Chapter- 10

Mechanical Properties of Fluids

Introduction: Fluids

- Fluids can be defined as any substance which is capable of flowing.
- They don't have any shape of their own.
- For example, water which does not have its shape, but it takes the shape of the container in which it is poured.
- Both liquids and gases can be categorized as fluids as they are capable of flowing.
- The volume of solids, liquids, and gas depends on the stress or pressure acting on it.
- In this chapter, we will study if we apply force on the fluid how does it affect the internal properties of fluids.
- Fluids offer very little resistance to shear stress.
- We will also study some characteristic properties of fluids.

Pressure

- The pressure is defined as force per unit area.
- Pressure = Force/Area
- For Example:

  - ✓ Consider a very sharp needle that has a small surface area and consider a pencil whose back is very blunt and has more surface area than the needle.
  - ✓ If we poke the needle in our palm it will hurt as the needle gets pierced inside our skin. Whereas if we poke the blunt side of the pencil into our hand it won't pain so much.
  - ✓ This is because the area of contact between the palm and the needle is very small therefore the pressure is large.
  - ✓ Whereas the area of contact between the pencil and the palm is more therefore the pressure is less.
- **Conclusion: Two factors which determine the magnitude of the pressure are:**
  - ✓ **Force – greater the force greater is the pressure and vice-versa.**
  - ✓ **Coverage area – greater the area less is the pressure and vice-versa.**

Example:

- Consider a stuntman lying on the bed of nails which means there are large numbers of nails on any rectangular slab. All the nails are identical and equal in height.
- We can see that the man is not feeling any pain and he is lying comfortably on the bed. This is because there is a large number of nails and all the nails are closely spaced with each other.

- All the small pointed nails make large surface area therefore the weight of the body is compensated by the entire area of all the nails.
- The surface area increases therefore pressure is reduced.
- But even if one nail is greater than the others then it will hurt. Because then the surface area will be less as a result pressure will be more.

Stuntman lying on a bed of nails.

Question: A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter of 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

Solution:
Mass of the girl, m = 50 kg
Diameter of the heel, d = 1 cm = 0.01 m
Radius of the heel, r = d/2 = 0.005m
Area of the heel = $\pi r^2 = \pi (0.005)^2 = 7.85 \times 10^{-5}\ m^2$
Force exerted by the heel on the floor:
F = mg = 50 × 9.8 = 490 N
Pressure exerted by the heel on the floor:
P = Force / Area
= $490 / (7.85 \times 10^{-5})$
= $6.24 \times 10^6\ N\ m^{-2}$
Therefore, the pressure exerted by the heel on the horizontal floor is $6.24 \times 10^6\ Nm^{-2}$.

Pressure in Fluids

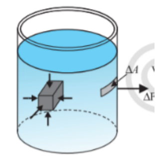
Pressure

The thrust experienced per unit area of the surface of a liquid at rest is called pressure.

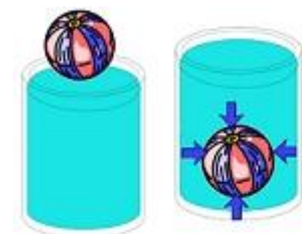
$$P = \frac{F}{A}$$

In the CGS system, the unit of pressure is dyne cm^{-2}. In SI, the unit of pressure in Nm^{-2} or Pascal (Pa).

- When a liquid is in equilibrium, the force acting on its surface is perpendicular everywhere.
- The pressure is the same at the same horizontal level.
- The pressure at any point in the liquid depends on the depth (h) below the surface, density.



- Consider a body submerged in the water, force is exerted by the water perpendicular to the surface of the body.
  - ✓ If there is no force applied perpendicularly but in the parallel direction, then there will be motion along the horizontal direction.
  - ✓ Since the fluid is at rest and the body is submerged in the fluid. Therefore, there cannot be motion along the horizontal direction.
  - ✓ Therefore, we always say the force is applied perpendicularly.



- The pressure is a scalar quantity because the force here is not a vector quantity but it is the component of force normal to the area.
- Dimensional Formula $[ML^{-1}T^{-2}]$
- I Unit: N/m^2 or Pascal (Pa).
- Atmosphere unit (atm) is defined as the pressure exerted by the atmosphere at sea level. It is a common unit of pressure.
- **1atm = 1.013 x 10^5 Pa**
- Fluids are the substances that can flow e.g., liquids and gases. It does not possess a definite shape.
- When an object is submerged in a liquid at rest, the fluid exerts a force on its surface normally. It is called the thrust of the liquid.

Question: The two thigh bones (femurs), each of cross-sectional area10 cm^2 support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs.

Solution:
Total cross-sectional area of the femurs is $A = 2 \times 10\ cm^2 = 20 \times 10^{-4}\ m^2$.
The force acting on them is F = 40 kg wt. = 400 N (taking $g = 10\ m\ s^{-2}$).
This force is acting vertically down and hence, normally on the femurs.
Thus, the average pressure is $= 2 \times 10^5\ N\ m^{-2}$

Pascal's Law

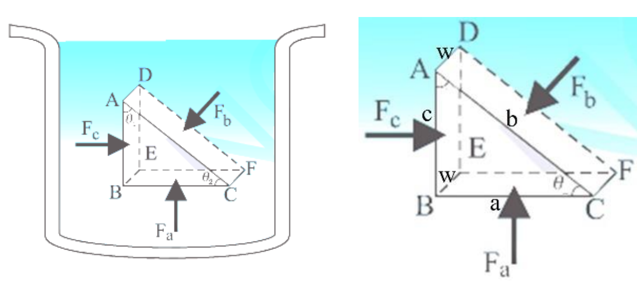
The pressure in a fluid at rest is the same at all points if they are at the same height.

PROOF

Fig shows an element in the interior of a fluid at rest. This element ABC-DEF is in the form of a right-angled prism.

The element is very small so that every part of it can be considered at the same depth from the liquid surface and therefore, the effect of gravity is the same at all these points.

F_a = force exerted normally on surface BEFC
F_b = force exerted normally on surface ADFC
F_c = force exerted normally on surface ADEB
P_a, P_b, P_c corresponding Pressures exerted



At equilibrium
$F_b \cos\theta = F_a$

$F_b \sin\theta = F_c$

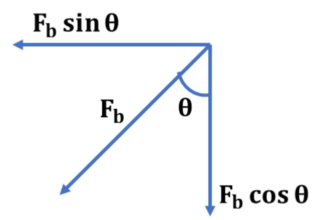
From trigonometry

$\sin\theta = \frac{c}{b}$

$c = b\sin\theta$

$cw = bw\sin\theta$ (multiplying w both the sides)

$A_c = A_b \sin\theta$ ($cw = A_c$ $bw = A_b$)



Similarly, we have

$A_a = A_b\cos\theta$

$\frac{F_b\cos\theta}{A_b\cos\theta} = \frac{F_a}{A_a}$

$\frac{F_b\sin\theta}{A_b\sin\theta} = \frac{F_c}{A_c}$

$\frac{F_b}{A_b} = \frac{F_a}{A_a}$

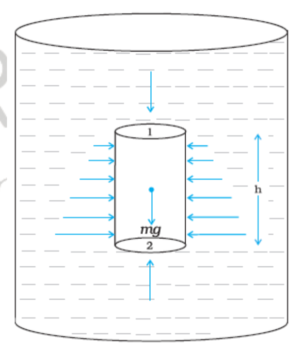
$\frac{F_b}{A_b} = \frac{F_c}{A_c}$

$\therefore P_a = P_b = P_c$

The pressure exerted is the same in all directions in a fluid at rest. Hence pressure is a scalar quantity.

Variation of Pressure with Depth

Consider a cylindrical object inside a fluid; consider 2 different positions for this object.



The fluid is at rest therefore the force along the horizontal direction is 0.

Forces along the vertical direction:

Consider two positions 1 and 2.

Force at position 1 is perpendicular to cross-sectional area A,

$F_1 = P_1$

Similarly, $F_2 = P_2$

Total force $F_{net} = -F_1 + F_2$ as

F_1 is along negative y-axis therefore it is –ve.

And F_2 is along the +ve y-axis.

$F_{net} = (P_2 - P_1)\,A$

This net force will be balanced by the weight of the cylinder(m).

Therefore, under equilibrium condition

F_{net} = mg = weight of the cylinder = weight of the fluid displaced.

$F_{net} = \rho V g$

where ρ=density=volume of the fluid

$F_{net} = \rho hAg$

where V=hA (h=height and A= area)

Therefore $(P_2-P_1)A = \rho hAg$

$P_2-P_1 = \rho hg$,

P_2-P_1 = gauge pressure

Consider the top of the cylinder exposed to air therefore

$P_1=P_a$ (where $P_a= P_1$ is equal to atmospheric pressure.)

$P_2=P$

Then

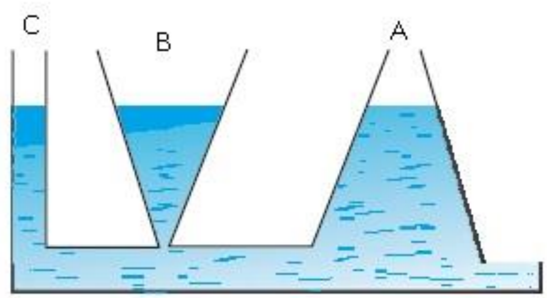
$$P = P_a + \rho hg$$

NOTE

- The pressure P, at depth below the surface of a liquid open to the atmosphere, is greater than atmospheric pressure by an amount ρhg.
- The pressure is independent of the cross-sectional or base area or the shape of the container.
- The difference in the pressure is dependent on the height of the cylinder.

