Chapter- 5

Laws of Motion

Force:

- The concept of force is used to explain mutual interaction between two material bodies as the action of one body on another in form of push or pull, which brings out or tries to bring out a change in the state of motion of the two bodies.
- A mutual interaction between two bodies, which creates a force on one body, also creates a force on the other body. Force on the body under study is known as ***action*** and the force applied by this body on the other is known as a ***reaction***.

Contact and Field Forces:

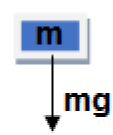
- When a body applies force on others by direct contact, the force is known as contact force. E.g.: Friction, Normal contact force, tension etc.
- When two bodies apply force on each other without any contact between them, the force is known as field force. E.g. : gravitational force, electrostatic force etc.

Basic Characteristics of a Force:

- Force is a vector quantity therefore has magnitude as well as direction.
- To predict how a force affects the motion of a body we must know its magnitude, direction and point on the body where the force is applied. This point is known as a ***point of application*** of the force.
- The direction and the point of application of a force both decide ***the line of action*** of the force.
- Magnitude and direction decide effect on translational motion and magnitude and line of action decides effects on rotational motion.

Some examples of forces :

1. Weight :



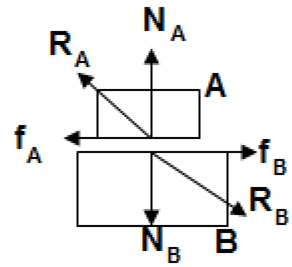
- It is the force with which earth attracts a body towards its centre.
- It is given by ;

W = mg

Where m = mass of the body and g = acceleration due to gravity

2. Contact forces :

When two bodies are in contact then each one exerts a contact force on the other. Contact force has two components :



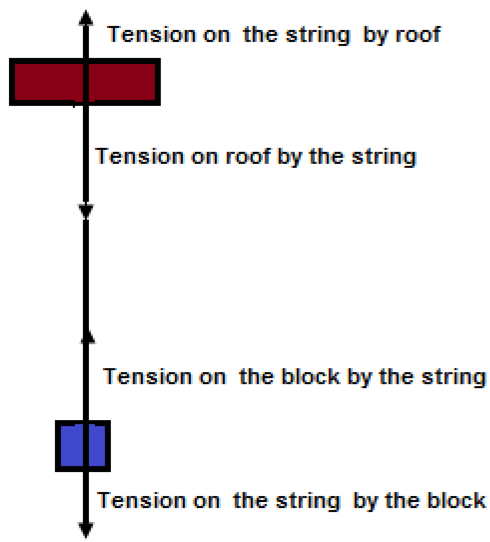
a. **Normal contact force(N):** It is the contact force along the normal to the surface of contact. It measures how strongly the surfaces in contact with each other are pressed together.

b. **Friction (f):** It is the component of the contact force along the tangent to the surface of contact. It opposes the relative motion (any attempted motion) between the surfaces in contact.

The magnitude of resulting contact force (R) is ;

$$R = \sqrt{N^2 + f^2}$$

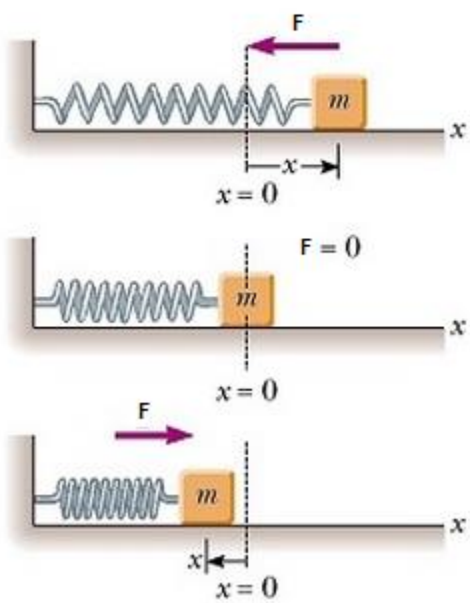
3. **Tension:** The force exerted by a taut string, chain or rope is called tension. The direction of tension is to pull the body where the normal reaction is to push the body.



4. **Spring force:** When a spring is compressed or elongated then it exerts a spring force on the body attached to its free end opposite to the deformation. **If displacement in spring is x then the spring force is**

$\vec{F} = -k\vec{x}$,

where k = spring constant.

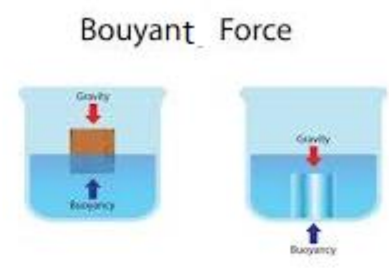


5. Buoyant force :

- When a body is dipped fully or partially in a fluid it experiences an upward Buoyant force (B).
- Its magnitude is given by, $B = V\rho g$

  Where V = volume of the fluid displaced by the portion of the body submerged in the liquid

  $\rho =$ the density of the fluid



- If the body is completely dipped in the fluid then

$$V = \text{volume of the body} = \frac{\text{its mass (m)}}{\text{its density } (\sigma)}$$

$$\Rightarrow B = \frac{m}{\sigma}\rho g$$

$$\Rightarrow B = mg\left(\frac{\rho}{\sigma}\right)$$

6. Viscous force: It is the resistance force in the opposite to the direction of motion of a body in a fluid. It is proportional to the speed of the body. Air resistance is an example of this force .

Newton's Laws of Motion:

Newton has published three laws, which describe how forces affect the motion of a body on which they act. These laws are fundamental in the sense that the first law gives a concept of force, inertia and the inertial frames; the second law defines force and the third law action and reaction as two aspects of mutual interaction between two bodies.

Newton's First Law:

- **Statement:** Everybody continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.
- Hence, if the net external force on a body is zero, its acceleration is zero. Then the body continues its state of rest or uniform motion.
- Acceleration can be non-zero only if there is a net external force on the body.
- For example, a spaceship out in interstellar space, far from all other objects and with all its rockets turned off, has no net external force acting on it. Its acceleration, according to the first law, must be zero. If it is in motion, it must continue to move with a uniform velocity.
- First law gives the qualitative definition of force given as, ***" Force is defined as an external agent which acting on a body can change the state of motion of the body"***

Question 1: Suppose we are standing on a stationary bus and the driver starts the bus suddenly. We get thrown backwards with a jerk. Why?

Answer :

- Our feet are in touch with the floor of the bus.

- If there were no friction, we would remain where we were, while the floor of the bus would simply slip forward under our feet and the back of the bus would hit us. However, fortunately, there is some friction between the feet and the floor.
- If the start is not too sudden, i.e. if the acceleration is moderate, the frictional force would be enough to accelerate our feet along with the bus. But our body is not strictly rigid. It is deformable, i.e. it allows some relative displacement between different parts. What this means is that while our feet go on the bus, the rest of the body remains where it is due to inertia. Relative to the bus, therefore, we are thrown backwards.
- As soon as that happens, however, the muscular forces on the rest of the body (by the feet) come into play to move the body along with the bus.

Question 2:An astronaut accidentally gets separated of his small spaceship accelerating in interstellar space at a constant rate of 100 m s^{-2}. What is the acceleration of the astronaut the instant after he is outside the spaceship? (Assume that there are no nearby stars to exert a gravitational force on him.)

Answer :

Since there are no nearby stars to exert a gravitational force on him and the small spaceship exerts a negligible gravitational attraction on him, the net force acting on the astronaut, once he is out of the spaceship, is zero. By the first law of motion, the acceleration of the astronaut is zero.

Inertia :

- The tendency of a material body to preserve its present state of uniform motion or rest is known as the inertia of the body. It was first conceived by Galileo.
- Inertia is a physical quantity and mass of a material body is a measure of its inertia in translational motion.

Linear momentum :

- The momentum of a body is defined to be the product of its mass *m* and velocity v, and is denoted by $\vec{p}$;

$$\vec{p} = m\vec{v}$$

- Momentum is a vector quantity.
- Momentum represents the amount of translational motion contained in a body.
- S.I. unit of linear momentum is newton second (Ns) or kg m s^{-1}
- Dimensional formula for linear momentum is $\left[M^1L^1T^{-1}\right]$

Newton's second law :

- **Statement:** The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.
  Mathematically ;

$$\frac{\Delta \vec{p}}{\Delta t} \,\alpha\, \vec{F}$$

$$\Rightarrow \frac{\Delta \vec{p}}{\Delta t} = k\, \vec{F}$$

Where k = proportionality constant

For simplicity; k is taken to be 1.

Hence; $\frac{\Delta \vec{p}}{\Delta t} = \vec{F}$

Now if ; $\Delta t \to 0$ then $\vec{F} = \frac{d\vec{p}}{dt}$

- **The expression for force or acceleration :**

  According to Newton's 2nd law, the net force on a particle is given by,

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m}$$

  Hence acceleration of a body is equal to the net force acting upon the body divided by its mass.

