Chapter- 3

Motion in a straight line

Mechanics

Mechanics is a branch of physics that deals with the motion of a body due to the application of force.

The two main branches of mechanics are:

(a) Statics and

(b) Dynamics

Statics

Statics is the study of the motion of an object under the effect of forces in equilibrium.

Dynamics

Dynamics is the study of the motion of the objects by taking into account the cause of their change of states (state of rest or motion).

Dynamics is classified into (i) Kinematics and (ii) Kinetics

Kinematics

The study of the motion of the objects without taking into account the cause of their motion is called kinematics.

Kinetics

Kinetics is the study of motion which relates to the action of forces causing the motion and the mass that is moved.

Concept of a Point Object

In mechanics, a particle is a geometrical mass point or a material body of negligible dimensions. It is only a mathematical idealization.

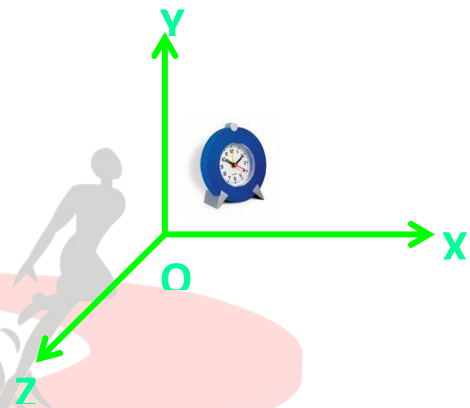
In practice, the nearest approach to a particle is a body, whose size is much smaller than the distance or the length involved.

Reference Point

Consider a rectangular coordinate system consisting of three mutually perpendicular axes, labelled X-, Y-, and Z- axes. The point of intersection of these three axes is called origin (O) and serves as the reference point. The coordinates (*x, y, z*) of an object describe the position of the object with respect to this coordinate system.

Frame of reference

The coordinate system along with a clock to measure the time constitutes a frame of reference.



Positive direction

The positive direction of an axis is in the direction of increasing numbers (coordinates).

Negative direction

The negative direction of an axis is in the direction of decreasing numbers (coordinates).

NOTE

While describing motion, we use reference point or origin w.r.t. which the motion of other bodies is observed. We can use any object as a reference point.

(For example, a car at rest or in motion can be used as a reference point.

When you travel on a bus or train you can see the trees, buildings, and the poles moving back.

To a tree, you are moving forward and to you, the trees are moving back.

Both, you and the trees, can serve as a reference point but motion can not be described without a reference point.

Rest

A body is said to be at rest if its position remains constant with respect to its surroundings or frame of reference.

Examples: Mountains, Buildings, etc.

Motion

A body is said to be in motion if its position is changing with respect to its surroundings or frame of reference.

Examples: 1. Moving cars, buses, trains, a cricket ball, etc.

2. All the planets revolving around the Sun

3. Molecules of gas in motion above 0 K

Rest and Motion are relative terms:

An object which is at rest can also be in motion simultaneously.

Eg. The passengers sitting in a moving train are at rest w.r.t. each other but they are also in motion at the same time w.r.t. objects like trees, buildings, etc.

MOTION IN ONE, TWO OR THREE DIMENSIONS

One Dimensional Motion

The motion of the object is said to be one dimensional if only one of the three coordinates is required to be specified with respect to time. It is also known as rectilinear motion.

In such a motion the object moves in a straight line.

Example: A train moving in straight track, a man walking in a narrow, levelled road, etc.

Two Dimensional Motion

The motion of the object is said to be two dimensional if two of the three coordinates are required to be specified with respect to time.

In such a motion the object moves in a plane.

Example: Ant moving on a floor, a billiard ball moving on a billiard table, etc.

Three Dimensional Motion

The motion of the object is said to be three dimensional if all the three coordinates are required to be specified with respect to time.

Such a motion takes place in space.

Example: A flying aeroplane, bird, kite, etc.

Motion in a Straight Line

DISTANCE

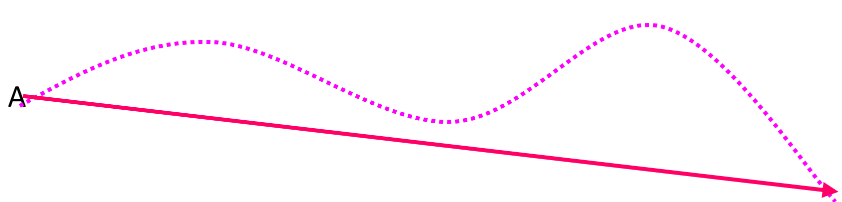
The line joining the successive positions of a moving body is called its path. The length of the actual path between the initial and final positions gives the distance travelled by the body. Distance is a scalar.

