Chapter – 9
SEQUENCES AND SERIES

01. Show that the sequence a_n defined by $a_n = 2n^2 + 1$ is not an AP

02. The nth term of a sequence is $3n - 2$. Is the sequence an AP? If it is an AP, then find its 6th term.

03. Write down the next term of the sequence $\dfrac{1}{6}, \dfrac{1}{3}, \dfrac{1}{2}$..........

04. How many terms are there in AP 20, 25, 30…….., 100?

05. Which term of the sequence 72, 70, 68, 66,…… is 40?

06. Which term of the sequence $20, 19\dfrac{1}{4}, 18\dfrac{1}{2}, 17\dfrac{3}{4}$.......... is the first negative term?

07. The product of three numbers in AP is 224, and the largest number is 7 times the smallest. Find the numbers.

08. If $a\left(\dfrac{1}{b} + \dfrac{1}{c}\right), b\left(\dfrac{1}{c} + \dfrac{1}{a}\right), c\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$ are AP, prove that a, b, c are in AP

09. If the first term if an AP is a and the sum of the first p terms is zero, then find the sum of its next q terms.

10. Find the sum to n terms of the sequence $\log a, \log ar, \log ar^2$.....

11. If the first term of an AP is 100 and the sum of the first six terms is five times the sum of the next six terms, then find the common difference.

12. How many terms of the AP $-6, -\dfrac{11}{2}, -5$...... are needed to give sum -25?

13. A man accepts a position with an initial salary of Rs. 5200 per month. It is understood that he will receive an automatic increase of Rs. 320 in the very next month and each month thereafter.
(a) Find his salary for the tenth month
(b) What are his total earnings during the first year?

14. The digit of a three-digit number is in AP and their sum is 21. The number obtained by reversing the digit is 396 less than the original number. Find the number.

15. Insert 6 arithmetic means between 3 and 24.

16. If the AM between pth and qth terms of an AP be equal to the AM between rth and sth terms of the AP, then show that $p + q = r + s$

17. If, $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is AM between a and b, then find the value of n.

18. If a, and b are the roots of $x^2 - 3x + p = 0$ and c and d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a GP prove that $(q + p) : (q - p) = 17 : 15$.

19. If the pth and qth terms of a GP are q and p respectively, then show that its $(p+q)$th term is

$$\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}.$$

20. Find the 8th term from the end of sequence 3, 6, 12,....... 25th term

21. Find four numbers forming a GP in which the third term is greater than the first term by 9 and the second term is greater than 4th by 18.

22. Find four numbers in GP, whose sum is 85 and the product is 4096.

23. If a, b, c are respectively the pth, qth and rth terms of a GP, show that

$$(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$$

24. Find the sum of the series $2+6+18+54+....+4374$

25. The sum of the first three terms of a GP is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio, and the sum to n terms of the GP.

26. Let S be the sum, P be the product R be the sum of reciprocals of n terms in a GP. Prove that $P^2 R^n = S^n$.

27. Find the sum of the series $9^{1/3}.9^{1/9}.9^{1/7}........\infty$

28. Find the sum of the series $4+44+444+.......$n terms

29. If the arithmetic mean and geometric mean between two numbers is 5 and 4 respectively, then find the two numbers.

30. Find the minimum value $4^x + 4^{1-x}, x \in R$.

31. If A and G be AM and GM respectively, between two positive numbers, then prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$

32. If the 4th, 10th and 16th terms of a GP are x, y, and z respectively, then prove that x, y, z are in GP

33. If a, b, c, d are in GP, then prove that $a^n + b^n, b^n + c^n, c^n + d^n$ are in GP

34. Prove that the product of n GMs between any two positive numbers is equal to the nth power of the GM between them.

35. Find the sum of the n terms of a series $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + ...$

36. Find the sum of the series $5^2 + 6^2 + 7^2 + ...+ 20^2$

37. If S_1, S_2, S_3 are the sum of first n natural numbers, their square and their cubes respectively, show that $9S_2^2 = S_3(1 + 8S_1)$.

38. Find the sum of n terms of series $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + ...$

39. Find the sum of the following series up to n terms $\dfrac{1^3}{1} + \dfrac{1^3+2^3}{1+3} + \dfrac{1^3+2^3+3^3}{1+3+5} + \dots\dots$