- **Definition of force as per Newton's 2nd law :**

  From Newton's 2nd law ; $\vec{F} = m\vec{a}$

  Hence net force acting on a particle is defined as the product of its mass and acceleration.

- **Unit of force :**

  S.I. unit of force is the newton (N).

  1 N = (1 kg)(1 ms^{-2}) = 1 kg m s^{-2}

  So 1 N is defined as the force which can accelerate 1kg body by an acceleration of 1ms^{-2}

- **Show that Newton's 1st law is contained in 2nd law :**

  In the second law, F = 0 implies a = 0.

  Hence the second law is consistent with the first law.

- **Newton's 2nd law for a body with variable mass :**

  According to Newton's 2nd law ;

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \left(\frac{dm}{dt}\right)\vec{v}$$

  If the body is moving with constant velocity, but its mass is varying then ;

$$\vec{F} = \left(\frac{dm}{dt}\right)\vec{v}$$

- **The second law of motion is a vector law :**

So it is equivalent to three equations, one for each component of the vectors :

$$F_X = \frac{dp_X}{dt} = ma_X$$

$$F_Y = \frac{dp_Y}{dt} = ma_Y$$

$$F_Z = \frac{dp_Z}{dt} = ma_Z$$

This means that if a force is not parallel to the velocity of the body, but makes some angle with it, it changes only the component of velocity along the direction of a force. The component of velocity normal to the force remains unchanged.

For example, in the motion of a projectile under the vertical gravitational force, the horizontal component of velocity remains unchanged.

- **Newton's 2nd law for a system of particles or rigid body :**

  Newton's 2nd law in the same form applies to a rigid body or, even more generally, to a system of particles. In that case, F refers to the total external force on the system and refers to the acceleration of the system as a whole. More precisely, a is the acceleration of the centre of mass of the system.

  Any internal forces in the system are not to be included in F.

- **The second law of motion is a local relation** which means that force **F** at a point in space (location of the particle) at a certain instant of time is related to **an** at that point at that instant. **Acceleration here and now is determined by the force here and now, not by any history of the motion of the particle.**

Numericals :

A bullet of mass 0.04 kg moving with a speed of 90 m s^{-1} enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet? (NCERT)

Solution: For the bullet ; $v^2 - u^2 = 2as$

$$\Rightarrow a = \frac{v^2 - u^2}{2s} = \frac{0^2 - (90)^2}{2(0.6)} = -6750ms^{-2}$$

The force on the bullet is $F = ma = (0.04kg)(-6750ms^{-2}) = -270N$

-ve sign represents that the force is resistive.

Numericals: **The motion of a particle of mass *m* is described by $y = ut + \frac{1}{2}gt^2$. Find the force acting on the particle. (NCERT)**

Solution: As $y = ut + \frac{1}{2}gt^2$

$$\Rightarrow v = \frac{dy}{dt} = \frac{d}{dt}\left(ut + \frac{1}{2}gt^2\right) = \frac{d}{dt}(ut) + \frac{d}{dt}\left(\frac{1}{2}gt^2\right)$$
$$\Rightarrow v = u + gt$$

So acceleration at any instant ;

$$\Rightarrow a = \frac{dv}{dt} = \frac{d}{dt}(u + gt) = \frac{d}{dt}(u) + \frac{d}{dt}(gt)$$
$$\Rightarrow a = g$$

So-net force on the body is ;

$$F = ma = mg$$

Numericals :

A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.

Solution :

The net force has magnitude ; $F = \sqrt{F_1^2 + F_2^2} = \sqrt{8^2 + 6^2}\,N = 10N$

Let net force makes angle α with the vector of magnitude 8N.

So; $\alpha = \tan^{-1}\left(\frac{6}{8}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 37^0$

So the magnitude of acceleration ; $a = \frac{F}{m} = \frac{10N}{5kg} = 2ms^{-2}$

Acceleration is making an angle 37^0 with the vector of magnitude 8N towards the other vector.

Impulse :

- The impulse of a force is defined as the product of force and time duration for which it acts.

  i.e. $\vec{I} = \vec{F}\Delta t$
- Impulse acts for a very short time.
- The force exerting impulse is called an impulsive force.

Impulse- momentum theorem :

- **Statement :**

  The impulse of a force on a body in an interval is equal to the change in momentum of the body.

  i.e. $\vec{I} = \Delta\vec{p}$

- **Proof :**

  By definition ; $\vec{I} = \vec{F}\Delta t$

  $\Rightarrow \vec{I} = \frac{\Delta\vec{p}}{\Delta t}.\Delta t = \Delta\vec{p}$. Hence proved.

Numericals :

A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 m s^{-1}. If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball. (Assume linear motion of the ball).

Answer :

Given that ; $u = 12ms^{-1}$ and $v = -12ms^{-1}$

m = 0.15 kg

Now ; $I = \Delta p = m(v-u)$

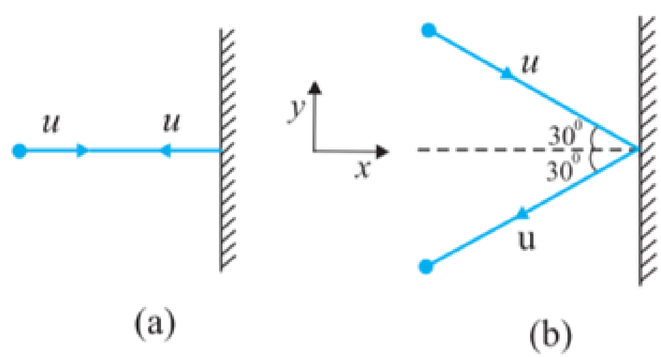
$\Rightarrow I = (0.15kg)(-12ms^{-1} - 12ms^{-1}) = -3.6kgms^{-1}$

Numericals :

Two identical billiard balls strike a rigid wall with the same speed but at different angles, and get reflected without any change in speed, as shown in the given figure.

What is (i) the direction of the force on the wall due to each ball?

(ii) the ratio of the magnitudes of impulses imparted to the balls by the wall?



Solution :

Let *u* be the speed of each ball before and after collision with the wall, and m the mass of each ball.

Case (a) : $\Delta p_X = -mu - mu = -2mu$

$\Delta p_Y = 0 - 0 = 0$

So force on the ball by the wall ; $F_X = \frac{\Delta p_X}{\Delta t} = \frac{-2mu}{\Delta t}$

$$F_Y = \frac{\Delta p_Y}{\Delta t} = \frac{0}{\Delta t} = 0$$

So force on a wall by the ball ; $F_1 = \frac{2mu}{\Delta t}$ (along +ve x-direction)

Case (b) : $\Delta p_X = -mu\cos\theta - mu\cos\theta = -2mu\cos\theta$

$$\Delta p_Y = -mu\sin\theta - (-mu\sin\theta) = 0$$

So force on the ball by the wall ; $F_X = \frac{\Delta p_X}{\Delta t} = \frac{-2mu\cos\theta}{\Delta t}$

$$F_Y = \frac{\Delta p_Y}{\Delta t} = \frac{0}{\Delta t} = 0$$

So force on a wall by the ball ; $F_2 = \frac{2mu\cos\theta}{\Delta t}$ (along +ve x-direction)

Now, the ratio ; $\frac{F_1}{F_2} = \frac{(2mu/\Delta t)}{(2mu\cos\theta/\Delta t)} = \frac{1}{\cos\theta} = \frac{1}{\cos 30^0} = \frac{2}{\sqrt{3}}$

Conceptual Question :

A bullet fired by a gun can easily pierce human tissue before it stops, resulting in casualty. The same bullet fired with moderate speed will not cause much damage. Why?

Ans:

For a given mass, the greater the speed, the greater is the momentum. The greater the change in the momentum in a given time, the greater is the force that needs to be applied or it exerts.

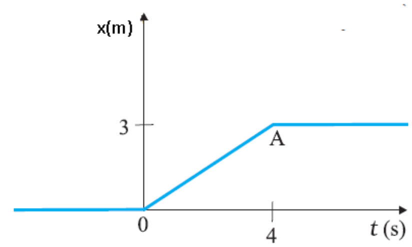
Conceptual Question :

A cricketer moves his hand backwards while catching a ball. Why?

Ans:

By moving hands backwards, the cricketer allows a longer time for his hands to stop the ball. So he experiences less force as, by Newton's second law $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$

Question : **Figure shows the position-time graph of a particle of mass 4 kg. What is the (a) force on the particle for *t* < 0, *t* > 4 *s*, 0 < *t* < 4 s? (b) impulse at *t* = 0 and *t* = 4 s ? (Consider one-dimensional motion only).**



Solution :

(a) **F = 0** in each case as for t<0 and >4s, the body is at rest and for 0<t<4s, the body has constant velocity.