Hydrostatic Paradox

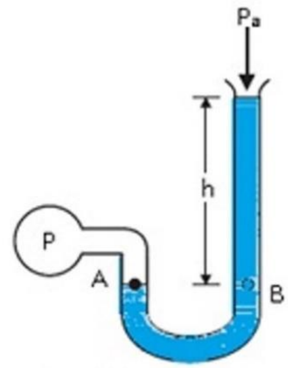
- Hydrostatic Paradox means: - hydro = water, static =at rest
- Paradox means that something taking place surprisingly.
- Consider 3 vessels of very different shapes (like thin rectangular shape, triangular, and some filter shape) and we have a source from which water enters into these 3 vessels.
- Water enters through the horizontal base which is the base of these 3 vessels we observe that the level of water in all the 3 vessels is the same irrespective of their different shapes.
- This is because the pressure at some point at the base of these 3 vessels is the same.
- The water will rise in all these 3 vessels till the pressure at the top is the same as the pressure at the bottom.
- As pressure is dependent only on height therefore in all the 3 vessels the height reached by the water is the same irrespective of the difference in their shapes.
- This experiment is known as Hydrostatic Paradox.
- The three vessels A, B, and C contain different amounts of liquids, all up to the same height.



How to measure Gauge pressure?

- Gauge pressure is measured by **Open Tube Manometer**.
- Open Tube Manometer is a U-shaped tube that is partially filled with mercury (Hg).

- One end is open and the other end is connected to some device where pressure is to be determined. This means it is like a system.
- The height to which the mercury column will rise depends on the atmospheric pressure. Similarly, depending on the pressure of the system the height of mercury in another tube rises.



Open Tube Manometer

Consider an open tube manometer

P_A = Pressure at A

= Pressure of the air in the vessel 'D'

= Pressure of system

P_a = atmospheric pressure

$P_A = P_B$ ....(1)

But $P_B = P_a + \rho gh$

$P_A = P_a + \rho gh$ .... (2) ($P_A = P_B$)

From equation (1) and (2)

$P_A = P_a + \rho gh$

$P = P_a + \rho gh$

$P - P_a = \rho gh$

$P = P_A$ = pressure of the system

NCERT

Question: At a depth of 1000 m in an ocean

a) What is the absolute pressure?

b) What is the gauge pressure?

c) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure.

(The density of seawater is 1.03×10^3 kg m^{-3}, g = 10 ms^{-2})

Solution: Here h = 1000 m and ρ = 1.03 × 103 kg m^{-3}.

a) absolute pressure $P = P_a + \rho gh$

$= 1.01 \times 10^5 P_a + 1.03 \times 10^3 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2} \times 1000 \text{ m}$

$= 104.01 \times 10^5 P_a$

≈ 104 atm

b) Gauge pressure is $P - Pa = \rho gh = P_g$

$P_g = 1.03 \times 10^3 \text{ kg m}^{-3} \times 10 \text{ ms}^2 \times 1000 \text{ m}$

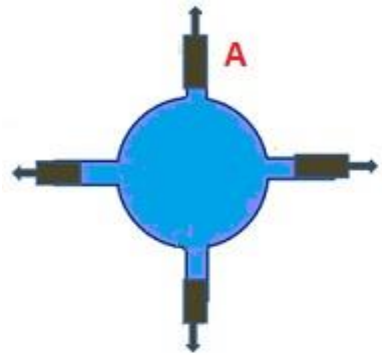
$= 103 \times 10^5 P_a$

≈ 103 atm

c) The pressure outside the submarine is $P = P_a + \rho gh$ and the pressure inside it is P_a. Hence, the net pressure acting on the window is gauge pressure, $P_g = \rho gh$. Since the area of the window is A = 0.04 m^2, the force acting on it is

P_g F = A = 103×10^5 Pa × 0.04 m^2 = 4.12×10^5 N.

Pascal's Law for Transmission of Fluid Pressure

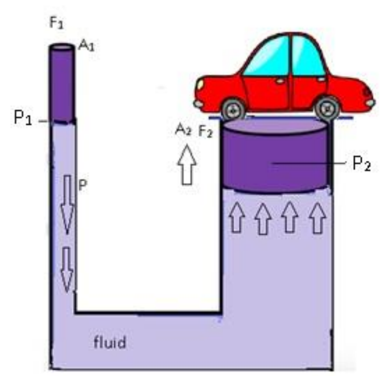


- **Pascal's law for transmission of fluid pressure states that the pressure exerted anywhere in a confined incompressible fluid is transmitted undiminished and equally in all directions throughout the fluid.**
- The above law means that if we consider a fluid that is restricted within a specific region in space and if the volume of the fluid doesn't change with the pressure, then the amount of pressure exerted will be the same as the amount of pressure transmitted.
  - ✓ Consider a circular vessel that has 4 openings and along these 4 openings, 4 pistons are attached.
  - ✓ When piston A is moved downwards pressure is exerted on the liquid in the downward direction, this pressure gets transmitted equally along with all the directions. As a result, all the other 3 pistons move equal distance outwards.
  - ✓ A circular vessel fitted with the movable piston at all the four ends and when piston A is moved downward pressure is exerted downward. An equal amount of pressure is exerted along with all the directions, as a result, they will move equal distances outward.

Applications: Pascal's Law for Transmission of Fluid Pressure

Hydraulic Lift

- A hydraulic lift is a lift which makes use of a fluid.
  - ✓ For example, Hydraulic lifts that are used in car service stations to lift cars.
- Inside a hydraulic lift, there are 2 platforms, one has a smaller area and the other one has a larger area.
- It is a tube-like structure that is filled with uniform fluid.
- There are 2 pistons (P1 and P2) which are attached at both the ends of the tube.



- The cross-sectional area of piston P1 is A1 and piston P2 is A2.
- If we apply force F1 on P1, the pressure gets exerted and according to Pascal's law the pressure gets transmitted in all the directions and the same pressure gets exerted on the other end. As a result, the Piston P2 moves upwards.
- The advantage of using the hydraulic lift is that by applying small force on the small area we can generate a larger force.
- Mathematically, $F_2 = PA_2$

Where F_2 = Resultant Force,

A_2 = area of cross-section

$F_2 = P A_2$

$F_2 = (F_1/A_1) A_2$

where $P = F_1/A_1$ (Pressure P is due to force F_1 on the area A_1)

$F_2 = (A_2/A_1) F_1$. This shows that the applied force has increased by A_2/A_1.

Because of Pascal's law, the input gets magnified.

NOTE

There is no gain in work. The work done by force F_1 is equal to the work done by F_2. The piston P1 has to be moved down by a larger distance compared to the distance moved up by piston P2.

NCERT Numerical

Question: A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 cm^2. What maximum pressure would the smaller piston have to bear?

Solution:

The maximum mass of a car that can be lifted, m = 3000 kg

Area of a cross-section of the load-carrying piston, A = 425 cm^2

$= 425 \times 10^{-4}$ m^2

The maximum force exerted by the load, F = mg

= 3000 × 9.8

= 29400 N

The maximum pressure exerted on the load-carrying piston, P = F/A

= 29400 / 425 x 10^5

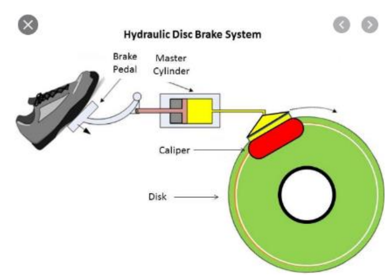
= 6.917 × 105 Pa

Pressure is transmitted equally in all directions in a liquid.

Therefore, the maximum pressure that the smaller piston would have to bear is 6.917×10^5 P_a.

Hydraulic Brakes

- Hydraulic brakes work on the principle of Pascal's law.
- According to this law whenever pressure is applied on fluid it travels uniformly in all the directions.
- Therefore, when we apply force on a small piston, the pressure gets created which is transmitted through the fluid to a larger piston. As a result of this larger force, uniform braking is applied on all four wheels.
- As braking force is generated due to hydraulic pressure, they are known as hydraulic brakes.
- Liquids are used instead of gas as liquids are incompressible.



Working

- When we press the brake pedal, the piston in the master cylinder forces the brake fluid through a linkage.
- As a result, pressure increases and gets transmitted to all the pipes and all the wheel cylinders according to Pascal's law.
- Because of this pressure, both the pistons move out and transmit the braking force on all the wheels.

NCERT Numerical

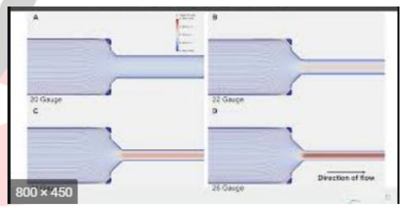
Question: Two syringes of different cross-sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0 cm and 3.0 cm respectively.

a) Find the force exerted on the larger piston when a force of 10 N is applied to the smaller piston.

b) If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out?

