Chapter- 09

MECHANICAL PROPERTIES OF SOLID

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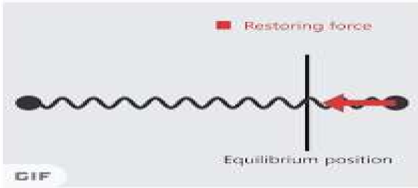
Elasticity:- the properties of matter by which the body regains its original configuration when the external force acting on it is removed is called elasticity.



Restoring force of a spring (vector ...
imgur.com

Cause of elasticity:- Applying the deforming force, the molecules are displaced from their equilibrium position. As a result, a restoring force is developed, which brings the molecules of solid to origin form.

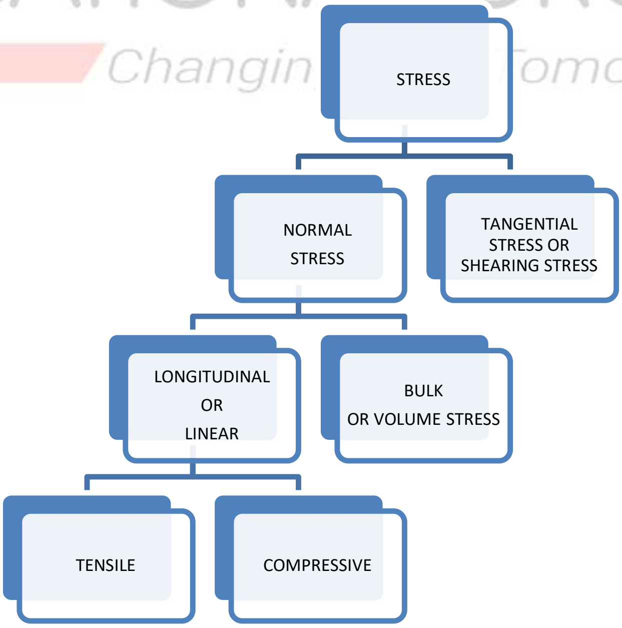
Plasticity:- it is the property due to which the body doesn't regain original shape and size after the removal of deforming force.

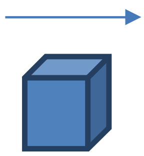
Question: What do you mean by perfectly plastics body? Give some examples.

The body does not have any tendency to recover its original configuration after the removal of deforming force.

Examples: - mud, wax

TYPE OF STRESS





Note:- Pressure is Scalar but Stress is tensor because it can be different in a different direction. So Stress is called Tensor.

Stress: it is defined as the internal restoring force acting per unit area of the cross-sectional deforming body.

STRESS = $\frac{F}{A}$

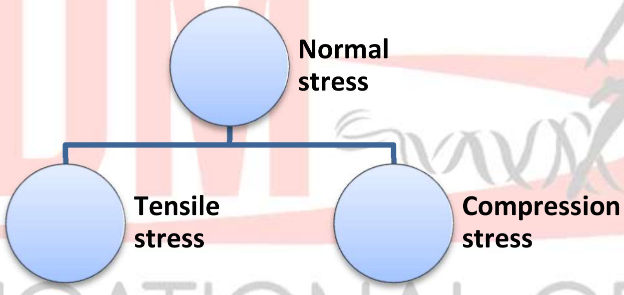
Note: At equilibrium, restoring force is equal in the magnitude of external force. Thus in equilibrium stress may be defined as external force per unit area of the body.

Type of stress: (i) Normal stress

(ii)Tangential stress

(iii) Hydrostatic stress

Normal stress: when the deforming force acts normal over the surface of the body, the internal restoring force per unit area is called normal stress.



Tensile Stress: this set up if there is an increase in the dimension of the body in the direction of applied force

Compression stress: this set up if there is a decrease in the dimension of the body due to the applied force.

(ii) **Tangential stress**: when the deforming force, acting tangentially to the surface of the body and changes the shape of the body the force per unit area is called tangential stress.

Hydrostatics Stress

If a body is subjected to a uniform and equilibrium pressure from all sides, it is under hydrostatic stress.