(b) **At t=0s ;** I = m(v-u) = 4kg [(3/4)m/s – 0] = 3 kg m/s

At t = 4s ; I = m(v-u) = 4kg [0 - (3/4)m/s] = - 3 kg m/s

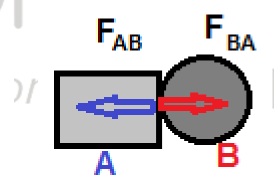
Newton's Third Law

- To every action, there is always an equal and opposite reaction.
- *For example, when you hold the ball, a force acts on the ball and an equal and opposite force act on your hand.*

- Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.
- $\vec{F}_{BA} = -\vec{F}_{AB}$

  $\vec{F}_{BA}$ = Force acting on A by B

  $\vec{F}_{AB}$ = Force acting on B by A



- Action & Reaction forces occur at the same instant. There is no cause-effect relation implied in the third law. So anyone can be chosen as action and anyone can be chosen as a reaction.
- Action & Reaction forces always act on different bodies.

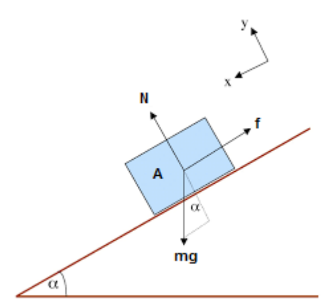
Free body diagram :

- In physics and engineering, a **free body diagram** (force **diagram**, or FBD) is a graphical illustration used to visualize the applied forces, moments, and resulting reactions on a **body** in a given condition.
- E.g. : Free body diagram for a block A kept on an inclined surface is shown in the figure

  Here mg = weight of the block

  N = normal contact force on the block by the incline.

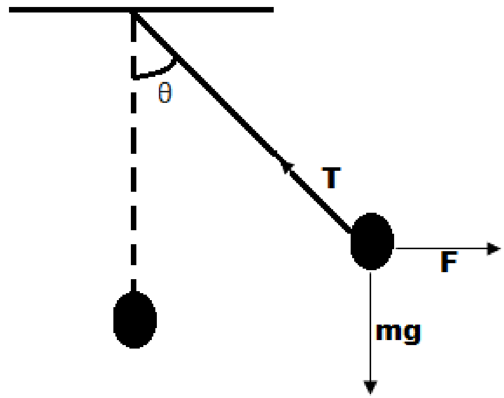
  F =frictional force



Conceptual Question :

A block is suspended from rigid support by a massless string. Then the block is pulled horizontally by a force F up to some distance keeping the string taut.

(a) Identify the forces on the block. (Neglect forces due to air)
(b) Draw the free body diagram.



Answer :

(a) The forces on the block are ;
  (i) Weight (mg) vertically downward
  (ii) Tension (T) due to string
  (iii) Applied horizontal force F

(b) The free-body diagram is as shown in the figure.

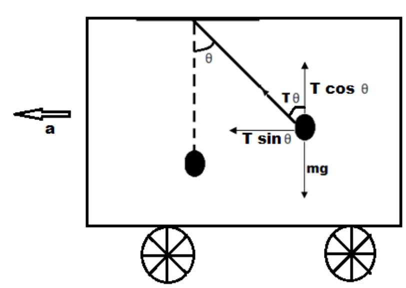
Inertial Frame of reference :

- In this frame Newton's laws of motion are valid.
- Generally, the earth is taken as an inertial frame i.e. observer is stationary w.r.t. earth.
- If a frame of reference or observer is moving with uniform velocity w.r.t. earth is also taken as an inertial frame.

Non-Inertial Frame of reference:

- In this frame, Newton's laws of motion are not obeyed.
- Generally, a frame accelerating or retarding w.r.t. earth is taken as a non - inertial frame.
- E.g. : Consider a pendulum bob suspended from the roof of a car accelerating horizontally. Then the bob will no longer be in a vertical position. Its equilibrium position will make an angle θ with vertical in the opposite direction of acceleration. Now let's analyse the motion of the bob.

  (i) **From earth frame:** The bob is accelerating along with the car.
  From the figure, it is clear that ;
  Weight (mg) is balanced by component Tcosθ.
  i.e. T cosθ = mg ….(i)
  The component T sinθ remains unbalanced and provides acceleration to the bob.
  i.e. T sinθ = ma ….(ii)
  So Newton's law is obeyed and hence earth frame is **an inertial frame**.



  (ii) **From car frame:** The bob is at rest.
  So by Newton's 1st law net force on bob must be 0.
  Here mg is balanced by T cosθ, but the component T sinθ remains unbalanced.
  Hence Newton's laws are failed here. So the car frame is non-inertial.

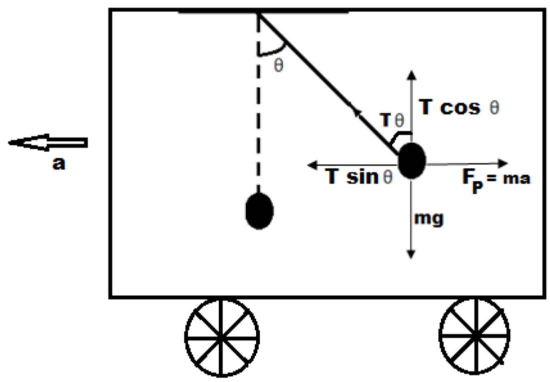
Pseudo force (F_P) :

- In a non- inertial frame, to make Newton's law obeyed, an additional force is considered called a pseudo force.
- The magnitude of the pseudo force on the body is equal to the product of the mass of the body and acceleration of the frame.
- The direction of the pseudo force is opposite to the direction of the acceleration of the frame.
- Mathematically ; $\vec{F}_P = -m\vec{a}_F$

  Where m = mass of the body

  $\vec{a}_F$ = acceleration of the frame

  $\vec{F}_P$ = Pseudo force



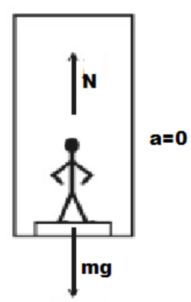
In the previous example, from the car frame, if we are considering pseudo force (F_P), then it balances Tsinθ. In the other hand, T cosθ balances mg and hence it satisfies Newton's law.

Apparent weight :

- *Apparent weight* is a property of objects that corresponds to how heavy an object is.
- The weight of an object is equal to the magnitude of the force of gravity acting on it.
- E.g. : If a body is kept on a surface, then the normal contact force it is exerting on the surface or it is experiencing by the surface is the apparent weight.
- The apparent weight of an object will be the same as the weight of the object whenever the force of gravity acting on the object is balanced by an equal but opposite normal force.
- The apparent weight of an object will differ from the weight of an object whenever the force of gravity acting on the object is not balanced by an opposite normal force.

Apparent weight in an elevator :

a. When the elevator is at rest or moving with uniform velocity :



(i)

In this case ; a = 0

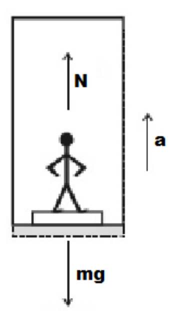
By force equation ; $N - mg = 0$

$\Rightarrow N = mg$

$\Rightarrow$ Apparent weight = Actual weight

So if a weighing machine is kept in the elevator, then the reading showed = m

b. When an elevator is accelerating up :



(ii)

In this case; acceleration = an (upward)

By force equation ; $N - mg = ma$

$$\Rightarrow N = mg + ma = m(g + a) > mg$$

$$\Rightarrow \text{Apparent weight} > \text{Actual weight}$$

So if a weighing machine is kept in the elevator,

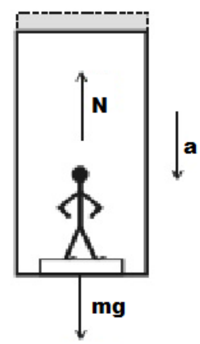
then the reading has shown = $\dfrac{m(g+a)}{g} = m + \dfrac{ma}{g}$

If elevator decelerates up with retardation a_{re} then in place of we use $-a_r$.

So, apparent weight = m(gea$_r$)

Reading of weighing machine = $m - \dfrac{ma}{g}$

c. When an elevator is accelerating down :



(iii)

In this case; acceleration = a (downward)

By force equation ; $mg - N = ma$

$$\Rightarrow N = mg - ma = m(g - a) < mg$$

$$\Rightarrow \text{Apparent weight} < \text{Actual weight}$$

So if a weighing machine is kept in the elevator,

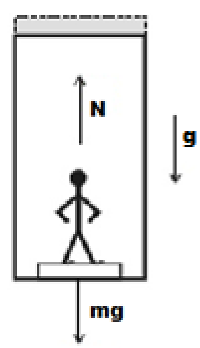
then the reading has shown = $\dfrac{m(g-a)}{g} = m - \dfrac{ma}{g}$

If elevator decelerates down with retardation a_{re} then in place of we use $-a_r$.

So, apparent weight = m(g+a$_r$)

Reading of weighing machine = $m + \dfrac{ma}{g}$

d. When an elevator is falling freely :



(iii)

In this case; acceleration = g (downward)

By force equation ; $mg - N = mg$

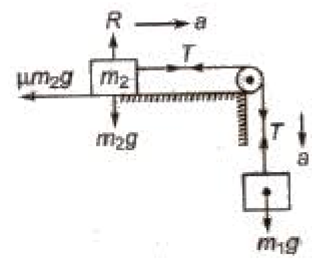
$$\Rightarrow N = mg - mg = 0$$

$$\Rightarrow \text{Apparent weight} = 0$$

So if a weighing machine is kept in the elevator, then the reading has shown = $\dfrac{m(g-g)}{g} = 0$

So the body feels weightlessness.