Solution

a) Since pressure is transmitted undiminished throughout the fluid,



$$F_2 = \frac{A_2}{A_1}F_1 = \frac{\pi\left(\frac{3}{2\text{x}10^{-2}\text{m}}\right)^2}{\pi\left(\frac{1}{2\text{x}10^{-2}\text{m}}\right)^2}\text{x}10\text{ N}$$

= 90 N

b) Water is considered to be perfectly incompressible. The volume covered by the movement of smaller piston inwards is equal to volume moved outwards due to the larger piston.

$$L_1A_1 = L_2A_2$$

$$L_2 = \frac{A_1}{A_2}L_1 = \frac{\pi\left(\frac{1}{2\text{x}10^{-2}\text{m}}\right)^2}{\pi\left(\frac{3}{2\text{x}10^{-2}\text{m}}\right)^2}\text{x}6\text{x}10^{-2}\text{ N}$$

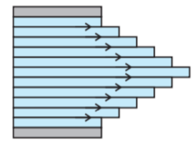
$$\simeq 0.67\text{x}10^{-2}\text{m} = 0.67\text{ cm}$$

Note, atmospheric pressure is common to both pistons and has been ignored.

Viscosity

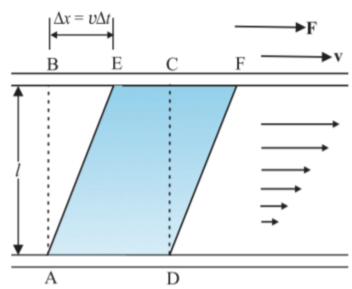
- Viscosity is the property of a fluid that resists the force tending to cause the fluid to flow.
- It is analogous to friction in solids.
- Example:
  - ✓ Consider 2 glasses one filled with water and the other filled with honey.
  - ✓ Water will flow down the glass very rapidly whereas honey won't. This is because honey is more viscous than water.
  - ✓ Therefore, to make honey flow we need to apply a greater amount of force. Because honey has the property to resist the motion.

- Viscosity comes into play when there is relative motion between the layers of the fluid.
- The different layers are not moving at the same pace.



Coefficient of Viscosity

- A layer of liquid sandwiched between two parallel glass plates in which the lower plate is fixed and the upper one is moving to the right with velocity v
- The coefficient of viscosity is the measure of the degree to which a fluid resists flow under an applied force.
- This means how much resistance does a fluid has to its motion.



- $\eta = \frac{\text{shearing stress}}{\text{strain rate}}$

  It is denoted by 'η'.
- Mathematically,

  Δt=time,

  Δx=displacement

  shearing stress = F/A

  shear strain = Δx/l where l= length

  shear strain rate=Δx/lΔt = v/l (Δx/Δt =v = velocity of top layer)

$$\eta = \frac{\text{shearing stress}}{\text{strain rate}}$$

$$\eta = \frac{\frac{F}{A}}{\frac{v}{l}}$$

$$\boxed{\eta = \frac{Fl}{vA}}$$

- Unit: Poiseiulle Nsm^{-2}
- Dimensional Formula: $[ML^{-1}T^{-1}]$

NOTE

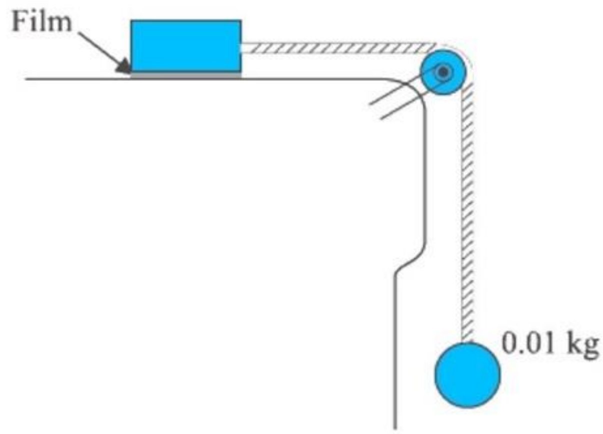
- The viscosity of a liquid decreases with the increase in temperature.
- The viscosity of gases increases with the increase in temperature.

NCERT Example

Question: A metal block of area 0.10 m^2 is connected to a 0.010 kg mass via a string that passes over an ideal pulley (considered massless and frictionless), as in Fig. A liquid with a film thickness of 0.30 mm is placed between the block and the table. When released the block moves to the right with a constant speed of 0.085 ms^{-1}. Find the coefficient of viscosity of the liquid.

Solution:

The metal block moves to the right because of the tension in the string. The tension T is equal in magnitude to the weight of the suspended mass m.



Thus, the shear force F is

$F = T = mg = 0.010 \text{ kg} \times 9.8 \text{ ms}^{-2} = 9.8 \times 10^{-2}$ N

(speed is constant, hence F=T)

Shear stress on the fluid $= \frac{F}{A} = \frac{9.8x10^{-2}}{0.10}$

Strain rate $= \frac{v}{l} = \frac{0.085}{0.030}$

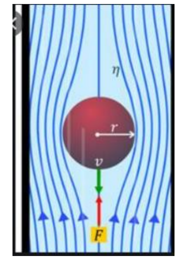
$$\eta = \frac{\text{stress}}{\text{strain rate}} = \frac{\frac{F}{A}}{\frac{v}{l}}$$

$$= \frac{(9.8x10^{-2}\text{N})(0.30x10^{-3}\text{m})}{(0.085 \text{ ms}^{-1})(0.10\text{m}^2)}$$

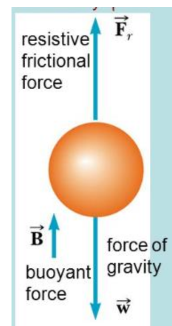
$$= 3.45x10^{-3}\text{Pa s}$$

Stokes Law

- The force that retards a sphere moving through a viscous fluid is directly proportional to the velocity and the radius of the sphere, and the viscosity of the fluid.



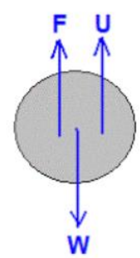
- Mathematically:
  F = 6πηrv where
  F=retarding force
  v =velocity of the sphere
  r=radius of the sphere
  6π=constant
- Stokes law applies only to the laminar flow of liquids. It does not apply to turbulent law.
- Derivation



  $F \propto \boldsymbol{\eta}^a$
  $F \propto \mathbf{r}^b$
  $F \propto \mathbf{v}^c$
  $F \propto \boldsymbol{\eta}^a \mathbf{r}^b \mathbf{v}^c$
  By dimensional analysis
  We get
  F = 6πηrv

Terminal Velocity

- Terminal velocity is the maximum velocity of a body moving through a viscous fluid.
- When viscous force plus buoyant force becomes equal to force due to gravity, the net force becomes zero and so does the acceleration.
- After that point velocity becomes constant. This velocity is known as terminal velocity.
- It is denoted by 'v_t'. Where t=terminal.
- Mathematically



  $W = mg = \frac{4}{3}\pi r^3 \rho g$
  ρ = density of solid
  V = $\frac{4}{3}\pi r^3$ = volume of solid
  U = weight of liquid displaced = $mg = \left(\frac{4}{3}\pi r^3\right)\sigma g = V\sigma g$
  σ = density of liquid

  F = $6\pi\eta r v$

  U + F = W

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

$$6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

$$6\pi\eta r v = \frac{4}{3}\pi r^3 g(\rho - \sigma)$$

$$V = \frac{2}{9} r^2 g \frac{(\rho - \sigma)}{\eta}$$ … … expression for terminal velocity

NOTE

- $v \propto r^2$
  Bigger raindrop fall with larger velocity.
- $v \propto (\rho - \sigma)$
  $\rho > \sigma, v$ downwards
  $\rho < \sigma, v$ upwards (the body will rise through the fluid eg.air bubble moves upwards)
  $\rho = \sigma, v = 0$
- $v \propto \frac{1}{\eta}$

NCERT Example

Question: The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20^{o} C is 6.5 cm s^{-1}. Compute the viscosity of the oil at 20^{o} C. Density of oil is 1.5×10^3 kg m^{-3}, the density of copper is 8.9×10^3 kg m^{-3}.

Solution: We have $v_t = 6.5 \times 10^{-2}$ ms^{-1},
$r = 2 \times 10^{-3}$ m,
$g = 9.8$ ms^{-2},
$\rho = 8.9 \times 10^3$ kg m^{-3},
$\sigma = 1.5 \times 10^3$ kg m^{-3}

$$\eta = \frac{2}{9} x \frac{(2x10^{-3})m^2 x9.8ms^{-2}}{6.5x10^{-2}ms^{-1}} x7.4x10^3 kgm^{-3}$$

$$= 9.9x10^{-1} kgm^{-1}s^{-1}$$

Question: Calculate the terminal velocity in air of an oil drop of radius $2x10^5$ m from the following data
$g=9.8m/s^2$;
coefficient of viscosity of air = $1.8x10^{-5}$ Pas;
density of oil=900 kg/m^3.
The upthrust of air may be neglected.

Solution: Radius $r = 2x10^{-5}$ m
$g=9.8m/s^2$
$\eta=1.8x10^{-5}$Pas
$\rho = 900kg/m^3$
$v=v_t$ when $6\pi\eta rv = mg$

$6\pi\eta rv = \rho x \frac{4}{3} \pi r^3 g$

Simplifying: - $v_t = 2r^2 g\rho/9\,\eta$ = (2 x (2x10^{-5})2x9.8x900)/9x1.8x10^{-5}

=4.36 cm/s

Question: Eight raindrops of radius 1mm each falling downwards with a terminal velocity of 5cm/s coalesce to form a bigger drop. Find the terminal velocity of a bigger drop.