Unit

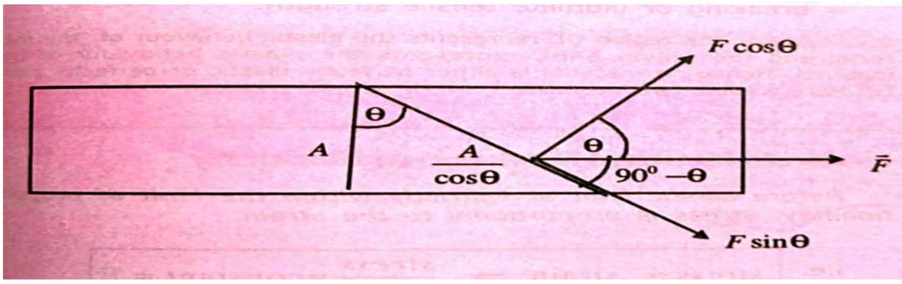
SI: - N/m^2

CGS: - dyne/cm^2

Dimension formula: - [M$L^{-1}T^{-2}$]

Question

Find out the value of normal stress and tangential stress from

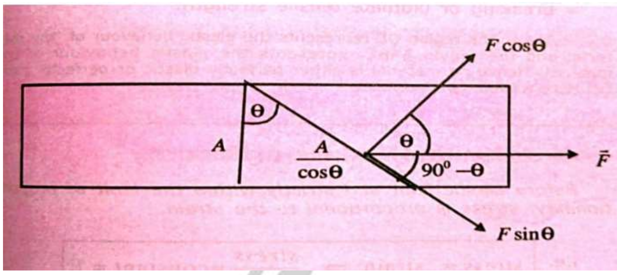


Normal stress = $\frac{F\cos\theta}{A}$

Tangential stress= $\frac{F\sin\theta}{A}$

Question

A bar is subjected to equal and opposite forces at its two ends. PQ plane making angle θ with the cross-section be A, then find the tensile stress and shearing stress on PQ. Also, discuss their maximum possible values?



Tensile stress = $\frac{F\cos\theta}{\frac{A}{\cos\theta}} = \frac{F\cos^2\theta}{A}$ (max $\theta = 0^\circ$)

Shearing Stress= $\frac{F\sin\theta}{\frac{A}{\cos\theta}} = \frac{F(\sin 2\theta)}{2A}$(max $\theta = 45^\circ$)

Strain: The ratio of change in configuration to its original configuration is known as strain.

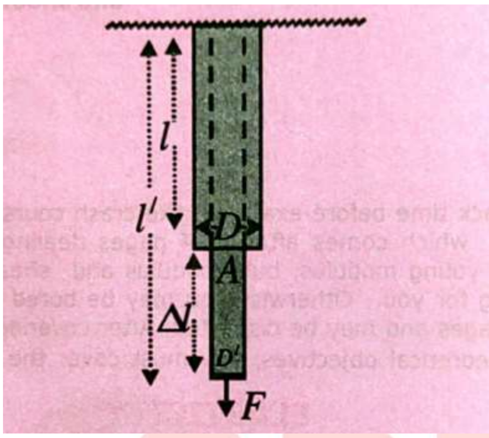
Mathematically, $\frac{Changeindimension}{originaldimision}$ **= strain**

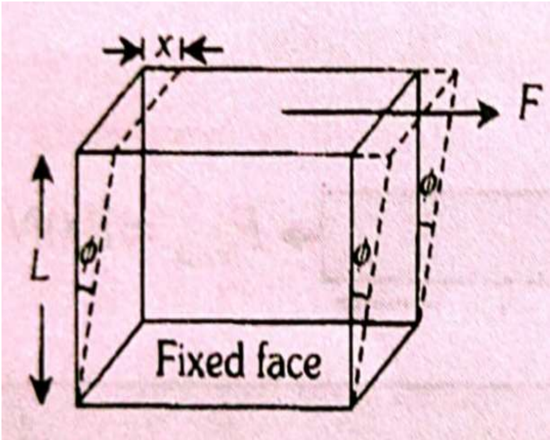
Type of strain

- STRAIN
  - TENSILE STRAIN
    - LINEAR/ LONGITUDINAL STRAIN($\frac{\Delta L}{L}$)
    - LATERAL STRAIN($\frac{\Delta D}{D}$)
  - VOLUME STRAIN
  - SHEAR STRAIN

Searing strain: - here the deforming force changes the shape of the body without changing its volume

It is the angle through which a plane perpendicular to the fixed surface of the body gets turned under the effect of tangential force.