DISPLACEMENT

Displacement is the directed line segment joining the initial and final positions of a moving body.

1. **The displacement is a vector quantity.**
2. **The displacement has units of length.**
3. **The displacement of an object in a given time interval can be positive, zero, or negative.**
4. **The actual distance travelled by an object in a given time interval can be equal to or greater than the magnitude of the displacement.**
5. **The displacement of an object between two points does not tell exactly how the object moved between those points.**
6. **The displacement of a particle between two points is a unique path, which can take the particle from its initial to the final position.**
7. **The displacement of an object is not affected due to the shift in the origin of the position-axis.**

If a body changes from one position x_1 to another position x_2, then the displacement Δx in time interval $\Delta t = t_2 - t_1$, is $\Delta x = x_2 - x_1$

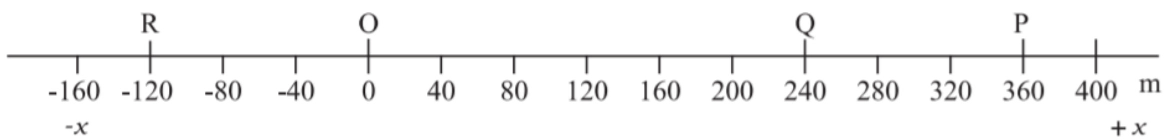


ILLUSTRATION

Consider the motion of a car along a straight line.

We choose the x-axis such that it coincides with the path of the car's motion and origin of the axis as the point from where the car started moving, i.e. the car was at x = 0 at t = 0

Let P, Q and R represent the positions of the car at different instants of time.



Consider two cases of motion.

In the first case, the car moves from O to P. Then the distance moved by car is

OP = +360 m. This distance is called the path length traversed by the car.

In the second case, the car moves from O to P and then moves back from P to Q.

During this course of motion, the path length traversed is

OP + PQ = + 360 m + (+120 m) = + 480 m.

NOTE

Distance is a scalar quantity — a quantity that has magnitude only and no direction

DISPLACEMENT:

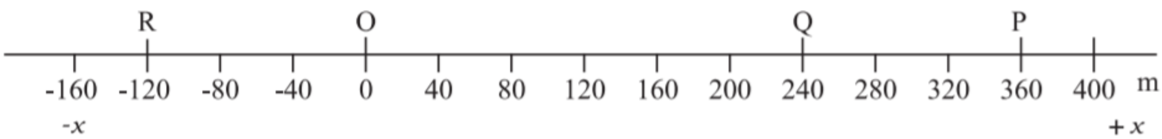
let x_1 and x_2 be the positions of an object at a time $t_1$and t_2.

Then its displacement, denoted by Δx, in time Δt = (t_1 - t_2),

Δx = x_2-x_1

If x_2> x_1, Δx is positive

if x_2 < x_1 , Δx is negative.



For example, displacement of the car is moving from O to P is :

Δx = = x_2-x_1 = (+360 m) – 0 m = +360 m

The displacement has a magnitude of 360 m and is directed in the positive x-direction as indicated by the + sign.

Similarly, the displacement of the car from P to Q is 240 m – 360 m = – 120 m.

The negative sign indicates the direction of displacement

The magnitude of displacement may or may not be equal to the path length traversed by an object.

For example, for the motion of the car from O to P, the path length is +360 m and the displacement is +360 m. In this case, the magnitude of displacement (360 m) is equal to the path length (360 m).

But consider the motion of the car from O to P and back to Q.

In this case, the path length = (+360 m) + (+120 m) = + 480 m.

The displacement = (+240 m) – (0 m) = + 240 m.

Thus, the magnitude of displacement (240 m) is not equal to the path length (480 m).

The magnitude of the displacement for a course of motion may be zero but the corresponding path length is not zero.

For example, if the car starts from O, goes to P, and then returns to O, the final position coincides with the initial position and the displacement is zero.

However, the path length of this journey is OP + PO = 360 m + 360 m = 720 m.

AVERAGE VELOCITY AND AVERAGE SPEED

Average velocity is defined as the change in position or displacement (Δx) divided by the time intervals (Δt), in which the displacement occurs :

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

where x_1 and x_2 are the positions of the object at a time t_1 and t_2 , respectively

The SI unit for velocity is m/s.