Solution:

$$\frac{4}{3}\pi R^3 = 8 \text{ x } \frac{4}{3}\pi r^3$$

$$R = 2r$$

$$R = 0.2 \text{ cm}$$

$$v = \frac{2}{9} r^2 \frac{(\rho - \sigma)}{\eta} g$$

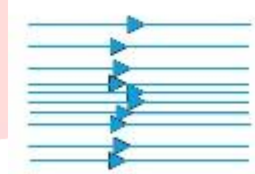
$$v \propto r^2$$

$$\frac{v_2}{v_1} = \frac{R_2}{R_1}$$

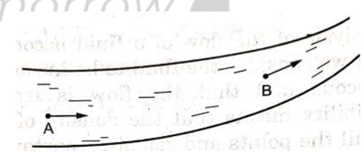
$$v_2 = v_1 \frac{R^2}{r^2} = 5 \text{ x } (\frac{2r}{r})^2 = 20 \text{cm/s}$$

STREAMLINE FLOW (Steady Flow)

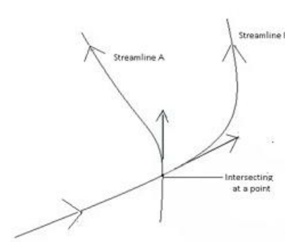
Some streamlines for fluid flow



- The flow of a fluid is said to be steady if, at any point, the velocity of each passing fluid particle remains constant within that interval of time.
- Streamline is the path followed by the fluid particle.
- If the velocity of liquid flowing is small
- As a particle goes from A to another point B its speed and direction may change, but all the particles reaching A will have the same speed at A and all particles reaching B will have the same speed at B.

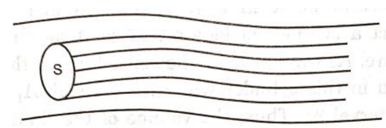


- If one particle passing through A has gone through B, then all the particles passing through
- Each particle follows the same path as taken by a previous particle passing through that point. All particles of the fluid pass through P with velocity v1, through Q with velocity v2, and through R with velocity v3, then the flow is steady.
- The path taken by a fluid particle under a steady flow is a streamline.



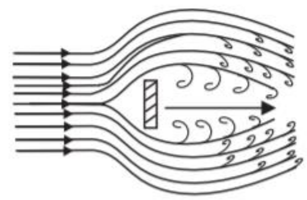
- The tangent to the streamline at a point gives the direction of the flow of fluid at that point.
- No two streamlines can intersect each other because, at the point of intersection, there will be 2 directions for their flow which will disturb the steady nature of the liquid flow.
- Steady flow is termed as ‘Streamline flow’.

Tube of flow



- Consider an area S in a fluid in a steady flow.
- Streamline from all the points of the boundary of S is drawn.
- These streamlines from the tube of cross-section S.
- This tube is called a Tube of flow.
- As the streamlines do not cross each other fluid flowing through different tubes of flow cannot intermix although there is no physical partition between the tubes.

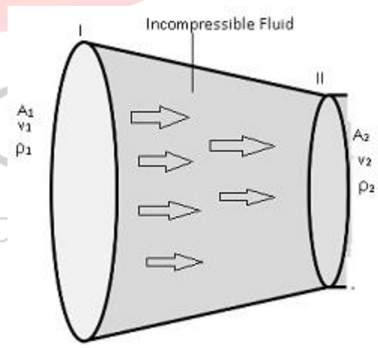
Turbulent flow



- In turbulent flow the path and the velocity of the particles of the liquid change continuously and haphazardly with time from point to point.
- When an obstacle is placed in the path of the part moving fluid, it causes turbulence.

Equation of Continuity

- According to the equation of continuity
  Av = constant.
  A =cross-sectional area
  v=velocity with which the fluid flows.
- **It means that if any liquid is flowing in streamline flow in a pipe of non-uniform cross-section area, then the rate of flow of liquid across any cross-section remains constant.**
- **Mathematical expression**



Consider a fluid flowing through a tube of varying thickness.
Let the cross-sectional area at one end (I) = A_1
cross-sectional area of other end (II)= A_2.
The velocity and density of the fluid at one end (I)=v_1,ρ_1 respectively,
velocity and density of fluid at other end (II)= v_2,ρ_2
The volume covered by the fluid in a small interval of time Δt
across left cross-sectional is Area (I) =$A_1 \times v_1 \times \Delta t$
The volume covered by the fluid in a small interval of time Δt
across right cross-sectional Area (II) = $A_2 \times v_2 \times \Delta t$
volume flowing in the tube in Δt = volume flowing out in Δt……………..(1)
The fluid inside is incompressible
(volume of fluid does not change by applying pressure)
density remains the same $\rho_1=\rho_2$……………………………………….(2)
Mass of liquid flowing at left end=$\rho_1 A_1 v_1 \Delta t$
Mass of liquid flowing at right end = $\rho_2 A_2 v_2 \Delta t$
From Eqn (1) and Eqn (2), we get

$$A_1 v_1 = A_2 v_2$$

Av=constant.
This is the equation of continuity.

- This equation is also termed as "**Conservation of the mass of incompressible fluids**".

NOTE

- The larger is the area of cross-section, the smaller will be the velocity of liquid flow and vice versa. The fluid is accelerated while passing from the wider cross-sectional area towards the narrower area.
- So the volume which is covered by the fluid at any cross-sectional area is constant throughout the pipe even if the pipe has different cross-sectional areas.

Question: The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2 one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m min^{-1}, what is the speed of the ejection of the liquid through the holes?

Solution:

Area of a cross-section of the spray pump, $A_1 = 8\ cm^2 = 8 \times 10^{-4}\ m^2$

Number of holes, $n = 40$

Diameter of each hole, $d = 1\ mm = 1 \times 10^{-3}\ m$

Radius of each hole, $r = d/2 = 0.5 \times 10^{-3}\ m$

Area of cross-section of each hole, $a = \pi r^2 = \pi\ (0.5 \times 10^{-3})^2\ m^2$

Total area of 40 holes,

$A_2 = n \times a = 40 \times \pi\ (0.5 \times 10^{-3})^2\ m^2$

$= 31.41 \times 10^{-6}\ m^2$

V_1 = Speed of flow of liquid inside the tube= 1.5 m/min = 0.025 m/s

V_2=Speed of ejection of liquid through the holes

According to the law of continuity, we have:

$A_1V_1 = A_2V_2$

$V_2 = A_1V_1/ A_2$

$= (8 \times 10^{-4} \times 0.025)\ 31.61 \times 10^{-6}$

$= 0.633$ m/s

Therefore, the speed of ejection of the liquid through the holes is **0.633 m/s.**

Poiseuille's Equation

According to Poiseuille, if a pressure difference (P) is maintained across the two ends of a capillary tube of length 'l' and radius 'r', then the volume of liquid coming out of the tube per second is directly proportional to the pressure difference (P).

- $V \propto r^4$ directly proportional to the fourth power of radius (r) of the capillary tube.
- $V \propto \frac{1}{\eta}$ inversely proportional to the coefficient of viscosity (ή) of the liquid.
- $V \propto \frac{P}{l}$ directly proportional to the pressure gradient.
- It is given as, $$V = \frac{\pi P r^4}{8\eta l}$$

Reynold's Number

Reynold number R_e is a dimensionless number whose value gives an approximate idea of whether the flow of fluid will be streamline or turbulent.

It is given by, $$R_e = \frac{\rho v d}{\eta}$$

where ρ = density of the fluid flow

d = the diameter of the pipe and

η = viscosity of the fluid

v = velocity of the fluid

- The value of R_e remains the same in any system of units.
- $R_e <= 1000$ flow is streamline or laminar
- $R_e >= 2000$ flow is turbulent
- The flow becomes unsteady for R_e between 1000 and 2000.

Reynold's no can also be defined as

$$R_e = \frac{inertial\ force}{viscous\ force}$$

$$R_e = \frac{\rho A v^2}{\left(\frac{\eta A v}{d}\right)}$$

$$R_e = \frac{\rho v^2}{\left(\frac{\eta v}{d}\right)}$$

Critical Velocity

The critical velocity is the limiting value of the velocity of liquid flow, up to which the flow is streamlined and above which the flow becomes turbulent.

$$v_c = \frac{K\eta}{\rho r}$$

where K = dimensionless constant,

q = coefficient of viscosity of liquid,

ρ = density of liquid

r = radius of tube.

NOTE

- The flow of liquids of higher viscosity and lower density through narrow pipes tends to be streamlined.
- The flow of liquids of lower viscosity and higher density through broad pipes tends to become turbulent because in that case, the critical velocity will be very small.

NCERT EXAMPLE

QUESTION: The flow rate of water from a tap of diameter 1.25 cm is 0.48 L/min. The coefficient of viscosity of water is 10^{-3} Pa s. After some time, the flow rate is increased to 3 L/min. Characterize the flow for both the flow rates.