Hooke's Law:-

Within the elastic limit, stress is directly proportional to the strain

i.e. stress α strain

mathematically, $\frac{stress}{strain}$ = E (modulus of elasticity or coefficient of elasticity) within the limit

Stress-Strain graph

The slope of the graph gives modulus of elasticity

Question:- Define the modulus of elasticity and state the factors on which the modulus of elasticity depends.

Answer: Definition, $\frac{stress}{strain}$ = E

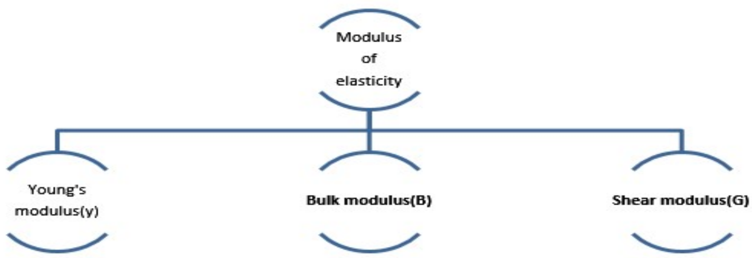
Depending on the following factors:

(i) Nature of material

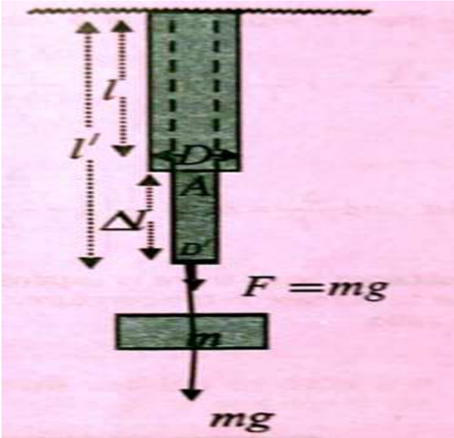
(ii) Temperature (in most cases temp causes a decrease in elasticity)

(iii) independent of dimension

(iv) nature of impurities presents in the material.



Young's Modulus(Y)



Y= $\frac{Normal stress}{longitudinal strain}$

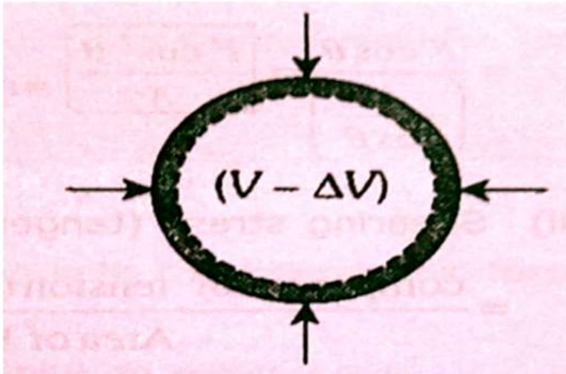
Y = $\frac{\frac{F}{A}}{\frac{\Delta L}{L}} = \frac{FL}{A\Delta L}$

Area of cross-section

Y= $\frac{mgl}{(\pi r^2)\Delta l}$ **(for the cylindrical body)**

Unit:- SI:- Nm^{-2} called pascal (pa)

Bulk modulus (B)



B= $\frac{Normal stress}{volume strain} = \frac{\frac{F}{A}}{\frac{-\Delta V}{V}} = \frac{-PV}{\Delta V}$

[ΔP = P- P' = $\frac{F}{A}$, Volume stress. Here $\Delta V$is took -ve due to its decrease in volume]