Note:

1. **Velocity is a vector quantity.**
2. **The direction of velocity is the same as the direction of displacement of the body.**
3. **Velocity can be either positive, zero, or negative.**
4. **Velocity can be changed in two ways:**

i) by changing the speed of the body or

ii) by keeping the speed constant but by changing the direction.

Uniform Velocity

A particle or a body is said to be moving with uniform velocity, if it covers equal displacements in equal intervals of time, howsoever small these intervals may be.

Variable Velocity

A particle or a body is said to be moving with variable velocity if its speed or its direction or both changes with time.

NOTE

a) If a particle undergoes a displacement s_1 along a straight line in time t_1 and a displacement s_2 in time t_2 in the same direction, then

$$v_{av} = \frac{s_1 + s_2}{t_1 + t_2}$$

b) If a particle undergoes a displacement s_1 along a straight line with velocity v_1 and a displacement s_2 with velocity v_2 in the same direction, then

$$v_{av} = \frac{(s_1+s_2)\, v_1 v_2}{s_1 v_2 + s_2 v_1}$$

c) If a particle travels first half of the displacement along a straight line with velocity v_1 and the next half of the displacement with velocity v_2 in the same direction, then

$$v_{av} = \frac{2 v_1 v_2}{v_1 + v_2} \quad \text{(in the case (b) put } s_1 = s_2\text{)}$$

d) If a particle travels first half of the time with velocity v_1 and the next half of the time with velocity v_2 in the same direction, then

$$v_{av} = \frac{v_1 + v_2}{2} \quad \text{(in the case (d) put } t_1 = t_2\text{)}$$

Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place :

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Total time interval}}$$

The average speed has the same unit (m/s) as that of velocity.

NOTE

- Average speed does not tell us in what direction an object is moving.
- It is always positive (in contrast to the average velocity which can be positive or negative).

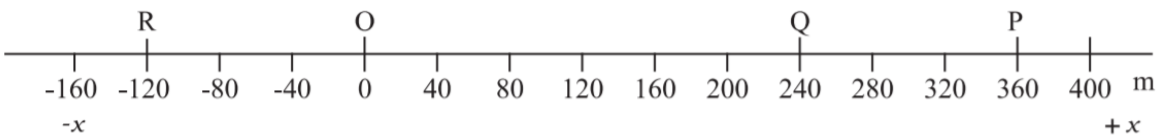
Difference between Speed and Velocity

Speed	Velocity
1. Speed is the time rate of change of distance of a body.	1. Velocity is the time rate of change of displacement of a body.
2. Speed tells nothing about the direction of motion of the body.	2. Velocity tells the direction of motion of the body.
3. Speed is a scalar quantity.	3. Velocity is a vector quantity.
4. Speed of the body can be positive or zero.	4. Velocity of the body can be positive, zero or negative.
5. Average speed of a moving body can never be zero.	5. Average velocity of a moving body can be zero.

NCERT Example 3.1

A car is moving along a straight line, say OP in Fig. It moves from O to P in 18 s and returns from P to Q in 6.0 s. What are the average velocity and average speed of the car in going

(a) from O to P ? and (b) from O to P and back to Q?



Answer (a)

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

$$\bar{v} = \frac{+360\ \text{m}}{18\ \text{s}} = +20\ \text{m s}^{-1}$$

$$\text{Average speed} = \frac{\text{Path length}}{\text{Time interval}}$$

$$= \frac{360\ \text{m}}{18\ \text{s}} = 20\ \text{m s}^{-1}$$

Thus, in this case, the average speed is equal to the magnitude of the average velocity.

(b) In this case,

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}} = \frac{+240\ \text{m}}{(18+6.0)\ \text{s}}$$

$$= +10\ \text{m s}^{-1}$$

$$\text{Average speed} = \frac{\text{Path length}}{\text{Time interval}} = \frac{\text{OP} + \text{PQ}}{\Delta t}$$

$$= \frac{(360+120)\ \text{m}}{24\ \text{s}} = 20\ \text{m s}^{-1}$$

DIFFERENTIATION

The derivative or differential coefficient of a dependent variable y w.r.t an independent variable x is defined as the ratio of the change in the variable y corresponding to the change in variable x when the change in x approaches zero.

