Chapter- 4

MOTION IN A PLANE

1 Mark Questions:

1. State for each of the following physical quantities if it is a scalar or a vector: volume, mass, work, velocity, acceleration, force, speed.

2. Read each statement below carefully and state with reasons, if it is true or false:

(a) The magnitude of a vector is always a scalar.

(b) each component of a vector is always a scalar.

(c) the total path length is always equal to the magnitude of the displacement vector of a particle.

(d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of the average velocity of the particle over the same interval of time.

(e) Three vectors not lying in a plane can never add up to give a null vector.

3. On open ground, a motorist follows a track that turns to his left by an angle of 30^0 after every 500m. Starting from a given turn, specify the displacement of the motorist at the third, sixth, and eight turns. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

4. Are the magnitude and direction of $\vec{A}+\vec{B}$ the same as that of $\vec{A}-\vec{B}$.

5. Can two vectors of different magnitudes be combined to give zero resultant?

6. Can $\vec{A}+\vec{B}=\vec{A}-\vec{B}$? If yes under what condition?

7. Under what condition the sum of three vectors is zero?

8. What is the maximum and minimum value $\left|\vec{a}-\vec{b}\right|$?

9. Can the magnitude of the rectangular component of a vector be greater than the magnitude of that vector?

10. If $\vec{a}=2\left(\hat{i}+\sqrt{3}\hat{j}\right)$, what is the angle made by $\vec{a}$ with $x-axis$?

11. What is the magnitude and direction of the vectors $\hat{i}+\hat{j}$ & $\hat{i}-\hat{j}$

12. If $\vec{A}=\frac{1}{4}\hat{i}-\frac{1}{3}\hat{j}+c\hat{k}$ is a unit vector, what is the value of c?

13. A particle starts from the origin at $t=0\,s$ with a velocity of $10.0\hat{j}\,m/s$ and moves in the x-y plane with a constant acceleration of $\left(8.0\hat{i}+2.0\hat{j}\right)m.s^{-2}$. (a) At what time is the x-coordinate of the particle16 m? What is the y-coordinate of the particle at that time? (b) What is the speed of the particle at the time?

14. If the magnitudes of two vectors are 3N, 8N, and their dot product is 12N, find the angle between the two vectors.

15. What are the components of a vector $A=2\hat{i}+3\hat{j}$ along with the directions of $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

16. Find the angle between the force $\vec{F}=\left(3\hat{i}+4\hat{j}-5\hat{k}\right)$ unit and the displacement $\vec{d}=\left(5\hat{i}+4\hat{j}+3\hat{k}\right)$ unit. Also, find the projection $\vec{F}\,on\,\vec{d}$.

17. For what value of x, the vector $\vec{A}=x\hat{i}+2\hat{j}-\hat{k}$ is perpendicular to $\vec{B}=3\hat{i}-\hat{j}+4\hat{k}$?

18. If $\left|\vec{A}\right| = 2, \left|\vec{B}\right| = 5,$ and the angle between $\vec{A}, \vec{B}$, 30^0 find the value of $\left|\vec{A} \times \vec{B}\right|$.

19. If $\left|\vec{A} \times \vec{B}\right| = \vec{A}.\vec{B}$, what is the angle between $\vec{A}$ and $\vec{B}$?

20. What is the angle between $\left(\vec{A} + \vec{B}\right)$ and $\left(\vec{A} \times \vec{B}\right)$?

21. If $\left|\vec{A}\right| = 2, \left|\vec{B}\right| = 5, \left|\vec{A} \times \vec{B}\right| = 8$, find the value of $\left|\vec{A}.\vec{B}\right|$.

22. Find the value of $\hat{i}.\hat{j}$ & $\hat{i} \times \hat{i}$

23. If $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$, find $|\vec{A} + \vec{B}|$.

24. Why does a projectile fire along the horizontal not follow a straight line path?

25. A body is thrown with a velocity of 9.8 m/s making an angle of 30^0 with the horizontal. After what time will it hit the ground?

26. What will be the effect on the horizontal range of a projectile when its initial velocity is doubled, keeping the angle of projection the same?

27. A hunter aims his gun on a monkey hanging from a branch of a tree. The instant the hunter fires, the monkey leaves the branch. Will the bullet hit the monkey?

28. Shells of different masses are fired from a cannon at the same angle at the same speed. Will their range and time of flight differ?