For a system in equilibrium, the value of B is always + ve

Compressibility:- Reciprocal of bulk modulus

K=$\frac{1}{B}$

Shear modulus(G)

G =$\frac{tangential stress\ (shearing stress)}{shearing strain}$

=$\frac{\frac{F}{A}}{\theta}$

Where $\theta = \frac{\Delta x}{l}$

then

G = $\frac{\frac{F}{A}}{\frac{\Delta x}{l}} = \frac{fl}{\Delta x A}$

SI unit: Nm^{-2} or Pa

Poison's Ratio(σ): The ratio between lateral strain and longitudinal strain.

Mathematically, (σ)=$\frac{lateral strain}{longitudinal strain} = \frac{\Delta D/D}{\Delta L/L} = \frac{L\Delta D}{D\Delta L}$

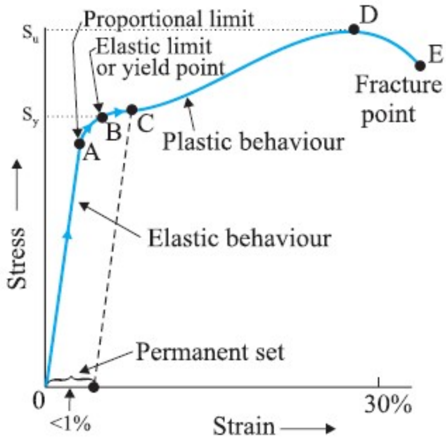
Note: The value of σ is always less than $\frac{1}{2}$. for most solids lines between $\frac{1}{4}$ and $\frac{1}{3}$.

Elastic Fatigue: the loss of strength of the material due to repeated strain on the same body is known as elastic fatigue.

QUESTION: why are the bridges declared unsafe after long use?

STRESS AND STRAIN CURVE

(1)when strain is small (region OA)



- **The curve is a straight line**
- stress α strain
- Hook's law obeys
- Point A is called the proportional limit
- The slope of OA = $\frac{Stress}{Strain}$>

(2)when strain is increasing a little bit(Region B)

- The strain is not proportional to Stress
- However, the wire still regains its original lens after the removal of the force.
- Point B is called Plastic limit or Yield point

(3)if the wire is stretched beyond the elastic line(region BC)

- The strain increases much more rapidly with a slight increase in stress.
- The wire does not come to its natural length even after stretching force is removed
- A permanent increase in length is called a permanent set.

(4) If the stress is increased further (Region CD)

- A small increase in stress causes a very large increase in strain.
- Max stress corresponding to D = Breaking Strength

(5) Region DE

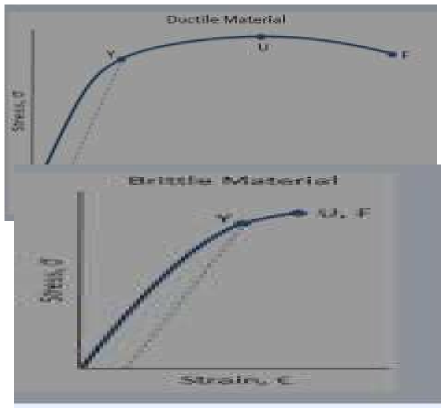
- The strain increases even if the wire is unloaded.
- The wire breaks at E.

Question: from Stress and Strain graph materials are classified as

(i) Ductile

(ii) Brittle

(iii) Elastomers

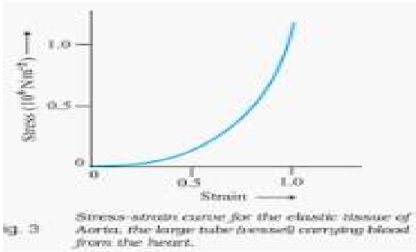


(i)Ductile:- the tensile strength and the breaking Points are apart from each other.

Eg:- Spring, copper

(ii)Brittle:- the tensile strength and the breaking point is nearer to others.

e.g Glass



(iii)Elastomers:- these who do not obey hook's law and the little stress is given to produce large strain.

