 1|$, show that $x^2 + y^2 = 2x$.
26. Find the square roots of $-5 + 12i$.
27. Find the square roots of $(1 + i)(3 + 2i)$.
28. Find the square roots of $\frac{1+2i}{1-2i}$
29. Find the square roots of $1 - i\sqrt{a^2 - 1}$

30. Find the square roots of $a^2 + \frac{1}{a^2} - 4i\left(a - \frac{1}{a}\right) - 6$
31. Solve the equation $9x^2 - 12x + 20 = 0$ by factorization method.
32. Solve the equation $x^2 - 5ix - 6 = 0$ by factorization method.
33. Solve: $x^2 - \sqrt{2}\,ix + 12 = 0$.
34. Solve: $6x^2 - 17ix - 12 = 0$.
35. Solve: $x^2 - (2 + i)x - (1 - 7i) = 0$.