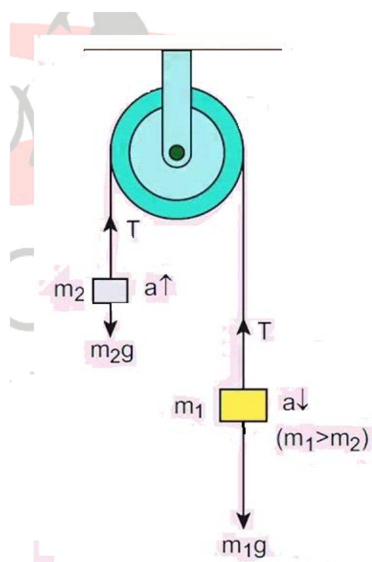
The motion of connected bodies :



- If bodies are connected by taut strings and move together then magnitudes of velocities and accelerations of all bodies are the same at every instant. They may move in different directions.
- E.g. : In the given figure m_1 moves downward but m_2 moves horizontal, but both moves with the same magnitude of acceleration.

Question: **In the given figure (simple Atwood machine), assume the string and pulley to be massless and frictionless. Find the magnitudes of accelerations of the objects and tension along the string.**

Answer :



Now from the figure;

Force equation for m_1 gives $m_1 g - T = m_1 a$ ....(i)

Force equation for m_2 gives $T - m_2 g = m_2 a$ ....(ii)

Adding equations (i) and (ii) we get ;

$$m_1 g - m_2 g = m_1 a + m_2 a$$
$$\Rightarrow a(m_1 + m_2) = (m_1 - m_2)g$$
$$\Rightarrow a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$

Using the expression for an in equation (ii) we get ;

$$T = m_2 g + m_2 a = m_2 g + \frac{m_2(m_1 - m_2)g}{m_1 + m_2}$$
$$\Rightarrow T = m_2 g\left[1 + \frac{m_1 - m_2}{m_1 + m_2}\right] = m_2 g\left[\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right]$$
$$\Rightarrow T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

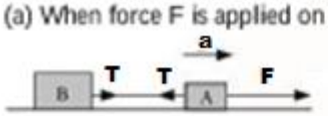
Question: Two bodies of masses 10 kg and 20 kg respectively kept on a smooth, horizontal surface are tied to the ends of a light string. a horizontal force F = 600 N is applied to (i) A, (ii) B along the direction of string. What is the tension in the string in each case?

Answer :

As F = ma

$$\therefore a = \frac{F}{m} = \frac{600}{30} = 20 m/s^2$$

(a) When force F is applied on body A:



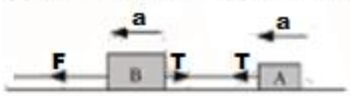
The equation of motion can be written as:

$F - T = m_1 a \therefore$

$T = F - m_1 a$

$= 600 - 10 \times 20 = 400$ N

(b) When force F is applied on body B:



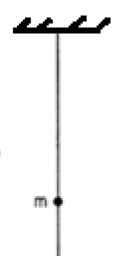
The equation of motion can be written as:

$F - T = m_2 a$

$T = F - m_2 a$

$\therefore T = 600 - 20 \times 20 = 200$ N

Question: A monkey of mass 40 kg climbs on a rope (as shown in the figure) which can stand a maximum tension of 600 N. In which of the following cases will the rope break: the monkey



(a) climbs up with an acceleration of 6 m s^{-2}

(b) climbs down with an acceleration of 4 m s^{-2}

(c) climbs up with a uniform speed of 5 m s^{-1}

(d) falls down the rope nearly freely under gravity

(Ignore the mass of the rope).

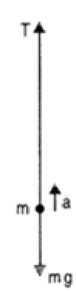
Answer: (a) From free body diagram for the monkey, the force equation can be

$$T - mg = ma$$

$\therefore$ $T = mg + ma = m(g + a)$

or $T = 40$ kg $(10 + 6)$ $ms^{-2} = 640$ N

But the rope can withstand a maximum tension of 600 N. So the rope will break



(a)

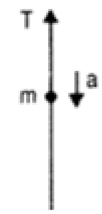
(*b*) When the monkey is climbing down with an acceleration, then

$$mg - T = ma \qquad \text{(Figure (b))}$$

$\Rightarrow \qquad T = mg - ma = m(g - a)$

or $\qquad T = 40 \text{ kg} \times (10 - 4) \text{ ms}^{-2} = 240 \text{ N}$

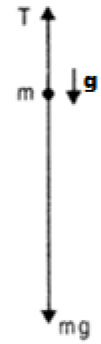
The rope will not break.



(c) As body rises up with uniform velocity a = 0

So, T = mg = (40 kg)(10m/s^2)=400N

So rope will not break



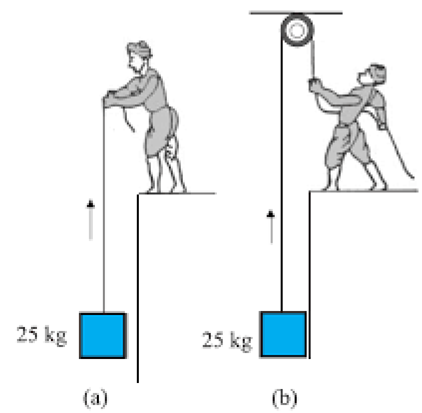
(d) When the monkey is climbing down **freely**

$$mg - T = mg$$

$\Rightarrow \qquad T = mg - mg = 0$

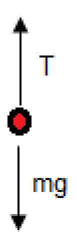
So the monkey will feel weightlessness.

Question: A block of mass 25 is raised by a 50 kg man in two different ways as shown in the figure. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding?

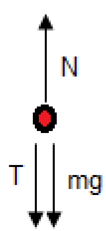


Answer :

In case (a) – Free body diagrams for block and man are ;



(For block)



(For man)

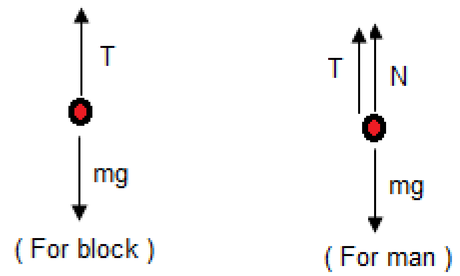
So ; for block ; T – mg =0

T = mg = (25kg) (10m/s^2) = 250 N

For man ; N = T + mg = 250 N + (50kg)(10m/s^2) = 250 N + 500N = 750 N

So the floor will yield.

In case (b) - Free body diagrams for block and man are ;



So ; for block ; $T - mg = 0$

$$T = mg = (25kg)(10m/s^2) = 250\ N$$

For man ; $N + T = mg$

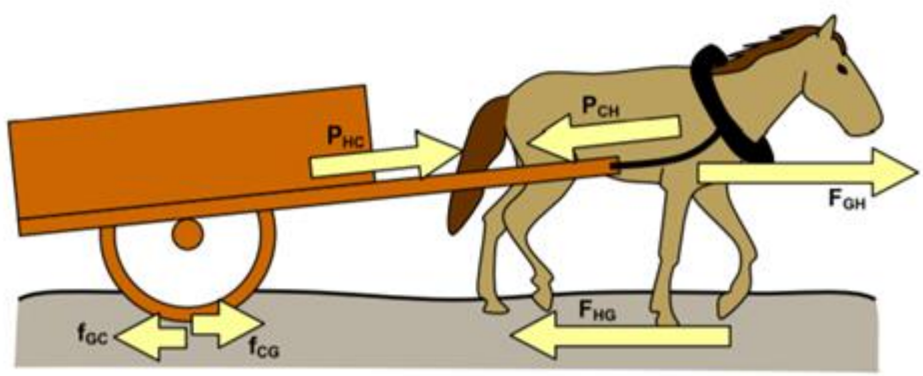
$\Rightarrow N + 250\ N = (50kg)(10m/s^2) = 500N$

$\Rightarrow N = 500\ N - 250\ N = 250\ N$

So the floor will not yield.

Horse – cart problem :

The horse and cart problem is an application of Newton's Third Law, which says: For every action, there is an equal and opposite reaction. If A exerts a force on B, then B will exert an equal and opposite force on A.



When the horse walks forward and pulls on the cart, the following forces are present:

	Action		Reaction
F_{HG}	The horse pushes against the ground.	F_{GH}	The ground pushes back on the horse.
P_{HC}	The horse's body pulls on the cart.	P_{CH}	The cart resists movement and pulls back on the horse's body.
f_{CG}	The cart wheel pushes against the ground.	f_{GC}	The friction with the ground resists the cart's movement.