(stress-strain curve for elastic tissue of Aorta)

Q)Show that potential energy(PE) in a stressed wire per unit volume = $\frac{1}{2}$ x stress x strain

Ans: y = $\frac{FL}{al}$

$\therefore$ F = $\frac{yal}{L}$ -------------(1)

If the wire is stressed through a small length dl,

$dw = F.\ dl = \frac{yal}{L}\ .\ dl$

$w = \int dw = \int_0^L \frac{yal}{L} dl$

$w = \frac{ya}{L}\int_0^l l dl$

$= \frac{ya}{L}\left[\frac{l^2}{2}\right]_0^l$

$= \frac{ya}{2l}\left[l^2 - 0^2\right]$

$\therefore\ w = \frac{yal^2}{2L}$ --------------(2)

$\frac{P.E}{Volume} = (\frac{1}{2}\frac{Yal^2}{L})$ /volume = $\frac{1}{2}$ y$\left[\frac{l}{L}\right]^2$ = $\frac{1}{2}$ Young's modulus x strain= $\frac{1}{2}$ x Stress x Strain

Elastics potential energy = energy density x volume.

Question: Steel is more elastic than rubber .why?

Let us consider two rods of steel and rubber each having length l and area of cross-section A. if they are subjected to the same deforming force, then

For Steel ; $Y_S = \frac{F_S L_S}{A_S \Delta L_S}$

for Rubber; $Y_R = \frac{F_R L_R}{A_R \Delta L_R}$

Then

$\frac{Y_S}{Y_R} = \frac{\Delta L_R}{\Delta L_S}$

since $\Delta L_R > \Delta L_S \Rightarrow Y_S > Y_R$

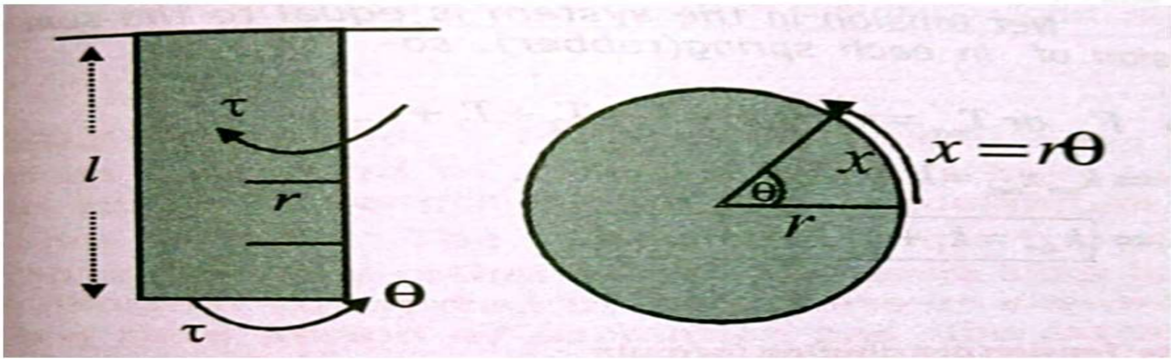
I .e Young's modulus of steel is more than Young's modulus of rubber.

so, Steel is more elastic than Rubber.

Question: The stretching of the coil is determined by its shear modulus, why?

The stretching of the coil simply changes its shape without any change in length. Due to its Shear modulus of elasticity is involved.

Question: When a wire of length l and radius R is twisted by an angle θ find out the value of Shear modulus.



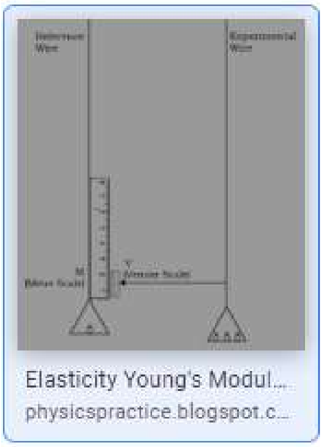
Question: what is the value of y for a perfectly rigid body?

Answer: ∞

Question: Why the liquid has zero moduli of rigidity?