Consider a function: $y = f(x)$

Its derivative is given by $f'(x)=\frac{dy}{dx}$

or $f'(x) = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$

$= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

Hence $f'(x) = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

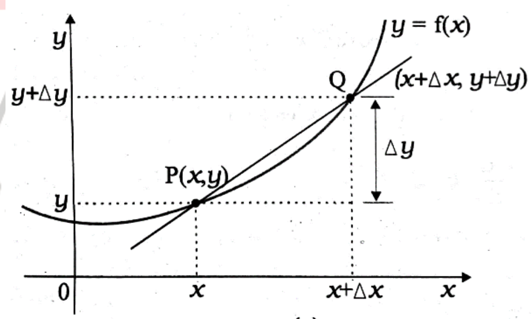
GEOMETRICAL INTERPRETATION

$y = f(x)$

$y + \Delta y = f(x + \Delta x)$

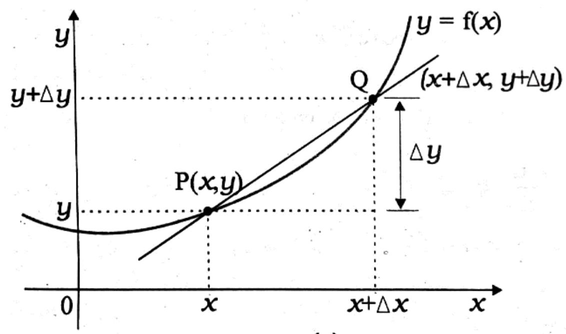
$P(x, y)$

$Q(x + \Delta x,\ y + \Delta y)$



The slope of chord PQ= $\tan\theta = \frac{QN}{PN}$

The slope of the chord $= \frac{\Delta y}{\Delta x}$



Taking the limit

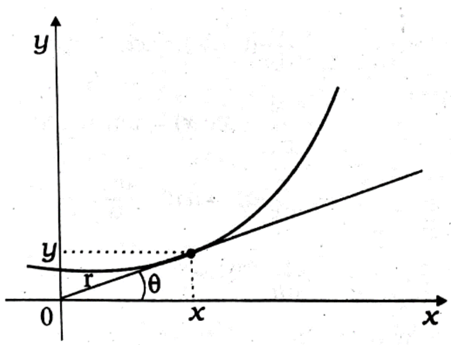
Let Q → P

i.e. $\Delta x \to 0$

When Q → P, chord PQ becomes tangent at 'P'

the slope of the tangent at P = $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

Thus, the derivative of a function $f(x)$ at a point is the slope of the tangent to the curve at that point.



BASIC DERIVATIVE FORMULAS

1. $\frac{d}{dt}(cf) = c\,\frac{d}{dt}(f)$
2. $\frac{d}{dt}(c_1 f_1 \pm c_2 f_2 \pm c_3 f_3 \pm \ldots) = c_1\frac{d}{dt}(f_1) \pm c_2\frac{d}{dt}(f_2) \pm c_3\frac{d}{dt}(f_3) \pm \ldots$
3. $\frac{d}{\mathrm{dt}}(uv) = \left\{\frac{d}{\mathrm{dt}}(u)\right\}v + u\left\{\frac{d}{\mathrm{dt}}(v)\right\}$
4. $\frac{d}{dt}\left\{\frac{u}{v}\right\} = \frac{\left\{\frac{d}{\mathrm{dt}}(u)\right\}v - u\left\{\frac{d}{\mathrm{dt}}(v)\right\}}{v^2}$
5. $\frac{d}{dt}f(x) = \left\{\frac{d}{dx}f(x)\right\}.\frac{dx}{dt}$

6. $\frac{d}{dt}(f(g(t)) = f'(g(t))\frac{d}{dt}g(t)$ ....**CHAIN RULE**

DERIVATIVE OF SIMPLE FUNCTIONS

1. $\frac{d}{dt}(t^n) = nt^{n-1}$ (For all values of n)

2. $\frac{d}{dt}(\text{constant}) = 0$
3. $\frac{d}{dt}(t) = 1.t^{1\text{-}1} = 1.t^0 = 1$
4. $\frac{d}{dt}(t^2) = 2t$
5. $\frac{d}{dt}(t^3) = 3t^2$