SOLUTION:

$D = 1.25 \text{ cm} = \frac{1.25}{100} \text{m}$

$\eta_{water} = 10^{-3} P_a \text{ sec}$

$\rho_{water} = 1000 \text{ kgm}^{-3}$

Q = volume of water flowing out per second

Q = v x a

$Q = v \times \frac{\pi D^2}{4}$

$v = \frac{4Q}{\pi D^2}$

$$R_e = \frac{\rho v D}{\eta} = \frac{\rho \frac{4Q}{\pi D^2} D}{\eta} = \frac{4\rho Q}{\pi D \eta}$$

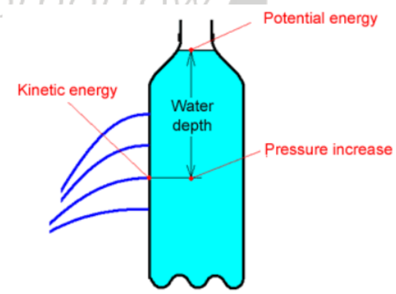
a) $Q = 0.48 \frac{L}{min} = \frac{.48x\left(\frac{1}{1000}\right)}{60} = 8x10^{-6} \frac{m^3}{sec}$

$R_e = \frac{4\rho Q}{\pi D \eta} = \frac{4x1000x8x10^{-6}}{3.14x1.25x10^{-2}x10^{-3}} = 815 \text{ (flow is steady)}$

b) $Q = 3 \frac{L}{min} = \frac{3x\left(\frac{1}{1000}\right)}{60} = 5x10^{-5} \frac{m^3}{sec}$

$R_e = \frac{4\rho Q}{\pi D \eta} = \frac{4x1000x5x10^{-6}}{3.14x1.25x10^{-5}} = 5096 \text{ (flow is turbulent)}$

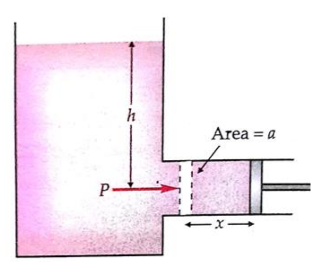
The energy of a Liquid



- A liquid in motion possess **three** types of energy

  Potential Energy
  - It is the energy possessed by the liquid by virtue of its height or position above the surface of earth or any reference level is taken as zero level. i.e. P.E = mgh
  - P.E per unit mass = $\frac{mgh}{m} = gh$
  - P.E per unit volume = $\frac{mgh}{V} = \rho gh$

  Pressure Energy



  - It is the energy possessed by the liquid by virtue of its pressure or work done in pushing the liquid in the vessel against pressure without imparting any velocity to it.
  - Mathematical expression:
    The liquid is pushed in with force F
    Pressure on the piston = P = ρgh

- The volume of liquid pushed into the tank = ax
  a = area of cross-section of the piston
  x = distance moved in by piston
  Mass of liquid pushed = $volume \times density = ax\rho$
  Force F = Pa
  Work done = Fx = Pax
  Pressure energy = Work done = Pax
  Pressure energy = P_r
  P_r.E per unit mass = $\frac{Pax}{ax\rho} = \frac{P}{\rho}$

  P_r.E per unit volume = $\frac{Pax}{ax} = P = \rho gh$

  Pressure energy per unit volume of the liquid is equal to the hydrostatic pressure due to the liquid.

Kinetic Energy

- It is the energy possessed by the liquid by virtue of its motion.
- K.E = $\frac{1}{2}mv^2$
- K.E per unit mass = $\frac{\frac{1}{2}mv^2}{m} = \frac{v^2}{2}$
-
- K.E per unit volume = $\frac{\frac{1}{2}mv^2}{V} = \frac{1}{2}\rho v^2$

NOTE

All three types of energy possessed by a liquid are manually convertible from one form to another.

Bernoulli's Theorem

This theorem states that for the streamline flow of an ideal fluid, the total energy i.e. the sum of pressure energy, potential energy, and kinetic energy per unit mass remains constant at every cross-section throughout the liquid flow.

i.e. $\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$

$\frac{P}{\rho}$ = Pressure energy per unit mass

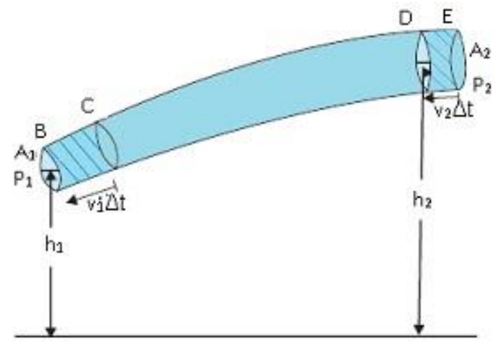
gh = Potential energy per unit mass

$\frac{1}{2}v^2$ = Kinetic energy per unit mass

Derivation

Perfectly incompressible, irrotational and non-viscous (Ideal Fluid)

Consider a tube of varying cross-section through which an ideal liquid is in streamline flow.



P_1 = Pressure applied on liquid at A_1

P_2 = Pressure at end B, against which liquid is to move out

a_1 = area of cross section at A_1

a_2 = area of cross section at A_2

h_1 = height of section A_1 from reference level

h_2 = height of section A_2 from reference level

v_1 = velocity of liquid flow at A_1

v_2 = velocity of liquid flow at A_2

ρ = density of ideal fluid remains constant (fluid is incompressible)

liquid flows from A to B so, $\mathbf{P_1 > P_2}$

from equation of continuity,

$$a_1 v_1 \rho = a_2 v_2 \rho = m$$

$$a_1 v_1 = a_2 v_2 = \frac{m}{\rho} = V$$

V = volume of liquid flowing

Force on liquid at $A_1 = P_1 a_1$

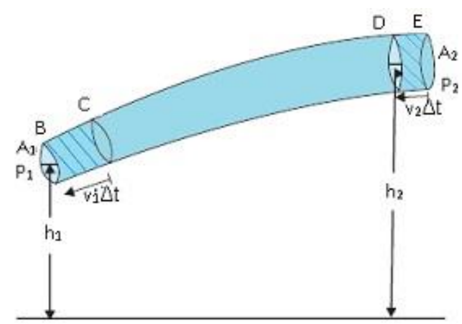
Force on liquid at $A_2 = P_2 a_2$

Work done/sec on liquid at $A_1 = (P_1 a_1) v_1 = P_1 V$

W=Fv

Work done/sec by liquid against pressure energy at $A_2 = (P_2 a_2) v_2 = P_2 V$

Net work done = $P_1V - P_2V$



Change in P.E = $mg (h_2 - h_1)$

Change in K.E = $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

According to the work-energy principle

$$P_1V - P_2V + mgh_1 - mgh_2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$P_1V - P_2V + mgh_1 - mgh_2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

$$P_1V + mgh_1 + \frac{1}{2}mv_1^2 = P_2V + mgh_2 + \frac{1}{2}mv_2^2$$

Dividing throughout by ‘m’

$$\frac{P_1V}{m} + \frac{mgh_1}{m} + \frac{\frac{1}{2}mv_1^2}{m} = \frac{P_2V}{m} + \frac{mgh_2}{m} + \frac{\frac{1}{2}mv_2^2}{m}$$

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$$

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$$

OR

$$\frac{P_r.E}{m} + \frac{P.E}{m} + \frac{K.E}{m} = \text{constant}$$

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

$$\frac{P_r.E}{\text{volume}} + \frac{P.E}{\text{volume}} + \frac{K.E}{\text{volume}} = \text{constant}$$

Note:

1. If the two ends of a tube are at the same level, then h = 0

i.e. $P + \frac{1}{2}\rho v^2 = \text{constant}$

if P increases, v decreases, and vice versa

Thus, Bernoulli's Theorem also states that in the streamline flow of an ideal liquid through a horizontal tube, the velocity increases where pressure decreases and vice versa.

P = Static Pressure

$\frac{1}{2}\rho v^2$ = Dynamic Pressure

2. Hydrostatics

Speed of the liquid is zero everywhere i.e. $v_1 = v_2 = 0$

$P_1 + \rho gh_1 = P_2 + \rho gh_2$

$P_1 - P_2 = \rho g(h_2 - h_1)$

Question: Water flows through a horizontal pipeline of varying cross-section. If the pressure of water equals 6 cm of mercury at a point where the velocity of flow is 30 cm/s, what is the pressure at the another point where the velocity of flow is 50 m/s?

Solution:

$P_1 = \rho g\, h = 6\times10^{-2} \times 13600 \times 9.8 = 7997\ N/m^2$

$v_1 = 30\ cm/s = 0.3\ m/s$

$v_2 = 50\ cm/s = 0.5\ m/s$

From Bernoulli's equation

$P + (1/2)\ \rho\ v^2 + \rho g\ h = \text{constant}$

$P_1 + (1/2)\ \rho\ v_1^2 = P_2(1/2)\ \rho\ v_2^2$

$7997 + 1/2 \times 1000 \times (0.3)^2 = P_2 + ½ \times 1000 \times (0.5)^2$

$P_2 = 7917\ N/m^2$

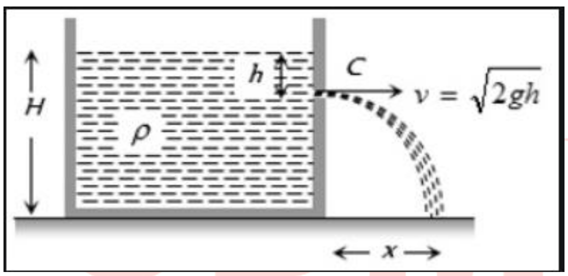
$P_2 = \rho g\ h_2$

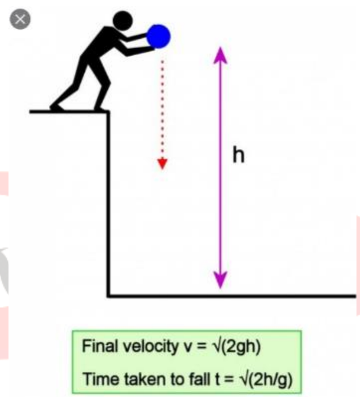
$P_2 = h_2 x13600x9.8$

$h_2 = 5.9$ cm Hg

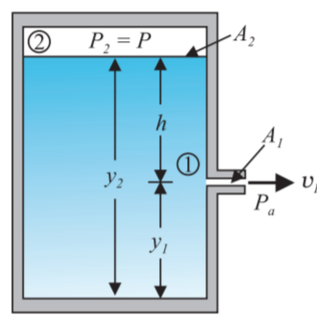
Torricelli's law

- Torricelli law states that the speed of flow of fluid from an orifice is equal to the speed that it would attain if falling freely for a distance equal to the height of the free surface of the liquid above the orifice.
- Consider any vessel which has an orifice (slit)filled with some fluid.
- The fluid will start flowing through the slit and according to Torricelli law, the speed with which the fluid will flow is equal to the speed with which a freely falling body attains such that the height from which the body falls is equal to the height of the slit from the free surface of the fluid.
- Let the distance between the free surface and the slit = h
- The velocity with which the fluid flows is equal to the velocity with which a freely falling body attains if it is falling from a height h i.e **$v = \sqrt{2gh}$**.