Answer: because it does not have a shape of itself.

Explain an experiment for the determination of Young's modulus of a material of the wire.



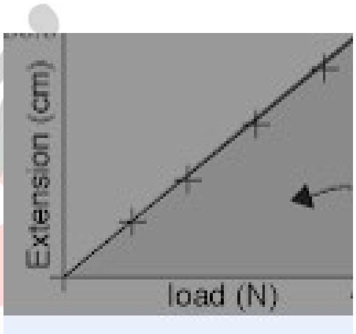
A – Reference wire

B – Experimental wire

(Both have the same length and same radius)

Fixed weight is given to move the wire straight. Weight is added gradually to pan attached to B. The difference of Vernier from Vernier scale which is attached to B gives the extension is no every time

The graph is plotted between load applied extension



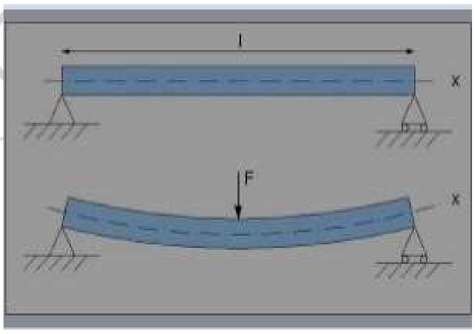
Slope = $\tan\theta = \frac{\Delta l}{Mg}$ ----------(1)

Young's Modulus, Y= $\frac{Stress}{Strain} = \frac{\frac{Mg}{\pi r^2}}{\frac{\Delta l}{l}} = \frac{Mgxl}{\Delta l \pi r^2}$

Y= $\frac{1}{\tan\theta}$ x $\frac{L}{\pi r^2}$

APPLICATIONS OF ELASTICITY

In designing a beam of the bridge the knowledge of elasticity is used such that, the bridge should not bend under its weight.



Why should the thickness of the beam be more than the breadth?

$\sigma = \frac{wl^3}{4bd^3y}$

Where y= young's modulus of rod

L= length,

b= breadth and

d= depth

it is obvious that

(i) To reduce bending one should use material of large young's modulus.

(ii) For a given load the bending may be reduced effectively by increasing thickness (d rather than breadth (b).

But on increasing the thickness too much the bar may bend. This is called bulking.

I-shape bar is used. This saves a lot of materials.

Application 2: the thickness of a metallic wire used in a crane to lift the heavy loads is decided from the knowledge of the elastic limit of the material of the rope.

Example:- A crane having a steel rope is required to lift a load of 1000 kg. The elastic limit of the steel is 3x 10^8 N/m^2. What should be the radius of the rope required for this purpose?

Answer: Ultimate stress =$\frac{F}{A}$ = $\frac{mg}{\pi r^2}$

⇒ 3x 10^8 = $\frac{mg}{\pi r^2}$

⇒ 3x 10^8= $\frac{10^3 x 9.8}{\frac{22}{7} x r^2}$

⇒ $r^2 = \frac{9.8x10^3x7}{3x10^8x2}$ ⇒ **r=**$\sqrt{\frac{9.8x10^3x7}{3x10^8x22}}$

Application 3: Using consideration of elasticity we can estimate the maximum height of a mountain. The Stress at the base of the mountain should be less than the critical shear stress at which the rocks begin to flow.

Let h_{max} – be the max height of mountain which produces the stress at the bottom causing the rock to flow = $\frac{Force}{Area} = \frac{mg}{a} = \frac{\rho v g}{a} = \frac{\rho a h \max}{a}$ = ρ h_{max}-------------------(1)

if the density of material of rock ρ = 3 x 10^3 kg/m^2

Elastic limit for typical rock = 30 x10^7 N/m^2

To avoid the flow of rock h_m ρ g< 30 x 10^7 N/ m^2

$h_m \approx \frac{30x1\ ^7}{\rho g} = \frac{3x10^8}{3x10^8x9.8}$ = 10 km

it is interesting to note that this is nearby the height of Mount Everest.