Note that the action-reaction pairs of forces affect the motions of different objects. With this in mind, the net force…

a) on the cart is	$P_{HC} - f_{GC}$	since the cart moves when the horse's pull overcomes friction ($P_{HC} > f_{GC}$)
b) on the horse is	$F_{GH} - P_{CH}$	since the horse moves when he pushes the ground enough to pull the cart ($F_{GH} > P_{CH}$)
c) on the ground is	$F_{HG} - f_{CG}$	since the horse-cart system moves when the horse overcomes friction ($F_{GH} > P > f_{GC}$).

In summary, although the action-reaction forces may cancel, the net force resulting from different interactions of forces allows for movement, relative to the ground.

Question :

Explain why

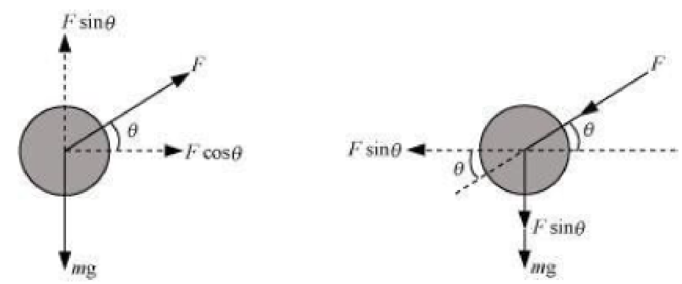
(a) a horse cannot pull a cart and run in space

(b) it is easier to pull a lawnmower than to push it

Answer :

(a) A horse-cart system moves due to the reaction force exerted by the ground on the horse. In space, a horse can't exert force on itself to start motion and hence can't pull the cart.

(b) In the case of pull, the effective weight is reduced due to the vertical component of the pull. In the case of a push, the vertical component increases the effective weight.



In pulling :
Effective weight =mg - Fsinθ

In Pushing:
Effective weight =mg + Fsinθ

Conservation of Momentum

- **Statement:** In an isolated system (i.e. when no external force is acting on the system), the total momentum of the system is conserved.
- But during interaction among different components of the system, the momentum of individual components may change, but for the whole system, there is no change in momentum.
- **Example 1:** In a Spinning top, total momentum = 0. For every point, there is another point on the opposite side that cancels its momentum.

- **Example 2:** Bullet fired from a Rifle,
  Initially, total momentum = 0
  Later, the trigger is pulled, bullet gains momentum in à direction, but this is cancelled by rifle's momentum. Therefore, total momentum = 0

During the process, the chemical energy in gunpowder gets converted into heat, sound and chemical energy.

Let after firing; the velocity of each bullet (of mass m) = u

The velocity of the gun (of mass M) = v

By conservation of linear momentum ; $Mv + mu = 0$

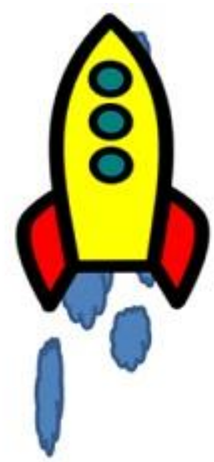
$$\Rightarrow v = -\frac{mu}{M} = \text{recoil velocity of gun}$$

If n bullets are fired per second,

then interaction time between gun and each bullet =(1/n) s

So the magnitude of the force between a gun and each bullet = $\left|\frac{mu}{1/n}\right| = |nmu| = |nMv|$

- **Example3:** Rocket propulsion;
  Initially, the mass of rocket: M. It just started moving with velocity v
  Initial momentum = Mv
  Later, gases are ejected continuously in opposite direction with a velocity relative to rocket in a downward direction giving a forward push to the rocket.
  Mass of the rocket becomes (M-m)
  Velocity of the rocket becomes (v + v')
  Final momentum = (M –m) (v + v') + mu
  Thus, Mass * velocity = constant

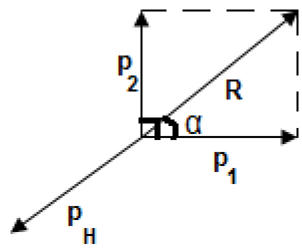


- **Example 4 : Explosion of a bomb ;**
  Before explosion momentum of a bomb = 0
  After explosion; the components of bomb move in different directions to make a resultant momentum equal to 0.

Question: **An explosive explodes into 3 components having masses in the ratio 1: 1 : 3. The lighter components move in perpendicular to each other with speeds equal to 30 m/s each. Find the magnitude and direction of motion of the heavier particle.**

Answer: Let the mass of each lighter particle = x. So the mass of the heavier particle = 3x

By a law of conservation of linear momentum;



$$\vec{p}_H + \vec{p}_1 + \vec{p}_2 = \vec{0}$$

$$\Rightarrow \vec{p}_H = -(\vec{p}_1 + \vec{p}_2)$$

$$\Rightarrow |\vec{p}_H| = \sqrt{p_1^2 + p_2^2} = \sqrt{(30x)^2 + (30x)^2}$$

$$\Rightarrow |\vec{p}_H| = 30\sqrt{2}x$$

$$\Rightarrow v_H = \frac{30\sqrt{2}x}{3} = 10\sqrt{2}ms^{-1}$$

$$\alpha = \tan^{-1}\left(\frac{p_2}{p_1}\right) = \tan^{-1}\left(\frac{10x}{10x}\right) = \tan^{-1}(1) = 45^0$$

The heavier particle must emerge in a direction opposite to the resultant vector of p_1 and p_2 i.e at an angle $(180^0\text{-}\alpha)=135^0$ with p_1 towards p_2.

Proof of law of conservation of linear momentum :

To explain conservation of momentum, let us take the following example. Consider two balls A and B having masses m_1 and m_2 respectively. Let the initial velocity of ball A be u_1 and that of ball B be u_2 $(u_1 > u_2)$. Their collision takes place for a very short interval of time t and after that A and B start moving with velocities v_1 and v_2 (now $v_1 < v_2$) respectively as shown in Figure.

Before collision $(u_1 > u_2)$ (a) After collision $(v_1 < v_2)$ (b)

The momentum of ball A before and after the collision is m_1u_1 and m_1v_1 respectively. If there are no external forces acting on the body, then the rate of change of momentum of ball A, during the collision will be

$$= \frac{m_1(v_1 - u_1)}{t}$$

and, similarly the rate of change in momentum of ball B

$$= \frac{m_2(v_2 - u_2)}{t}$$

Let F_{12} be the force exerted by ball A on B and F_{21} be the force exerted by ball B on A. Then, according to Newton's second law of motion

$$F_{12} = \frac{m_1(v_1 - u_1)}{t} \quad \text{and} \quad F_{21} = \frac{m_2(v_2 - u_2)}{t}$$

According to Newton's third law of motion, we have

$$F_{12} = -F_{21}$$

$$\frac{m_1(v_1 - u_1)}{t} = -\frac{m_2(v_2 - u_2)}{t}$$

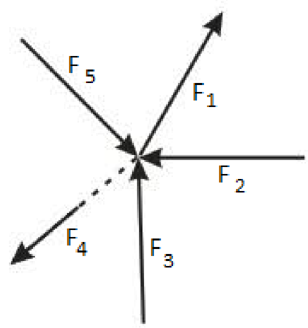
$$m_1 v_1 - m_1 u_1 = -m_2 v_2 + m_2 u_2$$

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

Hence Proved .

Concurrent forces :

- In a **concurrent** force system, all **forces** pass through a common point.
- The figure shows concurrent forces.

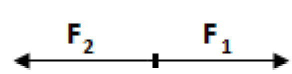


Equilibrium of a particle :

- Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero. According to the first law, this means that the particle is either at rest or in uniform motion.
- If two forces $\vec{F}_1$ and $\vec{F}_2$, act on a particle, equilibrium requires

  $$\vec{F}_1 = -\vec{F}_2$$

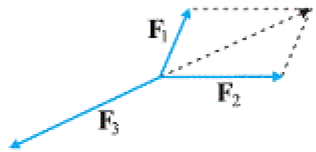
  i.e. the two forces on the particle must be equal and opposite.

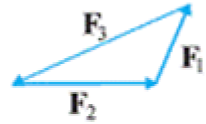


- Equilibrium under three concurrent forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ requires that the vector sum of the three forces is zero.

  $$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$$

  - Three vectors must be three sides of a closed triangle all in the same order.
  - They must obey Lami's theorem.





- Equilibrium under n concurrent forces $\vec{F}_1, \vec{F}_2 \ldots\ldots \vec{F}_n$ requires that the vector sum of the forces is 0.

$$\vec{F}_1 + \vec{F}_2 + \ldots\ldots + \vec{F}_n = \vec{0}$$

  - A particle is in equilibrium under the action of forces $\vec{F}_1, \vec{F}_2 \ldots\ldots \vec{F}_n$ if they can be represented by the sides of a closed n-sided polygon with arrows directed in the same sense.
  - As x , y and z components are independent of each other , hence for equilibrium sum of all x –components is 0 , sum of all y-components is 0 and sum of all z-components is 0 .

    i.e. $F_{1x} + F_{2x} + F_{3x} = 0$

    $F_{1y} + F_{2y} + F_{3y} = 0$

    $F_{1z} + F_{2z} + F_{3z} = 0$

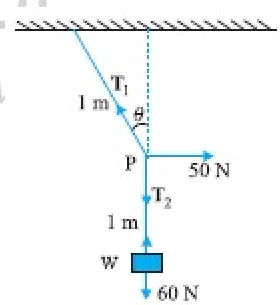
    where F_{1x}, F_{1y} and F_{1z} are the components of $\vec{F}_1$ along *x*, *y* and *z* directions respectively.