6. $\frac{d}{dt}(\sqrt{t}) = \frac{d}{dt}(t^{1/2}) = \frac{1}{2}t^{\frac{1}{2}-1} = \frac{1}{2}t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$
7. $\frac{d}{dt}\left(\frac{1}{t}\right) = \frac{d}{dt}(t^{-1}) = -1.t^{-1-1} = -1.t^{-2} = \frac{-1}{t^2}$
8. $\frac{d}{dt}\left(\frac{1}{t^2}\right) = \frac{d}{dt}(t^{-2}) = -2.t^{-2-1} = -2.t^{-3} = \frac{-2}{t^3}$
9. $\frac{d}{dt}sin(at+b) = a\,cos(at+b)$ and $\frac{d}{dt}cos(at+b) = -a\,sin(at+b)$
10. $\frac{d}{dt}(e^{at+b}) = a.e^{at+b}$ and $\frac{d}{\text{dt}}\ln(\text{at}+b) = \frac{a}{t}$

EXAMPLES ON DIFFERENTIATION

The SUM and DIFFERENCE Rules

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

ILLUSTRATION

Question-1: $f(x) = x^3 - 4x + 5$

$$f'(x) = \frac{d}{dx}(x^3 - 4x + 5)$$

$$= \frac{d}{dx}(x^3) - 4\frac{d}{dx}(x) + \frac{d}{dx}(5)$$

$$= 3x^2 - 4 + 0$$

$$= 3x^2 - 4$$

Question-2: $f(x) = -\frac{x^4}{2} + 3x^3 - 2x$ .....left to Students

Derivatives of Sine and Cosine

ILLUSTRATION

1) $\frac{d}{dx}[\sin x] = \cos x$

2) $\frac{d}{dx}[\cos x] = -\sin x$

Chain Rule Differentiation

ILLUSTRATION

Find the derivative of $\sin x^2$ w.r.t x $\frac{d}{dx}\sin(x^2)$

$= \cos(x^2)\frac{d}{dx}(x^2)$

$= \cos(x^2).(2x)$

$= (2x)\cos(x^2)$

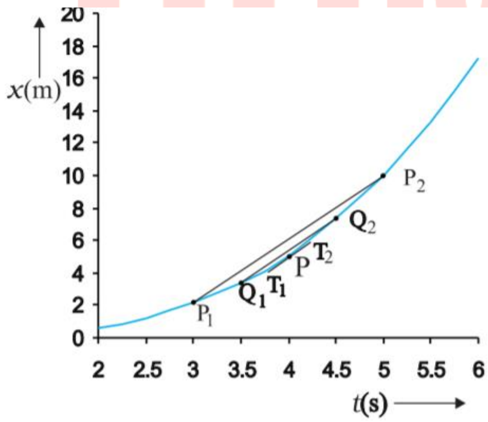
Instantaneous Velocity

When a body is moving with variable velocity, the velocity of the body at any instant is called instantaneous velocity.

The velocity at an instant is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta x}}{\Delta t}$$

Or $\vec{v} = \frac{\overrightarrow{dx}}{dt}$



Suppose we want to calculate the instantaneous velocity at the point P at an instant t.

The slope of P_1P_2 at t_1 and t_2 with intervals of **Δt** from t, (i.e. t_1 = t- Δt and t_2 = t+ Δt) gives the average velocity at P.

The slope of Q_1Q_2 at t_3 and t_4 with intervals of **Δt/2** from t, (i.e. t_3 = t- Δt/2 and t_4 = t+Δt/2) gives the average velocity at P which is the closest value to the instantaneous velocity.

Proceeding this way, Δt may be gradually reduced to approach zero, i.e. $\Delta t \rightarrow 0$ to get the actual value of the instantaneous velocity.

NOTE

- Average speed over a finite interval of time is greater than or equal to the magnitude of the average velocity.
- The instantaneous speed at an instant is equal to the magnitude of the instantaneous velocity at that instant.

NCERT EXAMPLE

The position of an object moving along x-axis is given by x = a + bt^2 where a=8.5 m, b=2.5m$/s^2$ and t is measured in seconds.

1. What is its velocity at t = 0 s and t = 2.0 s?
2. What is the average velocity between t = 2.0 s and t = 4.0 s?

SOLUTION

1. $v = \frac{dx}{dt}$

$$v = \frac{d}{dt}(a + bt^2)$$
$$v = \frac{d}{dt}(a) + b\frac{d}{dt}(t^2)$$
$$v = b(2t)$$
$$v = 2.5 \times 2 \times t$$
$$v = 5t$$

At t = 0 s, v = 0 m/s and at t = 2.0 s, v = 10 m/s.