Derivation of the Law



Let A_1 = area of the slit (it is very small),
v_1 = Velocity with which fluid is flowing out.
A_2 = Area of the free surface of the fluid,
v_2=velocity of the fluid at the free surface.

From Equation of Continuity, Av=constant
Therefore $A_1v_1 = A_2v_2$
From the figure, $A_2 >>> A_1$, this implies $v_2 << v_1$.
This means fluid is at rest on the free surface. Therefore $v_2 \sim 0$

Using Bernoulli's equation,
$P + (1/2)\rho v^2 + \rho gh$ = constant.

Applying Bernoulli's equation at the slit:
$P_a + (1/2)\rho v_1^2 + \rho g y_1$ …………………(1)
where P_a=atmospheric pressure,
y_1=height of the slit from the base

Applying Bernoulli's equation at the surface:

$P+\rho g y_2$ .... (2)

$v_2=0$ therefore $(1/2)\ \rho\ v_1^2=0$,

y_2=height of the free surface from the base.

By equating (1) and (2),

$$P_a+ (1/2)\ \rho\ v_1^2+ \rho g y_1= P+\rho g y_2$$

$$(1/2)\ \rho\ v_1^2 = (P-P_a) + \rho g(y_2-y_1)$$

$$(1/2)\ \rho\ v_1^2 =(P-P_a)\ \rho g h \qquad \text{(where } h=(y_2-y_1))$$

$$v_1^2= 2gh + \frac{2(P-Pa)}{\rho}$$

$$\boxed{v_1 = \sqrt{2gh + \frac{2(P-Pa)}{\rho}}}$$

v_1 is known as Speed of Efflux. This means the speed of the fluid outflow.

Case1:

When the tank is open

$P = P_a$

$v_1 =$**$\sqrt{2gh}$**

This is the speed of a freely falling body.

This is per Torricelli's law which states that the speed by which the fluid is flowing out of a small slit of a container is the same as the velocity of a freely falling body.

Case2:

The tank is not open to the atmosphere but $P>>P_a$.

Therefore 2gh is ignored as it is very very large, hence **$v_1= \sqrt{2P/\rho}$.**

The velocity with which the fluid will come out of the container is determined by the pressure at the free surface of the fluid alone.

Question: Calculate the velocity of the emergence of a liquid from a hole in the side of a wide cell 15cm below the liquid surface?

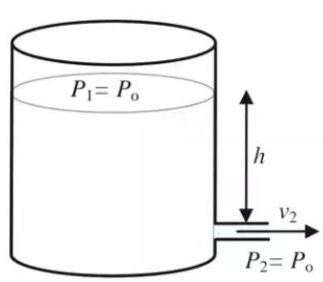
Solution:

h = 15cm

By using Torricelli's law $v_1 = \sqrt{2gh}$

$= \sqrt{2 \times 9.8 \times 15 \times 10^{-2}}$ m/s

$= 1.7$ m/s

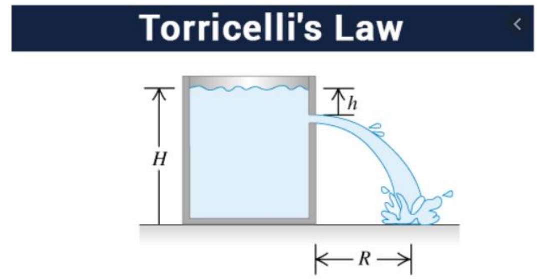


Question: Water stands at a height 'H' in a tank whose side walls are vertical. A hole is made in one of the walls at a depth 'h' below the water surface.

a) Find at what distance from the foot of the wall does the emerging stream of water strike the floor?

b) For what value of 'h'. this range is maximum?

c) Can a hole be made at another depth so that the second stream has the same range?



Solution:

a) $v = \sqrt{2gh}$

$$s = ut + \frac{1}{2}at^2$$

$$H - h = 0 \text{ x } t + \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2(H-h)}{g}}$$

$$R = vt = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$$

$$\boldsymbol{R = 2\sqrt{h(H-h)}}$$

b) for 'R' to be maximum $\frac{dR}{dh} = 0$

$$\frac{dR}{dh} = 2\frac{d}{dh}(h(H-h))^{\frac{1}{2}}$$

$$= 2x\frac{1}{2}(h(H-h^2)^{\frac{1}{2}-1}x\frac{d}{dh}(hH-h^2)$$

$$= \frac{1}{h\sqrt{H-h}}(H-2h)$$

$$\frac{dR}{dh} = 0$$

$$\mathbf{h} = \boldsymbol{\frac{H}{2}}$$

c) $2\sqrt{h(H-h)} = 2\sqrt{x(H-x)}$

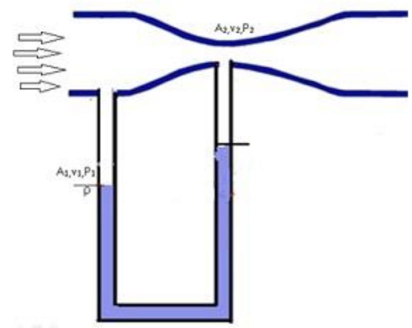
$h(H-h) = x(H-x)$

x^2 -Hx + (Hh-h^2) = 0

x = h or (H-h)

Venturimeter

- Venturimeter is a device to measure the flow of incompressible liquid.
  It consists of a tube with a broad diameter having a larger cross-sectional area but there is a small constriction in the middle.
  It is attached to the U-tube manometer. One end of the manometer is connected to the constriction and the other end is connected to the broader end of the Venturimeter.
  The U-tube is filled with a fluid whose density is ρ.
  A_1= cross-sectional area at the broader end,
  v_1 = velocity of the fluid.
  A_2=cross-sectional area at constriction,
  v_2= velocity of the fluid.



- By the equation of continuity, wherever the area is more velocity is less and vice-versa. As A_1 is more this implies v_1 is less and vice-versa.
- Pressure ∝ A
  Therefore at A_1 pressureP_1 is less as compared to pressure P_2 at A_2.
- This implies $P_1<P_2$ as $v_1>v_2$.
- As there is a difference in the pressure the fluid moves, this movement of the fluid is marked by the level of the fluid increase at one end of the U-tube.

Venturimeter: determining the fluid speed

By Equation of Continuity:

$A_1v_1 = A_2v_2$

This implies $v_2 = \frac{A_1}{A_2}v_1$ ………………………………………………(1)

By Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho (\frac{A_1}{A_2})^2 v_1^2$$

$P_1 - P_2 = \frac{1}{2}\rho v_1^2((\frac{A_1}{A_2})^2 - 1)$……………………………………(2)

$$v_1^2 = \frac{2(P_1 - P_2)}{\rho[(\frac{A_1}{A_2})^2 - 1]}$$

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho[(\frac{A_1}{A_2})^2 - 1]}}$$

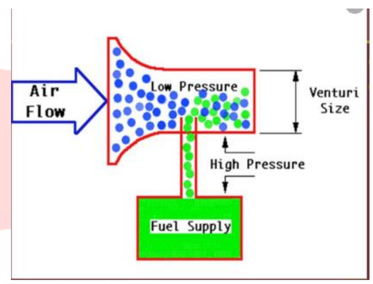
As there is pressure difference the level of the fluid in the U-tube changes.

- $(P_1-P_2) = h\rho_m g$ where ρ_m (density of the fluid inside the manometer).
- $1/2\rho v_1^2(A_1^2/A_2^2-1) = h\rho_m g$

- $$v_1 = \sqrt{\frac{2h\rho mg}{\rho[(\frac{A_1}{A_2})^2 - 1]}}$$

Practical Application of Venturimeter

- Spray Gun or perfume bottle- They are based on the principle of Venturimeter.
  - ✓ Consider a bottle filled with fluid and have a pipe that goes straight till constriction. There is a narrow end of the pipe which has a greater cross-sectional area.
  - ✓ The cross-sectional area of constriction which is in the middle is less.
  - ✓ There is pressure difference when we spray as a result some air goes in, the velocity of the air near constriction increases, and hence pressure decreases.
  - ✓ The difference in pressure between the top and bottom of the tube draws the perfume upward.
- The carburetor of the automobile has a Venturi channel (nozzle) through which air flows at large speed. The pressure is then lowered at the narrow neck and the petrol (gasoline) is sucked up in the chamber to provide the correct mixture of air to fuel necessary for combustion.



Question: The flow of blood in a large artery of an anesthetized dog is diverted through a Venturimeter. The wider part of the meter has a cross-sectional area equal to that of the artery. A = 8 mm^2. The narrower part has an area a = 4 mm^2. The pressure drop in the artery is 24 Pa. What is the speed of the blood in the artery?