    F_{2x}, F_{2y} and F_{2z} are the components of $\vec{F}_2$ along *x*, *y* and *z* directions respectively.

    F_{3x}, F_{3y} and F_{3z} are the components of $\vec{F}_3$ along *x*, *y* and *z* directions respectively.

Numerical :

A mass of 6 kg is suspended by a rope of length 2 m from the ceiling. A force of 50 N in the horizontal direction is applied at the midpoint P of the rope, as shown. What is the angle the rope makes with the vertical in equilibrium? (Take *g* = 10 m s-2). Neglect the mass of the rope.



Answer :

Consider the equilibrium of weight W.

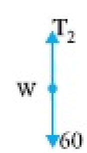
Clearly, $T_2 = 6 \times 10 = 60$ N. ...(i)

Consider the equilibrium of the point P under the action of three forces - the tensions T_1 and T_2, and the horizontal force 50 N.

The horizontal and vertical components of the resultant force must vanish separately :

$T_1 \cos\theta = T_2 = 60$ N

$T_1 \sin\theta = 50$ N

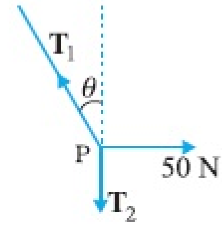


(Free body diagram of the block)

$$\frac{T_1 \sin\theta}{T_1 \cos\theta} = \frac{50N}{60N} = \frac{5}{6}$$

$$\Rightarrow \tan\theta = \frac{5}{6}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{5}{6}\right)$$



Free body diagram of point P

which gives that;

Note the answer does not depend on the length of the rope (assumed massless) nor on the point at which the horizontal force is applied.

Friction :

- Friction is a contact force that opposes **relative motion**.
- No friction exists until an external force is applied.

i. Static friction

- A force that resists initiation of motion of one body over another with which it is in contact
- Opposes Impending motion
- Denoted by f_s
- Self-adjusting.
- Let the ball be at rest initially
- Applied force, $F_a = 0$; Static friction, $f_s = 0$
- Later, applied force, $F_a = F$, then f_s also increases but only up to a certain limit. As soon as F_a becomes greater than the maximum fs, the ball starts to move.
- The maximum value of static friction is called **limiting friction** $(F_s)_{max}$.

So; $f_s \le (F_s)_{max}$

f_s acts when a body is at rest. Hence called as **Static friction.**

ii. Kinetic Friction :

- When the body starts moving on a surface then frictional force resisting the relative motion of the body w.r.t. the surface is called the **kinetic friction**.

Laws of friction :

- The limiting value of static friction $(f_s)_{max}$ is independent of the area of contact.
- The limiting value of static friction $(f_s)_{max}$ varies with the normal force *(N)* approximately as :

$$(f_s)_{max} \;\alpha\; N$$

$$\Rightarrow (f_s)_{max} = \mu_s N$$

where μ_s is a constant of proportionality depending only on the nature of the surfaces in contact. The constant μ_s is called the coefficient of static friction.

- The law of static friction may thus be written as

$$f_s \le (f_s)_{max}$$

- If the applied force *F* exceeds $(f_s)_{max}$ the body begins to slide on the surface. It is found experimentally that when relative motion has started, the frictional force decreases from the static maximum value $(f_s)_{max}$. The frictional force that opposes relative motion between surfaces in contact is called **kinetic or sliding friction** and is denoted by FK.
- Kinetic friction, like static friction, is found to be independent of the area of contact. Further, it is nearly independent of the velocity.
- It satisfies a law similar to that for static friction: FK = μ_k N
  where μ_k= the coefficient of kinetic friction, depends only on the surfaces in contact.
- As

$$f_k < (f_s)_{max}$$
$$\Rightarrow \mu_k N < \mu_s N$$
$$\Rightarrow \mu_k < \mu_s$$

- When the relative motion has begun, and applied force = F, then the acceleration of the body according to the second law is

$$a = \frac{F - f_k}{m}$$

  - For a body moving with constant velocity, *F = FK*.
  - If the applied force on the body is removed, its acceleration is

$$a = \frac{-f_k}{m}$$

  and it eventually comes to a stop.

Numerical :

A particle of mass 5 kg moving horizontally with a speed 20 ms^{-1} enters into a rough patch and stops after 20 m. Calculate the (i) kinetic friction between the surfaces and (ii) coefficient of kinetic friction. (Take g = 10 m s^{-2})

Solution :

$$a = \frac{v^2 - u^2}{2s} = \frac{0^2 - 20^2}{2(20)} ms^{-2} = -10ms^{-2}$$

$$\Rightarrow f_k = ma = 5kg(-10ms^{-2}) = -50N$$

As on horizontal path ; so N = mg

$$\text{As}; |f_k| = \mu_k N = \mu_k mg$$

$$\Rightarrow \mu_k = \frac{|f_k|}{mg} = \frac{50}{5 \times 10} = 1$$

Numericals :

Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary (given that the coefficient of static friction between the box and the train's floor **is 0.1 and take g = 10 ms^{-2}).**

Solution:

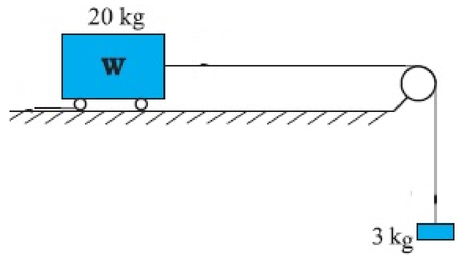
Since the acceleration of the box is due to the static friction,

$ma = f_s \leq \mu_s N = \mu_s m g$

$\Rightarrow a \leq \mu_s g$

$\Rightarrow {}_{max} = \mu_s g = 0.15 \times 10 \text{ m s}^{-2} = 1.5 \text{ m s}^{-2}$

Numericals :



(i) What is the acceleration of the block and trolley system shown in the figure, if the coefficient of kinetic friction between the trolley and the surface is 0.04?

(ii)What is the tension in the string? (Take g =10 m s^{-2}). Neglect the mass of the string.

Solution :

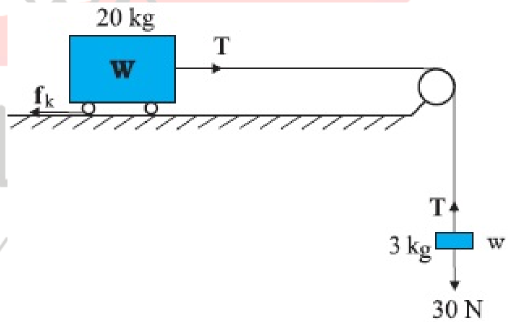
As the string is inextensible, and the pully is smooth, the 3 kg block and the 20 kg trolley both have the same magnitude of acceleration.

Applying second law to the motion of the block,

$30 - T = 3a$....... *(i)*

Apply the second law to the motion of the trolley

$T - f_k = 20\,a$.



Now $f_k = \mu_k N$,

Here $\mu_k = 0.04$,

N =mg = 20 x 10 = 200 N.

Thus the equation for the motion of the trolley is;

$T - 0.04 \times 200 = 20\,a$

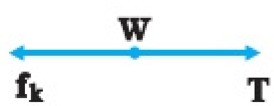
Or $T - 8 = 20a$. ....(ii)

The equations (i) and (ii) give ;

a = (22/23)m s^{-2} = 0.96 m s^{-2}

And $T = 27.1$ N.

[Free body diagraam of 3kg block]

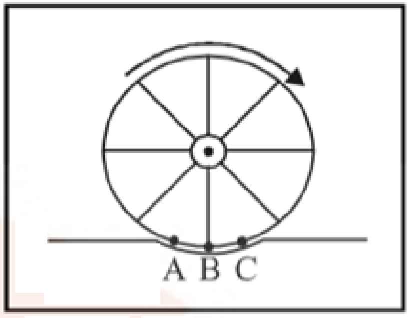


[Free body diagrm for 20 kg block]

Rolling Friction :

The opposing force that comes into play when a body rolls over the surface of another body is called the rolling friction.

Cause of rolling friction. Let us consider a wheel which is rolling along a road. As the wheel rolls along the road, it slightly presses into the surface of the road and is itself slightly compressed as shown in Fig.



Thus, a rolling wheel :

(i) constantly climbs a 'hill' (BC) in front of it, and

(ii) has to simultaneously get itself detached from the road (AB) behind it.

The force of adhesion between the wheel and the road opposes this process.

Both these processes are responsible for rolling friction.

