Chapter- 2

Relations and Functions

1. If $(x + 3, 4 - y) = (1, 7)$ then $(x - 3, 4 + y)$ is equal to

   a) $(-2, -3)$ b) $(-5, 1)$ c) $(3, 4)$ d) $(1,5)$

2. If $A = \{1, 2, 4\}, B = \{2, 4, 5\}, C = \{2, 5\}$ then $(A - C) \times (B - C)$ is equal to

   a) $\{(1, 4)\}$ b) $\{(1,4), (4, 4)\}$ c) $\{(4, 1), (4, 4)\}$ d) $\{(1,2), (2, 5)\}$

3. Let $A = \{x: x^2 - 5x + 6 = 0\}, B = \{2, 4\}, C = \{4, 5\}$ then $A \times (B \cap C)$ is

   a) $\{(2, 4), (3, 4)\}$
   b) $\{(4.2), (4, 3)\}$
   c) $\{(2, 4), (3, 4), (4, 4)\}$
   d) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

4. If $A \times B = \{(a, 1), (a, 5), (a, 2), (b, 2), (b, 5), (b, 1)\}$, find A, B, and $B \times A$

5. Let A and B be two sets, such that $A \times B$ consists of 6 elements. If three elements of $A \times B$ are $(1, 4), (2, 6), (3, 6)$. Find $A \times B$ and $B \times A$.

6. Let $A = \{a, b\}$ and $B = \{c, d\}$. Write down all subsets of $A \times B$

7. If $A = \{2, 4, 6\}$ and $B = \{1, 4\}$, show that $A \times B \neq B \times A$

8. Let $A = \{1, 2\}, B = \{1, 2, 3, 4\}, C = \{4, 5\}, D = \{5, 6, 7, 8\}$. Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ and $A \times C \subseteq B \times D$

9. Let A and B be two sets such that $|A| = 3$ and $|B| = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A, B, and $A \times B$ where x, y, z are distinct elements.

10. If $A = \{1, 2, 3\}, B = \{3, 4\}$ and C = $\{4, 5, 6\}$, find $(A \times B) \cap (B \times C)$.

11. Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by $\{(a, b): a, b \in A \text{ and } a \text{ divides } b\}$. Write R in roster form.

12. The relation R is defined as $R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$. Write R in roster form.

13. Let $A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by $R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}, x \in A \text{ and } y \in B\}$. Write R in roster form.

14. Let $A = \{1, 2, 3, 4\}$ and R is a relation in set A given by

$R = \{(a, \ b) \in A \times A : a \ is \ a \ multiple \ of \ b\}$. Represent the relation R explicitly. Express the relation in the pictorial diagram.

15. Determine the domain and range of the relation $R = \left\{ \left(x + 4, \ \frac{2+x}{2-x} \right), 4 \leq x \leq 6, \ x \in N \right\}$

16. Determine the domain and range of the relation R, where $R = \{(x, x^3): x \ \text{is a prime}$ number less than 10}.

17. If a set A has m elements and another set B has n elements, what is the number of relations from B to A?

18. Sets A and B have respectively m and n elements. The total number of relations from A to B is 64. If $m < n$ and $m \neq 1$, write the values of m and n respectively.

19. Determine the domain and range of the relation $R = \left\{ \left(x + 4, \ \frac{2+x}{2-x} \right), 4 \leq x \leq 6, \ x \in N \right\}$

20. If $R = \{(x, y): x, y \in N, x < 5 \ and \ y = 3\}$ is a relation, then find domain and range of R.

21. Is the following relation a function? $R = \{(2, 3), \left(\frac{1}{2}, 0 \right), (2, 7), (-4, 6)\}$

22. Is the following relation a function? $R = \{ (x, |x|) : x \ \text{is a real number} \}$

23. Let $f: R \ \to R$ be given by $f(x) = x^2 + 3$. Find $\{x : f(x) = 28\}$.

24. If $f(x) = 3x^4 - 5x^2 + 9$, find $f(x - 1)$.

25. If $f(x) = \frac{1}{x+1}, x \neq -1$, then find $f(f(x))$.

26. Find the domain and the range of the real function f defined by $f(x) = \sqrt{x - 1}$.

27. Find the domain of the function $f(x) = \sqrt{x - 1} + \sqrt{2 - x}$

28. Find the range of the function $f(x) = 1 - |x - 1|$.

29. Find the domain of the function $f(x) = [x] + x$

30. Find the domain and range of $f(x) = \frac{x-2}{x-1}$

31. Draw the graph of $f(x) = |x - 2|, x \in R$. What are the domain and range of $f(x) = |x - 2|$.

32. If $f: R \rightarrow R$ is defined by $f(x) = [x]$, the greatest integer function, find its domain, range, and draw its graph.

33. Find domain and range of the real function $f(x)$, defined by $f(x) = \begin{cases} 1 - x, & x < 0 \\ 1, & x = 0 \\ x - 1, & x > 0 \end{cases}$ and

draw its graph.

34. Find the values of x for which the functions $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal.

35. Let f and g be real functions defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$. For what real

numbers, $f(x) < g(x)$?

36. Let $f, g : R \rightarrow R$ be defined respectively by $f(x) = x + 1$ and $g(x) = 2x - 3$.

Find $f + g, f - g, fg$.

37. Let f and g be two functions given by $f = \{(2, 4), (5, 6), (8, -1), (10, -3)\}$ and

$g = \{(2, 5), (7, 1), (8, 4), (10, 13), (11, -5)\}$ then find the domain of $f + g$.

38. If $f(x) = x^2$, find $\frac{f(1.1) - f(1)}{1.1 - 1}$

39. Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions. Find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x), \left(\frac{f}{g}\right)(x)$.

40. Find the domain and range of the function f defined by $f(x) = \frac{1}{\sqrt{9 - x^2}}$

41. Find the domain of $\log_e (x + 1)$.

42. Find the domain of $e^x + 2^x$.