Solution:
The density of blood is 10.1 to be $1.0^6 \times 10^3$ kg m^{-3}
The ratio of the areas is (A/a) = 2
Using Equation **= $2h\rho_m g/\rho[A_1^2/A_2^2-1]^{-1/2}$**
$v_1 = \sqrt{2x24P_a}/(1060kgm^{-3}x\,(2^2-1)) = 0.125ms^{-1}$

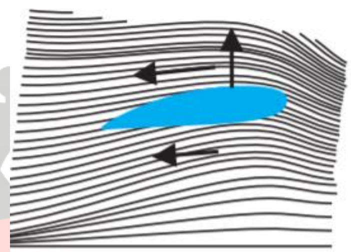
Dynamic Lift

- Dynamic lift is the normal force that acts on a body by virtue of its motion through a fluid.
- Consider an object which is moving through the fluid, and due to the motion of the object through the fluid, there is a normal force that acts on the body.

- This force is known as a dynamic lift.
- Dynamic lift is most popularly observed in airplanes.
  - ✓ Whenever an airplane is flying in the air, due to its motion through the fluid (fluid is air in the atmosphere). Due to its motion through this fluid, there is a normal force that acts on the body in the vertically upward direction.
  - ✓ This force is known as Dynamic lift.
- Examples:
  - ✓ Airplane wings
  - ✓ Spinning ball in the air(Magnus effect)

Dynamic lift on airplane wings

- Fig. shows an aerofoil, which is a solid piece shaped to provide an upward dynamic lift when the airplane moves horizontally through the air.
- The cross-section of the wings of an airplane looks somewhat like the aerofoil shown in Fig. with streamlines around it.
- When the aerofoil moves against the wind, the orientation of the wing relative to flow direction causes the streamlines to crowd together above the wing more than those below it.
- The flow speed on top is higher than that below it.
- The pressure at the top is less than the pressure below.
- This difference in pressure creates an upward force resulting in a dynamic lift of the wings and this balances the weight of the plane.



Question: In a test experiment on a model airplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m s^{-1} and 63 m s^{-1} respectively. What is the lift on, the wing if its area is 2.5 m^2? Take the density of air to be 1.3 kg m^{-3}.

Solution:
Speed of wind on the upper surface of the wing, V_1 = 70 m/s
Speed of wind on the lower surface of the wing, V_2 = 63 m/s

Area of the wing, A = 2.5 m^2
The density of air, ρ = 1.3 kg m^{-3}

According to Bernoulli's theorem, we have the relation:

Where,

$P_1+1/2 (\rho V_1^2) = P_2+1/2(\rho V_2^2)$

$P_2-P_1 = 1/2 \rho (V_1^2 - V_2^2)$

P_1 = Pressure on the upper surface of the wing
P_2 = Pressure on the lower surface of the wing

The pressure difference between the upper and lower surfaces of the wing provides lift to the airplane.

Dynamic Lift on the wing $= (P_2 - P_1)\,A$

$= 1/2\,\rho\,(V_1^2 - V_2^2)\,A$

$= 1.3((70)^2 – (63)^2)\,x2.5$

$= 1512.87$

$= 1.51 \times 10^3$ N

Therefore, the lift on the wing of the airplane is 1.51×10^3 N.

Question: A fully loaded Boeing aircraft has a mass of $3.3x10^5$ kg. Its total wing area is 500 m^2. It is a level flight with a speed of 960 km/h. Estimate the pressure difference between the lower and upper surfaces of the wings.

Solution:

Weight of the aircraft = Dynamic lift

$mg = (P_1 - P_2)\,A$

$mg/A = \Delta P$

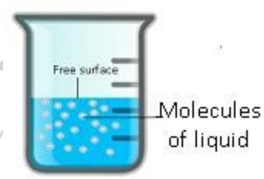
$\Delta P = 3.3x10^5 x9.8/500$

$= 6.5x10^3 N/m^2$

Surface Tension

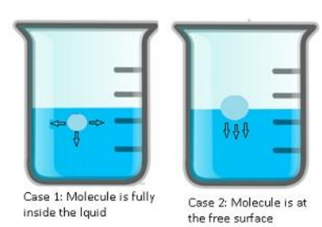
- Surface tension is the property of the liquid surface which arises because surface molecules have extra energy.
- Surface energy is the extra energy that the molecules at the surface have.

Case 1: When molecules are inside the liquid



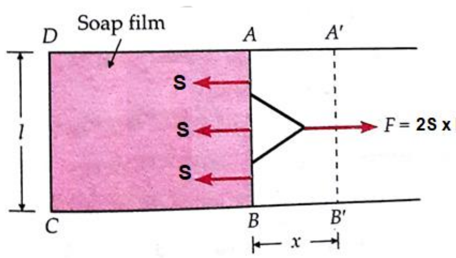
- Suppose there is a molecule inside the water, there will be several other molecules that will attract that molecule in all the directions.
- As a result, this attraction will bind all the molecules together.
- This results in the negative potential energy of the molecule as it binds the molecule.
- To separate this molecule huge amount of energy is required to overcome potential energy.
- Some external energy is required to move this molecule and it should be greater than the potential energy.
- Therefore, to separate this molecule, a huge amount of energy is required.

Case2: When the molecules are at the surface



- When the molecule is at the surface, half of it will be inside and half of it is exposed to the atmosphere.
- For the lower half of the molecule, it will be attracted by the other molecules inside the liquid.
- But the upper half is free. The negative potential energy is only because of the lower half.

- But the magnitude is half as compared to the potential energy of the molecule which is fully inside the liquid.
- So, the molecule has some excess energy, because of this additional energy which the molecules have at the surface different phenomena happen like surface energy, surface tension.
- Liquids always tend to have the least surface area when left to itself.
- As more surface area will require more energy as a result liquid tend to have the least surface area.
- Surface energy is defined as surface energy per unit area of the liquid surface.
- Denoted by 'S'.
- Mathematically
  - ✓ Consider a case in which liquid is enclosed in a movable bar.
  - ✓ Slide the bar slightly and it moves some distance ('d').
  - ✓ There will be an increase in the area, (dl) where l=length of the bar.
  - ✓ Liquids have two surfaces one on the bar and other below the bar. Therefore area=2(dl)
  - ✓ Work done for this change =F x Displacement
  - ✓ Surface tension(S)=Surface Energy/area
  - ✓ Or Surface Energy=S x area
  - ✓ U=Sx2dl
  - ✓ $S=\frac{U}{A}$
  - ✓ Therefore, S x2l× d =F x d



- **S = F/l**
- Surface tension is the surface energy per unit area of the liquid surface.
- It can be also defined as Force per unit length on the liquid surface.

Drops and Bubbles

Why water and bubbles are drops?

- Whenever liquid is left to itself it tends to acquire the least possible surface area so that it has the least surface energy, so it has the most stability.
- Therefore, for more stability, they acquire the shape of a sphere, as a sphere has the least possible area.

Spherical Shape

Distinction between Drop, Cavity and Bubble

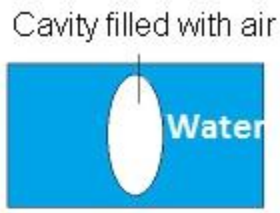
Drop: Drop is a spherical structure filled with water.

- There is only one interface in the drop.
- The interface separates water and air.

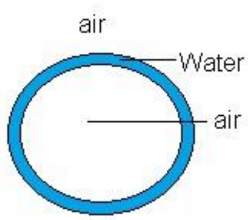
- Example: Water droplet.

water droplet

Cavity: Cavity is a spherical shape filled with air.
- In the surroundings there is water and in the middle, there is a cavity filled with air.
- There is only one interface which separates air and water.
- Example: - bubble inside the aquarium.



Bubble: In a bubble, there are two interfaces. One is air-water and another is water and air.
- Inside a bubble there is air and there is air outside.
- But it consists of a thin film of water.



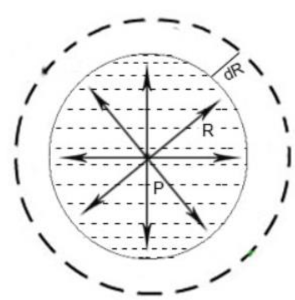
soap bubbles

Excess of pressure on the curved surface of the liquid

- For the curved surface of the liquid in equilibrium, the pressure on the concave side of liquid will be greater than the pressure on its convex side.

Excess pressure inside a liquid drop

- Consider a liquid drop of radius 'R'.
- Due to surface tension, drop experiences a resultant force acting inwards, perpendicular to the surface
- As the size of the drop cannot be zero, the force due to excess pressure acts outwards perpendicular to the surface to counterbalance the resultant force due to surface tension.



Mathematical expression

p = excess pressure = $P_i - P_o$

S = surface tension

P_o= atm pressure

P_i = pressure inside the drop.

Let there be an increase in the radius of the drop by a small quantity ΔR.

W = Work done

W =F x ΔR

W = (excess pressure x area) x ΔR

W = p x $4\pi R^2$ x ΔR……………………………………..(1)

This work done against the force of surface tension is stored inside the drop in the form of its PE.

$\Delta A = A_2 - A_1$

$= 4\pi(R+\Delta R)^2 - 4\pi R^2$

$= (8\pi R)\, \Delta R$

U = S ΔA = S (8πR) ΔR……………………………(2)

(Increase in PE of the drop)

Drop is in equilibrium, so the increase in surface energy is at cost of work done by the excess pressure

From Eqn (1) and (2) we get

W = U

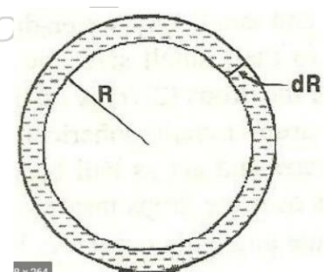
p x $4\pi R^2$ x ΔR = S (8πR) ΔR

$(P_i - P_o) = \frac{2S}{R}$

$$p = \frac{2S}{R}$$

Excess pressure inside a soap bubble

In a bubble, there are two interfaces. One is air-water and another is Water- air.