The angle of friction :

The angle of friction between any two surfaces in contact is defined as the angle which the resultant of the force of limiting friction F and normal reaction N makes with the direction of normal reaction N

It is represented by θ.

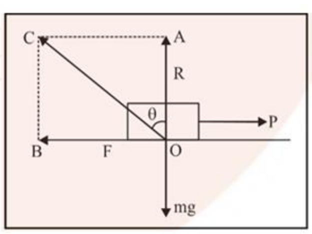
In fig. OA represents the normal reaction N which balances the weight mg of the body.

OB represent F, the limiting force of sliding friction, when the body tends to move to the right.

Complete the parallelogram OACB. Join OC.

This represents the resultant of N and F.

By definition, $\angle AOC = \theta$ is the angle of friction between the two bodies in contact.



Relation between μ and θ

$$\text{In } \Delta \text{ AOC}, \quad \tan\theta = \frac{AC}{OA} = \frac{OB}{OA} = \frac{F}{N} = \mu \qquad \text{……………………(i)}$$

The angle of repose :

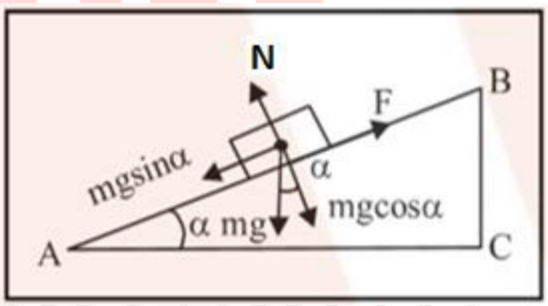
Angle of repose or angle of sliding is defined as the minimum angle of inclination of a plane with the horizontal, such that a body placed on the plane just begins to slide down.

It is represented by α.

Its value depends on material and nature of the surfaces in contact.

In fig., AB is an inclined plane such that a body placed on it just begins to slide down.

∠BAC α = angle of repose.



Now from the figure ; N = mg cos α

F = mg sin α

Since ; F = μ N ⇒ mg sin α = μ mg cos α

⇒ μ = tanα ……….(ii)

From equations (i) and (ii) we have ; θ = α

⇒ Angle of friction = Angle of repose

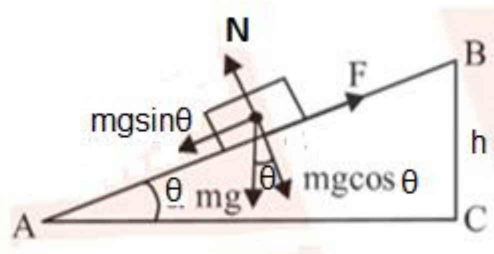
Sliding of a block along an inclined plane :

Let h = height of an incline

θ = Angle of inclination of the incline .

μ= coefficient of friction between block and incline (assuming both kinetic and static friction coefficient nearly equal).

- If θ ≤ α i.e. angle of repose ($\tan^{-1}\mu$); then the body will not slide.
- If θ ≥ α ; then the body slides down the incline



Now using Newton's laws ;

N = mg cosθ

mgsinθ – F = ma

⇒ mg sin θ - μ N = ma

⇒ mg sin θ - μmg cosθ = ma

⇒ a = g (sin θ - μ cosθ) = g sinθ (1 - μ cotθ) ……(i)

Displacement of the block ; s = AB = h / sinθ

Time to reach ground = t

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$$

$$\Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2h/\sin\theta}{g\sin\theta(1-\mu\cot\theta)}}$$

$$\Rightarrow t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g(1-\mu\cot\theta)}} \;\ldots\ldots\ldots\ldots\text{(ii)}$$

Speed at ground = v

$$v^2 - u^2 = 2as$$

$$\Rightarrow v^2 - 0^2 = 2g\sin\theta(1-\mu\cot\theta)\frac{h}{\sin\theta}$$

$$\Rightarrow v = \sqrt{2gh(1-\mu\cot\theta)} \;\ldots\ldots\ldots\text{(iii)}$$

Special case (for smooth incline) : μ = 0

So equation (i) becomes ; a = g sinθ

Equation (ii) becomes ; $t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$

Equation (iii) becomes ; $v = \sqrt{2gh}$

So it can be concluded that **on smooth incline speed at the ground is independent on the angle of inclination for a given height, but time to reach the ground is less if the angle of inclination is more and vice-versa**.

Dis-advantages of friction :

- In a machine with different moving parts, friction does have a negative role. It opposes the relative motion and thereby dissipates power in the form of heat, etc.
- Due to friction wear and tear is produced in tyres of a vehicle.

Advantages of friction :

- Kinetic friction that dissipates power is nevertheless important for quickly stopping relative motion. It is made use of by brakes in machines and automobiles.
- We can walk, able to write etc because of friction.
- On an ordinary road, the friction between the tyres and the road provides the necessary external force to accelerate the car.

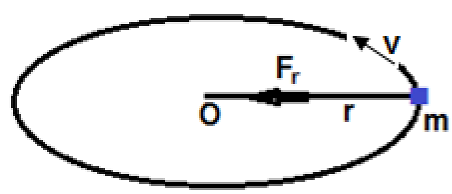
[Due to both advantages and disadvantages, friction is a necessary evil]

Methods to reduce friction :

- Lubricants are a way of reducing kinetic friction in a machine.
- Use ball bearings between two moving parts of a machine reduce friction. Since the rolling friction between ball bearings and the surfaces in contact is very small, power dissipation is reduced.
- A thin cushion of air maintained between solid surfaces in relative motion is another effective way of reducing friction.

Dynamics of circular motion :

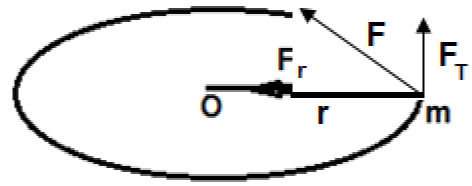
- **In uniform circular motion :**
  Only speed is constant, but velocity changes as direction changes at every instant.
  So the acceleration of the particle is perpendicular to the direction of motion i.e. along the radius and towards the centre. Hence this is called centripetal acceleration.
  Its magnitude is ; $a = \frac{v^2}{r} = \omega^2 r$
  By Newton's 2nd law; net force is F = ma
  Hence net force on the particle acts along radius towards the centre.
  Its magnitude is given by ; $F = \frac{mv^2}{r} = m\omega^2 r$



  This force is called a centripetal force.
  Where m = mass of the particle
  v = speed of the particle, ω = angular speed of the particle.
  r = radius of the circular path.
- **In non-uniform circular motion :**
  Acceleration has two components.
  (i) Centripetal acceleration; $a_r = \frac{v^2}{r} = \omega^2 r$ along radius towards the centre.
  (ii) Tangential acceleration ; $a_T = \frac{dv}{dt} = r\alpha$

So the magnitude of net acceleration ; $a = \sqrt{(a_r)^2 + (a_T)^2}$

Hence by Newton's 2nd law ;



Centripetal force ; $F_r = \frac{mv^2}{r} = m\omega^2 r$

The tangential component of force ; $F_T = m\frac{dv}{dt} = mr\alpha$

So the magnitude of net force in a non-uniform circular motion is ; $F = \sqrt{(F_r)^2 + (F_T)^2}$

MCQ :

One end of a string of length *l* is connected to a particle of mass *m* and the other to a small peg on a smooth horizontal table. If the particle moves in a circle with speed *v* the net force on the particle (directed towards the centre) is :

(a) *T*, (b) $T - \frac{mv^2}{l}$ (c) $T + \frac{mv^2}{l}$ (d) 0

Answer : (a) T
Because only force along radius towards centre is the tension long string and this provides centripetal force .

Numericals :

A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev./min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N?

Solution :
Given ; m = 0.25 kg , r = 1.5 m , ω = 40rev/min = (40 x 2π rad)/60s = (4π/3) rad/s
Here the tension (T) provides the necessary centripetal force

$$\therefore \ T = m\omega^2 r = 0.25kg \times \left(\frac{4\pi}{3} rad/s\right)^2 \times 1.5m = 6.6N$$

As the speed of stone increases then more tension acts to provide the required centripetal force.
As tension has a maximum limit, so the stone can be whirled at maximum speed ($_{max}$).

$$\therefore \ T_{max} = \frac{mv_{max}^2}{r}$$

$$\Rightarrow v_{max} = \sqrt{\frac{rT_{max}}{m}} = \sqrt{\frac{1.5 \times 200}{0.25}} ms^{-1} = 20\sqrt{3} ms^{-1}$$

Conceptual MCQ :

If, while whirling a stone in a horizontal circle by a massless string, the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks :

(a) the stone moves radially outwards,
(b) the stone flies off tangentially from the instant the string breaks,
(c) the stoneflies off at an angle with the tangent whose magnitude depends on the speed of the particle?
(d) the stone just falls there.
Answer: b

Conceptual MCQ :

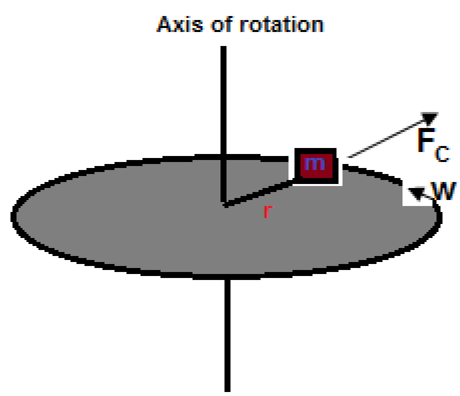
A stone of mass *m* tied to the end of a string revolves in a vertical circle of radius *R*. The net forces at the lowest and highest points of the circle directed vertically downwards are :
[Choose the correct alternative]

Lowest Point	Highest Point
(a) $mg - T_1$	$mg + T_2$
(b) $mg + T_1$	$mg - T_2$
(c) $mg + T_1 - (m v_1^2) / R$	$mg - T_2 + (m v_2^2) / R$
(d) $mg - T_1 - (m v_1^2) / R$	$mg + T_2 + (m v_2^2) / R$

Answer : (a)

Centrifugal force :

If a particle is at rest w.r.t. a rotating frame , then to explain its state in this frame a pseudo force is considered to act on the particle along radius away of the centre . This pseudo force is called as centrifugal force .