2. Average velocity

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1}$$
$$v_{avg} = \frac{a + 16b - a - 4b}{4 - 2}$$
$$v_{avg} = 6 \times b$$
$$v_{avg} = 15m/s$$

SOME NUMERICALS :

1. Position at any instant is ; $x = t^3 - 12t$
   Find the magnitude of acceleration when body comes to rest .

2. Position at any instant is ; $x = 12t - t^2$
   Find (i) the initial velocity or velocity at the starting of motion .
   (ii) acceleration at the instant body comes to rest .
   (iii) distance travelled by the body in the interval 0 to 8s .

3. Position at any instant is ; $x = t^2 - 4t$
   Find (i)the displacement in 0 to 4 s .
   (ii) average velocity in 0 to 4s

4. Velocity at any instant is ; $v = 4 - t^2$
   What is the acceleration at the instant body comes to rest .
5. Velocity at any instant is ; $v = 4t - t^2$
   Find the average acceleration in the interval 0 to 4s .

SOLUTION:1

$x = t^3 - 12t$

$$\Rightarrow v = \frac{dx}{dt} = \frac{d}{dt}(t^3 - 12t) = 3t^2 - 12$$
$$\Rightarrow a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 12) = 6t$$

The instant body comes to rest

$v = 0$

$$\Rightarrow 3t^2 - 12 = 0 \Rightarrow t = 4s$$

At this instant, a $= 6 \times 4 = 24$ms^{-2}

SOLUTION:2

$x = 12t - t^2$

$$\Rightarrow v = \frac{dx}{dt} = \frac{d}{dt}(12t - t^2) = 12 - 2t$$
$$\Rightarrow a = \frac{dv}{dt} = \frac{d}{dt}(12 - 2t) = -2$$

(i) Initial velocity i.e. at t = 0;

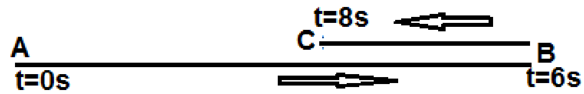
$v_0 = 12 - 2 \times 0 = 12m/s$

(ii) The instant body comes to rest

$v = 0$

$\Rightarrow 12 - 2t = 0 \Rightarrow t = 6s$

At this instant, a $= -2$ms^{-2}



(iii) At t=0, v= 12m/s

At t=8, v = 12 – 2x8 = -4 m/s

At t = 6s, v = 0

From these, we conclude that from t=0s to t=6s body is travelling in +ve direction and then returning in –ve direction as shown in the figure.

As the direction is changing distance is not equal to the magnitude of displacement.

$$\therefore d = |\Delta x_{0\to 6s}| + |\Delta x_{6s\to 8s}| = |x_{6s} - x_{0s}| + |x_{8s} - x_{6s}|$$
$$\Rightarrow d = |(12 \times 6 - 6^2) - (12 \times 0 - 0^2)| + |(12 \times 8 - 8^2) - (12 \times 6 - 6^2)|$$
$$\Rightarrow d = |36| + |-4| = 40m$$

NON-UNIFORM MOTION

The particle is said to have non-uniform motion if it covers unequal displacements in equal intervals of time, however small these time intervals may be.

Acceleration

If the velocity of a body changes either in magnitude or in direction or both, then it is said to have acceleration.

- For a freely falling body, the velocity changes in magnitude and hence it has acceleration.
- For a body moving around a circular path with a uniform speed, the velocity changes in direction and hence it has acceleration.
- For a projectile, whose trajectory is a parabola, the velocity changes in magnitude and direction, and hence it has acceleration.
- The acceleration and velocity of a body need not be in the same direction.
- Eg.: A body is thrown vertically upwards.

The average acceleration over a time interval is defined as the change of velocity divided by the time interval:

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

where v_1 and v_2 are the instantaneous velocities or simply velocities at time t_1 and t_1. It is the average change of velocity per unit time.

The SI unit of acceleration is ms^{-2}.

1. A body can have zero velocity and non-zero acceleration.

Eg.: For a particle projected vertically up, the velocity at the highest point is zero, but acceleration is -g.

2. If a body has a uniform speed, it may have acceleration.

Eg.: Uniform circular motion

3. If a body has uniform velocity, it has no acceleration.

4. Acceleration of free fall in a vacuum is uniform and is called acceleration due to gravity (g) and it is equal to 980 cms^{-2} of 9.8 ms^{-2}.