W = F x ΔR

$= [(P_L - P_o)\, 4\pi R^2]\, \Delta R$

U= Increase in P. E

= S x ΔA

$= S \times 2(4\pi(R+\Delta R)^2 - 4\pi R^2)$

= S x 2(8πRΔR)

= S x 16πRΔR

Work done = Increase in PE

W = U

$p(4\pi R^2)\, \Delta R = S \times 16\pi R \Delta R$

$$p = \frac{4S}{R}$$

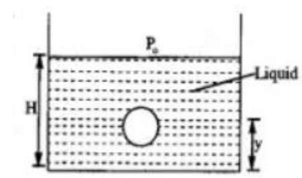
Note

- When the air bubble of radius r is at depth 'h' below the free surface of the liquid of density ρ and surface tension S, then the
  $p = P_i - P_o = 2S/R + h\rho g$

Numerical:

QUESTION: suppose an air bubble of radius 1 mm is formed 10cm below the free surface of the liquid of density 10^3 kg/m^3 and surface tension 0.075 N/m. find the excess pressure inside the air bubble?

SOLUTION



Air bubble R = 1 mm

S = 0.075 N/m

$\rho = 10^3$ kg/m^3

p =?

$p = 2S/R + h\rho g = 1130$ N/m^2

Question:

The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of the water in a beaker. What is the pressure required in the tube to blow a hemispherical bubble at its end in the water? The surface tension of water at temperature of the experiments is 7.30×10^{-2} Nm^{-1}.1 atmospheric pressure = 1.01×10^5 Pa, density of water = 1000 kg/m^3, g = 9.80 m s^{-2}. Also, calculate the excess pressure.

Solution

The excess pressure in a bubble of gas in a liquid is given by 2S/r, where S is the surface tension of the liquid-gas interface. As there is only one liquid surface, therefore using the formula pressure is 2S/r. The radius of the bubble is r. Now the pressure outside the bubble P_o equals atmospheric pressure plus the pressure due to 8.00 cm of the water column. That is

$P_o = (1.01 \times 105 \text{ Pa} + 0.08 \text{ m} \times 1000 \text{ kg m}^{-3} \times 9.80 \text{ m s}^{-2})$

$= 1.01784 \times 10^5$ Pa

Therefore, the pressure inside the bubble is

$P_i = P_o + 2S/r$

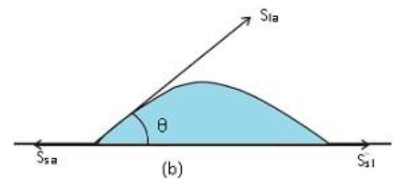
$= 1.01784 \times 10^5 \text{ Pa} + (2 \times 7.3 \times 10^{-2} \text{ Pa m}/10^{-3} \text{ m})$

$= (1.01784 + 0.00146) \times 10^5$ Pa

$= 1.02 \times 10^5$ Pa

where the radius of the bubble is taken to be equal to the radius of the capillary tube since the bubble is hemispherical!

Angle of Contact

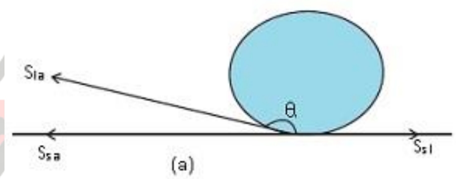


- The angle between the tangent to the liquid surface at the point of contact and solid surface inside the liquid is termed as Angle of contact
- The angle of contact is the angle at which a liquid interface meets a solid surface.
- It is denoted by θ.
- It is different at interfaces of different pairs of liquids and solids.
- For example - Droplet of water on lotus' leaf. The droplet of water (Liquid) is in contact with the solid surface which is the leaf

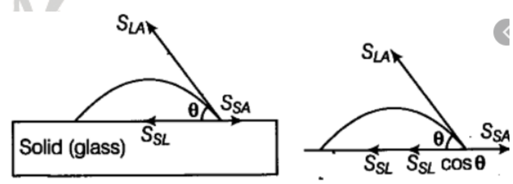
Significance of Angle of Contact

The angle of contact determines whether a liquid will spread on the surface of a solid or it will form droplets on it.

- If the Angle of contact is **obtuse**: then droplet will be formed.
- If the Angle of contact is **acute**: then the water will spread.



- S_{la} = liquid-air surface tension
  S_{sa} = solid air surface tension
  S_{sl} = solid-liquid interface tension.



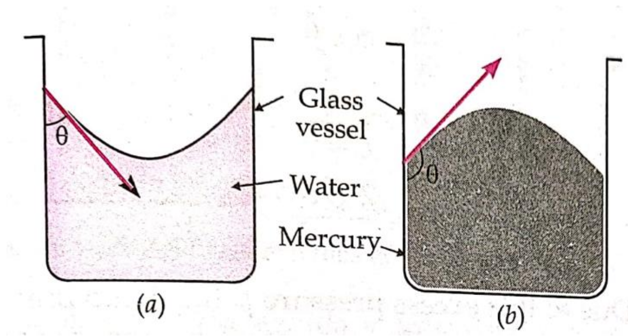
$$S_{SL} + S_{LA}\cos\theta = S_{SA}$$

$$\Rightarrow \quad S_{LA}\cos\theta = S_{SA} - S_{SL}$$

- At equilibrium
  $S_{sa} = S_{la}\cos\Theta + S_{sl}$
  $S_{la}\cos\Theta = S_{sa} - S_{sl}$

$$\cos\Theta = (S_{sa} - S_{sl})/S_{la}$$

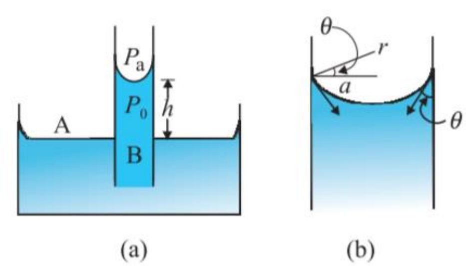
- $S_{sl} > S_{sa}$, then cosΘ is obtuse.
  E_{sl} is more i.e. more energy required to create the solid-liquid surface. Molecules of liquid are attracted strongly to themselves and weakly to those of solid.
  e.g. Waterleaf interface
- $S_{sl} < S_{sa}$, then cosΘ is acute.
  E_{sl} is less i.e. less energy required to create the solid-liquid surface. Molecules of liquids are strongly attracted to those of the solids.
  e.g. Water spreads on glass or plastic.



$\theta < 90^0$, then the liquid spreads on the surface of solid
$\theta > 90^0$, then the liquid forms droplets on the surface of solid

Capillary Rise

- In Latin the word Capilla means hair.
- Due to the pressure difference across a curved liquid-air interface, the water rises up in a narrow tube in spite of gravity.
- Consider a vertical capillary tube of circular cross-section (radius a) inserted into an open vessel of water.
- The contact angle between water and glass is acute. Thus, the surface of the water in the capillary is concave. As a result, there is a pressure difference between the two sides of the top surface. This is given by
  - ✓ $(P_i - P_o) = (2S/r) = 2S/(a \sec\theta) = (2S/a)\cos\theta$ (i)
- Thus, the pressure of the water inside the tube, just at the meniscus (air-water interface) is less than the atmospheric pressure.
- Consider the two points A and B. They must be at the same pressure,
  - ✓ $P_0 + h\rho g = P_i = P_A$ (ii)
  - ✓ where ρ is the density of water, and h is called the capillary rise
  - ✓ $h\rho g = (P_i - P_0) = (2S\cos\theta)/a$ (By using equations (i) and (ii))
  - ✓ $h = \frac{2S}{r\rho g}$ ($r = a\sec\theta$)
- Therefore, the capillary rise is due to surface tension. It is larger, for a smaller radius.



Capillary rise,
a) Schematic picture of an arrow tube immersed water.
b) The enlarged picture near the interface.

Rise of liquid in a capillary tube of insufficient height:

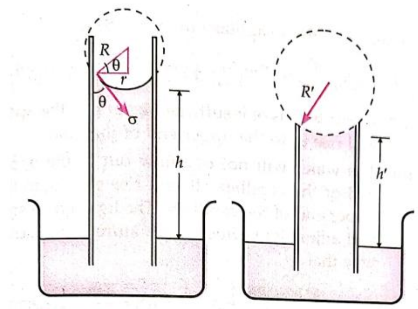


Fig. 10.59 Rise of liquid in a tube of insufficient height.

R = r sec θ
h = 2S/Rρg
hR = 2S/ ρg
S, ρ, g constant
hR = constant
hR = h'R'
R decreases R increases.

Liquid meniscus becomes more and more flat, but the liquid does not overflow to some extent.

Numerical

Question: Calculate the height to which what are will rise in the capillary tube of 1.5 mm diameter. The surface tension of water is 7.4×10^{-3} Nm^{-1}.

Solution:

Here, $r = \frac{1.5}{2} = 0.75\text{ mm} = 0.75 \times 10^{-3}\text{m}$

$S = 7.4 \times 10^{-3}\ \text{Nm}^{-1}$

For water, $\rho = 10^3\ \text{kg m}^{-3}$

The angle of contact $\theta = 0^0$

$$\therefore h = \frac{2S\cos\theta}{rg} = 2\frac{7.4 \times 10^{-3} \times \cos 0^0}{0.75 \times 10^{-3} \times 10^3 \times 9.8}$$

$$h = 0.002014\text{ m}$$