Its magnitude is given by ; $F_C = \frac{mv^2}{r} = m\omega^2 r$

Where; m = mass of the particle.
v = speed of the frame and ω = angular speed of the frame.
r = distance of the particle from the axis of rotation of the frame.

Numericals :

A disc revolves with a speed of $33\frac{1}{3}$ rev/min, and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the coefficient of friction between the coins and the record is 0.15, which of the coins will revolve with the record?

Solution :

As the disc is rotating around an axis, so all points on the disc have the same angular speed ω. The coin for which centrifugal force is balanced by static friction remains at rest.

$\omega = \frac{100}{3} rev/\min = \frac{100 \times 2\pi\ rad}{3 \times 60\,s} = \frac{10\pi}{9} rad/s$

i.e. $m\omega^2 r = f_s$

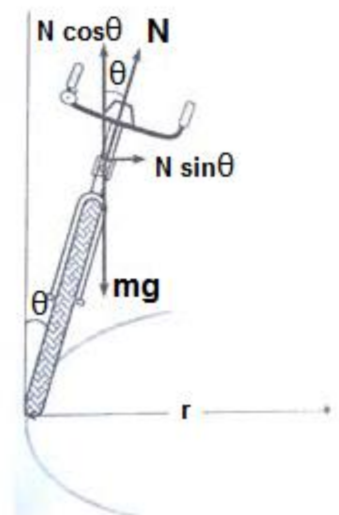
$\Rightarrow m\omega^2 r \le (f_s)_{\max} = \mu_s mg$

$\Rightarrow r \le \frac{\mu_s g}{\omega^2} = \frac{0.15 \times 10}{(10\pi/9)^2} m = 0.1215m = 12.15cm$

$\Rightarrow r \le 12.15cm$

So coin at 4cm will continue rotating along with the disc and the coin at 14 cm will fall away.

Bending of cyclist :



- On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips.
- To avoid this cyclist bends a little from the normal position so that the component of normal contact force along radius towards the centre provides the necessary centripetal force.
- Now from figure;
  N cosθ balances mg and N sinθ provides the necessary centripetal force.
  So, N cosθ = mg ……… (i)
  $N \sin\theta = mv^2/r$ …….(ii)
  Now dividing equation (ii) by equation (i) we get

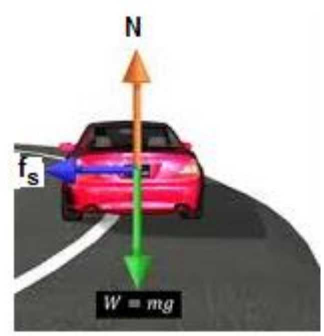
$$\frac{N\sin\theta}{N\cos\theta} = \frac{mv^2/r}{mg}$$

$$\Rightarrow \tan\theta = \frac{v^2}{rg}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Hence angle of bending depends on (i) speed (v) of the cyclist
(ii) The radius of the path

The motion of a vehicle in a circular level road :



While negotiating along circular level road three forces act on the car
(i) The weight of the car, mg
(ii) Normal reaction, N
(iii) Frictional force, f_s
As there is no acceleration in the vertical direction

$N - mg = 0$

$\Rightarrow$ $N = mg$ …………………..(i)

The centripetal force required for circular motion is along the surface of the road and is provided by the component of the contact force between the road and the car tyres along the surface. This by definition is the frictional force. Note that it is the static friction that provides the centripetal acceleration. Static friction opposes the impending motion of the car moving away from the circle.

Hence $f_s = \frac{mv^2}{r}$ ....(ii)

Since $f_s \le (f_s)_{max} = \mu_s N = \mu_s mg$ ....(iii)

$\Rightarrow \frac{mv^2}{r} \le \mu_s mg$ (Using equation (i) I equation (iii))

$\Rightarrow v \le \sqrt{\mu_s rg}$

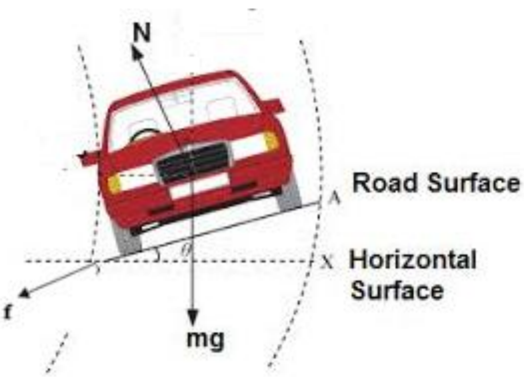
Hence $v_{max} = \sqrt{\mu_s rg}$ ....(iv)

Equation (iv) gives a maximum safe speed of a vehicle in a circular level road.

The motion of a vehicle on a banked circular road :

We can reduce the contribution of friction to the circular motion of the car if the road is banked.

So a component of normal reaction acts along radius towards the centre to provide the necessary centripetal force.

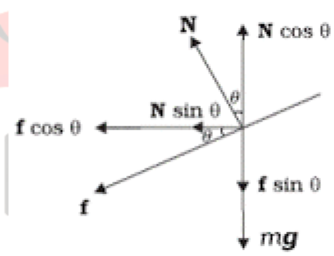


Since there is no acceleration along the vertical direction, the net force along this direction must be zero. Hence,

$N \cos \theta = mg + f \sin \theta$ ....(i)

The centripetal force is provided by the horizontal components of *N* and *f*.

$N \sin \theta + f \cos \theta = \frac{mv^2}{r}$ .....(ii)



[Free body diagram for the vehicle on banked road]

From equation (ii) it is evident that if speed increases , then more frictional force arises . Hence, at maximum safe speed , the static friction becomes maximum i.e. limiting friction , given by ;

$f_{max} = \mu_s N$ ....(iii)

Using the condition for maximum safe speed (v_{max}) in equations (i) and (ii) we get;

Equation (i) $\Rightarrow N \cos \theta = mg + \mu_s N \sin \theta$

$\Rightarrow N(\cos \theta - \mu_s \sin \theta) = mg$ ....(iv)

Equation (i) $\Rightarrow N \sin \theta + \mu_s N \cos \theta = \frac{mv_{max}^2}{r}$

$\Rightarrow N(\sin \theta + \mu_s \cos \theta) = \frac{mv_{max}^2}{r}$ ....(v)

Dividing equation (v) by equation (iv) we get

$$\frac{N(\sin \theta + \mu_s \cos \theta)}{N(\cos \theta - \mu_s \sin \theta)} = \frac{mv_{max}^2 / r}{mg} \Rightarrow \frac{(\sin \theta + \mu_s \cos \theta)}{(\cos \theta - \mu_s \sin \theta)} = \frac{v_{max}^2}{rg}$$

$$\Rightarrow \frac{\cos \theta(\tan \theta + \mu_s)}{\cos \theta(1 - \mu_s \tan \theta)} = \frac{v_{max}^2}{rg}$$

$$\Rightarrow v_{max} = \sqrt{rg\left(\frac{\tan\theta + \mu_s}{1 - \mu_s \tan\theta}\right)} \quad .....(vi)$$

- Putting $\mu_s = 0$ i.e. neglecting friction in equation (vi) we have ;

  $$\Rightarrow v_0 = \sqrt{rg\tan\theta} \quad ....(vii)$$

  At this speed, the frictional force is not needed at all to provide the necessary centripetal force. Driving at this speed on a banked road will cause little wear and tear of the tyres.
- If $v < v_0$; the vehicle would slip down the banked road. So friction will act opposite to the direction of the previous case.

  So proceeding as above we can find the minimum safe speed on the banked road is;

  $$\Rightarrow v_{min} = \sqrt{rg\left(\frac{\tan\theta - \mu_s}{1 + \mu_s \tan\theta}\right)}$$
- A car can be parked only if $\tan\theta \le \mu_s$.