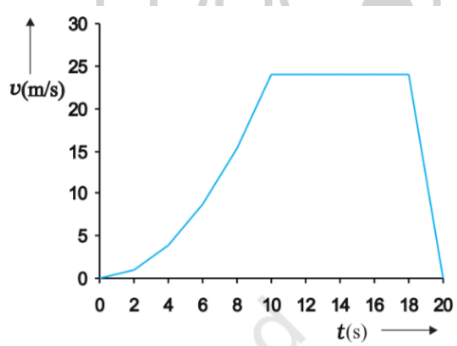
NCERT NUMERICAL

Find the average acceleration for velocity-time graph shown in Fig for different time intervals

I. 0 s - 10 s,

II. 10 s – 18 s,

III. 18 s – 20 s:



SOLUTION

On a plot of velocity versus time, the average acceleration is the slope of the straight line connecting the points corresponding to (v_1, t_1) and (v_2, t_2).

I. $a_{avg} = \frac{(24-0)}{(10-0)}$ =2.4m/s^2

II. $a_{avg} = \frac{(24-24)}{(18-0)} = 0 m/s^2$

III. $a_{avg} = \frac{(0-24)}{(20-18)} = -12 m/s^2$

INSTANTANEOUS ACCELERATION

The acceleration of a particle at any instant or any point is called instantaneous acceleration.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The acceleration at an instant is the slope of the tangent to the v–t curve at that instant

NOTE:

- Acceleration is a vector quantity.
- The direction of acceleration is the same as the direction of the velocity of the body.
- Acceleration can be either positive, zero or negative.
- Acceleration of a body is zero when it moves with uniform velocity.
- Acceleration is measured in
  - ✓ cm/s^2 ($cm\ s^{-2}$) in cgs system of units
  - ✓ m/s^2 ($m\ s^{-2}$) in the SI system of units and
  - ✓ km/h^2 ($km\ h^{-2}$) in practical life when distance and time involved are large.

UNIFORM ACCELERATION

If equal changes of velocity take place in equal intervals of time, however small these intervals may be, then the body is said to be in uniform acceleration.

or

A body has uniform acceleration if its velocity changes at a uniform rate.

Eg.1: The motion of a freely falling body is uniformly accelerated motion.

INTEGRATION

Integration is also called as antiderivatives

i.e. if $\frac{d}{dx}f(x) = g(x) \Rightarrow \int g(x)dx = f(x) + c$

For eg.

$$\frac{d}{dx}(x^2) = 2x \text{ or } \frac{d}{dx}\left(\frac{x^2}{2}\right) = x$$

$$\Rightarrow \int x dx = \frac{x^2}{2} + c$$

2. Integration formulae for functions

1. $\int (t^n)dt = \frac{t^{n+1}}{n+1} + c$ (For all real values of n except n = -1)
2. $\int \frac{1}{t}dt = ln(t) + c$
3. $\int a dt = at + c$
4. $\int (at+b)^n dt = \frac{(at+b)^{n+1}}{a(n+1)} + c$
5. $\int \sin(at+b)dt = \frac{-\cos(at+b)}{a} + c$ and $\int \cos(at+b)dt = \frac{\sin(at+b)}{a} + c$
6. $\int e^{at+b}dt = \frac{e^{at+b}}{a} + c$

3. Integration formulae for some algebraic operations:

1. $\int (cf)dt = c\int (f)dt$
2. $\int (c_1f_1 \pm c_2f_2 \pm c_3f_3 \pm \ldots)dt = c_1\int (f_1)dt \pm c_2\int (f_2)dt \pm c_3\int (f_3)dt \pm ..$

3. $\int t^2 dt = \frac{t^3}{3} + c$
4. $\int \frac{1}{t^2} dt = \frac{-1}{t} + c$
5. $\int \sqrt{t} dt = \frac{2t^{3/2}}{3} + c$

4. Finite integration or integration with limit:

If $\int f(x)dx = g(x) + c$

$$\Rightarrow \int_a^b f(x)dx = \{g(b) + c\} - \{g(a) + c\} = g(b) - g(a)$$

So finite integral is a value where the indefinite integral is a function.