(a) w.r.t train (b) w.r.t ground

29. A ball is dropped from the window of a train moving along horizontal rails. Along what path the ball travels to reach the ground as viewed by a person standing on the ground ?

30. A projectile is fired with kinetic energy K. If the range is maximum, what is its KE at the highest point?

31. What is the angle between velocity and acceleration vector in uniform circular motion?

32. How an acceleration must act on a body to make it move with a constant speed along a circular path?

2 Marks Questions:

33. A man can swim with a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

34. Prove that when the angle between two vectors of equal magnitude is $\frac{2\pi}{3}$ the magnitude of their resultant is equal to either of them.

35. Find the angle made by the vector $2\hat{i}+3\hat{j}$ with X-axis.

36. Find the value of m such that $\vec{A}=m\hat{i}-2\hat{j}+\hat{k}$ and $\vec{B}=2m\hat{i}+m\hat{j}-4\hat{k}$ are perpendicular.

37. Prove that vectors $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$ and $2\hat{i}-\hat{j}$ are perpendicular to each other.

38. If two vectors $18\hat{i}+4\hat{j}+5\hat{k}$ and $9\hat{i}+2\hat{j}+c\hat{k}$ are parallel, find the value of c.

39. If the maximum range of a gun along horizontal is 16 km, then find the muzzle velocity of the shell.

40. A body is projected with a speed v at an angle θ with the horizontal to have a maximum range. Find the velocity at the highest point.

41. Prove that the maximum horizontal range is four times the maximum height attained by the projectile when fired at an inclination to have a maximum horizontal range.

42. A particle is projected with a velocity 'u' so that its horizontal range is thrice the greatest height attained. What is its horizontal range?

43. Which is greater: the angular velocity of the hour hand of a watch or angular velocity of the earth around its axis? Calculate.

3 Marks Questions:

44 Derive an expression for the acceleration of a body moving in a circular path of radius r with uniform speed 'v'.

45. Rain is falling vertically with a speed of 35 ms^{-1}. Winds start blowing after some time with a speed of 12 ms^{-1} in the east to west direction. In which direction should a boy wait at a bus stop holding his umbrella?

46. The position vector of a particle at the time t is given by $\vec{r} = 3t\,\hat{i} + 2t^2\,\hat{j} + 5\,\hat{k}$ where t is seconds and r is in the meter. Find the magnitude and direction of velocity and acceleration at the time t = 1s.

47. A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 ms^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take g = 9.8 ms^{-2})

48. A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired, for the shell with muzzle speed 600 ms^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit?

(Take g = 10 ms^{-2})

49. The ceiling of a long hall is 25m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall?

50. A particle starts from the origin at t = 0 with a velocity $5\hat{i}$ m/s and moves in x - y plane under the action of a force which produces a constant acceleration of $(3\hat{i}+2\hat{j})$ m/s^2.

(a) What is the y-coordinate of the particle at the instant its x-coordinate is 84m?

(b) What is the speed of the particle at this time?

51. A projectile is thrown with a velocity of 200 ms^{-1} in the upward direction making an angle of 60^0 with the horizontal. Find the time after which the inclination is 30^0 with horizontal.

52. A plane is flying horizontally at a height of 1000 m with a velocity of 100 ms^{-1} when a bomb is released from it, find

(a) time is taken by it to reach the ground.

(b) the velocity with which the bomb hits the target on the ground.

(c) the distance of the target.

53. A hill of 500m high. Supplies are to be set across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The cannon is located at a distance of 800m from the foot of the hill and can be moved on the ground at a speed of 2m/s; so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reaches on the ground across the hill? (Take g = 10 m/s^2).

54. (a) Using the cross product of two vectors. Find the area of a triangle and rectangle.

(b) Find a unit vector that is perpendicular to both $\vec{A}$ *and* $\vec{B}$, where $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$.

5 marks questions:

55. A body is projected with velocity v_0 at an angle with the vertical. Derive an equation for the trajectory of the body. Calculate :

(i) time of flight (ii) maximum height (iii) range of the projectile

56. State parallelogram law of vector additions. Derive the expression of the magnitude and direction of the resultant vector. A man is going due east with a velocity of 3 km/h. Rain falls vertically downwards of a speed of 10 km/h. Calculate the relative velocity of rain with respect to man, and the angle at which he should hold his umbrella to save himself from rain.