Chapter – 13

Limit and Derivatives

01. $\lim_{x\to 2}\frac{x^2-3x+2}{x^2+x-6}$

02. $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

03. $\lim_{x\to 1}\frac{x^3+1}{x^5+1}$

04. $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$

05. $\lim_{x\to 0}\frac{(1+x)^n-1}{x}$

06. $\lim_{x\to b}\frac{(x+3)^{5/2}-(b+3)^{5/2}}{x-b}$

07. $\lim_{x\to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right)$

08. $\lim_{x\to 1}\frac{x^4-\sqrt{x}}{\sqrt{x}-1}$

09. If $\lim_{x\to(-a)}\frac{x^7+a^7}{x+a}=7$ then find the value of a

10. If $\lim_{x\to b}\frac{x^3-b^3}{x-b}=\lim_{x\to 1}\frac{x^4-1}{x-1}$, find all possible values or b.

11. $\lim_{x\to 0}\frac{\sin 3x}{\sin 7x}$

12. $\lim_{x\to 0}\frac{\sin^2 5x}{x^2}$

13. $\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$

14. $\lim_{x\to \pi/2}\frac{\tan 2x}{x-\frac{\pi}{2}}$

15. $\lim_{x\to \pi/2}\left(\frac{\pi}{2}-x\right)\tan x$

16. $\lim_{x\to 0}\frac{\sin^2 2x}{\sin^2 4x}$

17. $\lim_{x\to 0}\frac{1-\cos 5x}{x}$

18. $\lim_{x\to \pi/2}\frac{\cos^2 x}{1-\sin x}$

19. $\lim_{x\to 0}\frac{\tan 2x - x}{3x-\sin x}$

20. $\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$

21. $\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}$

22. $\lim_{x\to 0}\frac{\operatorname{cosec} x - \cot x}{x}$

23. $\lim_{x\to 0}\frac{(\sin 3x+\sin 5x)}{(\sin 6x-\sin 4x)}$

24. $\lim_{x\to a}\frac{\sin\sqrt{x}-\sin\sqrt{a}}{x-a}$

25. $\lim_{x\to a}\frac{\cos x-\cos a}{\sqrt{x}-\sqrt{a}}$

26. $\lim_{x\to 0}\frac{e^{bx}-1}{x}$

27. $\lim_{x\to 0}\frac{\log(1+cx)}{x}$

28. $\lim_{x\to 0}\frac{e^{x}-e^{-x}}{x}$

29. $\lim_{x\to 0}\left(\frac{e^{x}-e^{4}}{x-4}\right)$

30. $\lim_{x\to 0}\frac{a^{x}-a^{-x}}{x}$

31. $\lim_{x\to 0}\frac{e^{3x}-e^{2x}}{x}$

32. $\lim_{x\to 1}\frac{x-1}{\log_e x}$

33. $\lim\limits_{x\to 3}\dfrac{\log x-\log 5}{x-5}$

34. $\lim\limits_{x\to 0}\dfrac{\log(5+x)-\log(5-x)}{x}$

35. $\lim\limits_{x\to 0}\dfrac{3^x-2^x}{\tan x}$

36. $\lim\limits_{x\to 0}\dfrac{\log(1+x^2)}{\sin^3 x}$

37. Suppose the function is defined by $f(x)=\begin{cases}\dfrac{|x-3|}{x-3}, & x\neq 3\\ 0, & x=3\end{cases}$

(a) Find the left-hand limit of $f(x)$ at $x=3$ (b) Find the right-hand limit of $f(x)$ at $x=3$

38. Find the left-hand limit and right-hand limit of the greatest integer function $f(x)=[x]=$ greatest integer less than or equal to x at x = k where k is an integer. Also, shown that $\lim\limits_{x\to k}f(x)$ does not exist.

39. $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sqrt{1-\cos 2x}}$

40. Evaluate the left hand and right-hand limits of the following functions at x = 2, $f(x)=\begin{cases}2x+3, \text{if} & x\le 2\\ x+5, \text{if} & x>2\end{cases}$. Does $\lim\limits_{x\to 2}f(x)$ exist?

41. Evaluate the left hand and right-hand limits of the function defined by $f(x)=\begin{cases}1+x^2, & \text{if } 0\le x\le 1\\ 2-x^2, & \text{if } x>1\end{cases}$ at $x=1$. Also, show that $\lim\limits_{x\to 1}f(x)$ does not exist.

42. For what integers m and n does $\lim\limits_{x\to 0}f(x)$ and $\lim\limits_{x\to 1}f(x)$ exist, if $f(x)=\begin{cases}mx^2+n, & x<0\\ nx+m, & 0\le x\le 1\\ nx^2+m, & x>1\end{cases}$

43. If $f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$ for what values of a does $\lim\limits_{x\to a}f(x)$ exist?

44. Find the derivative of the following functions using the 1st principle of the derivative.

(a) x^3-27 (b) $\sqrt{4-x}$ (c) $\cos\sqrt{x}$ (d) $\operatorname{cosec} x$

(e) $\sin(2x-3)$ (f) $\sqrt{\sin x}$ (g) $x\cos x$ (h) e^{2x}

(i) $\log x$ (j) xe^x

45. Find $f'(x)$ if $f(x)=(x-2)^2(2x-3)$

46. Find the derivative of the following functions

(a) $\dfrac{x^4+x^3+x^2+1}{x}$ (b) $\dfrac{x^2+x+1}{\sqrt{x}}$ (c) $(5x^3+3x-1)(x-1)$

(d) $x^{-4}(3-4x^{-5})$ (e) $\dfrac{2}{x+1}-\dfrac{x^2}{3x-1}$ (f) $\dfrac{3x+4}{5x^2-7x+9}$

47. Find the derivative of $x^n+ax^{n-1}+a^2x^{n-2}+....+a^{n-1}x+a^n$ for some fixed real number a.

48. If $f(x)=1-x+x^2-x^3+.....+x^{99}+x^{100}$, then find $f'(1)$

49. If $f(x)=x^{100}+x^{99}+.....+x+1$, then find $f'(1)$

50. Find the derivative of the following functions.

(a) $5\sec x+4\cos x$ (b) $2\tan x-7\sec x$ (c) $(\sec x-1)(\sec x+1)$

(d) $(ax^2+\cot x)(p+q\cos x)$ (e) $\sin(x+a)$

51. Find the derivative of the following functions

(a) $(x+\cos x)(x-\tan x)$ (b) $x^4(5\sin x-3\cos x)$ (c) $\dfrac{\sec x-1}{\sec x+1}$ (d) $\dfrac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$

(e) $\dfrac{x}{1+\tan x}$ (f) $(3x+5)(1+\tan x)$ (g) $\dfrac{x^5-\cos x}{\sin x}$ (h) $\dfrac{a+b\sin x}{c+d\cos x}$

(i) $\dfrac{a}{x^4}-\dfrac{b}{x^2}+\cos x$ (j) $\dfrac{\cos x}{1+\sin x}$ (k) $\dfrac{\cos(x-a)}{\cos x}$

52. Find the derivative of $\dfrac{1-\cos x}{1+\cos x}$

53. Find the derivative of $x^2\sin x+\cos 2x$