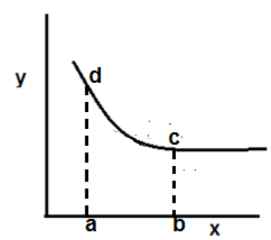
Eg. $\int_1^2 t^2 dt = \left[\frac{t^3}{3}\right]_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{7}{3}$

5. Geometrical meaning of integration

If a function y(x) is represented graphically by plotting graph between y ~ x then area below the graph bounded by x-axis, x=a and x=b is equal to the integration of y dx within x = a and x = b

i.e. Area (abcd) = is as integral

$\int_a^b y dx$ = area below the graph



Example *Integrate the following functions w.r.t. x,*

(i) x^3 (ii) $x^2 + \frac{1}{x}$ (iii) e^{3x} (iv) $\left(x - \frac{1}{x}\right)^2$

(v) $\frac{1}{\sqrt{x}}$ (vi) $4e^{5x}$

Solution. (i) $\int x^3 dx = \frac{x^{3+1}}{3+1} = \frac{x^4}{4}$

(ii) $\int\left(x^2 + \frac{1}{x}\right)dx = \int x^2 dx + \int \frac{1}{x} dx = \frac{x^3}{3} + \log_e x$

(iii) $\int e^{3x} dx = \frac{e^{3x}}{\frac{d}{dx}(3x)} = \frac{e^{3x}}{3}$

(iv) $\int\left(x - \frac{1}{x}\right)^2 dx = \int\left(x^2 - 2 + \frac{1}{x^2}\right)dx = \int x^2 dx - \int 2dx + \int x^{-2} dx$

$$= \frac{x^{2+1}}{2+1} - 2\int dx + \frac{x^{-2+1}}{-2+1} = \frac{x^3}{3} - 2x - x^{-1} = \frac{x^3}{3} - 2x - \frac{1}{x}$$

(v) $\int \frac{1}{\sqrt{x}} d\ x = \int \frac{1}{x^{1/2} dx} = \int x^{-1/2} dx = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = 2x^{\frac{1}{2}} = 2\sqrt{x}$

(vi) $\int 4e^{5x} dx = 4\int e^{5x} dx = 4\ \frac{e^{5x}}{\frac{d}{dx}(5x)} = \frac{4}{5}e^{5x}$

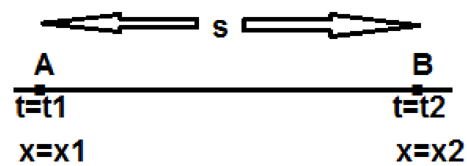
USES OF INTEGRATION IN KINEMATICS

- **Position, displacement as integral of velocity:**

$$v = \frac{dx}{dt} \Rightarrow dx = vdt \Rightarrow \int dx = \int vdt \Rightarrow x = \int vdt$$

Integrating within the interval t_1 to t_2

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} vdt \Rightarrow (x_2 - x_1) \text{ or } \Delta x \text{ or } s = \int_{t_1}^{t_2} vdt$$



- So position at any instant is the indefinite integral of velocity
- Displacement as a definite integral of velocity in an interval.

Velocity as integral of acceleration:

$$a = \frac{dv}{dt} \Rightarrow dv = adt \Rightarrow \int dv = \int adt \Rightarrow v = \int adt$$

Integrating with the interval t_1 to t_2 we get

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} adt \Rightarrow (v_2 - v_1) \text{ or } \Delta v = \int_{t_1}^{t_2} adt$$

- So velocity at any instant is the indefinite integral of acceleration
- Velocity change in an interval is the definite integral of acceleration in the interval.

NUMERICAL

Q1. Velocity at any instant is; Find $v = 4 - 2t$

a) position at t = 2s if the body starts from origin
b) displacement in the interval 2s to 4s
c) average velocity in the interval 2s to 4s

Solution

Velocity at any instant is; $v = 4 - 2t$

(a) Position at t = 2s :

As if the body starts from origin hence at t=0 , x =0

Using the condition in equation (i) we get ;

$$v = \frac{dx}{dt} \Rightarrow dx = vdt \Rightarrow \int dx = \int vdt \Rightarrow x = \int (4-2t)dt$$
$$\Rightarrow x = 4\int dt - 2\int tdt = 4t - t^2 + C$$

(b) displacement in the interval 2s to 4s

(c) Average velocity in the interval 2s to 4s

$$s = x_{4s} - x_{2s} = (4\times 4 - 4^2) - (4\times 2 - 2^2) = 0 - 4 = -4m$$
$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_{4s} - x_{2s}}{4s - 2s} = \frac{-4m}{2s} = -2m/s$$

NUMERICALS

Q2. Acceleration at any instant is; $a = 2t - 4$ Find

a) the expression for velocity at any instant t if body is starting from rest .
b) the expression for position at any instant if body is starting from x=2m.

Solution

Acceleration at any instant is; $a = 2t - 4$

(a) the expression for velocity at any instant t if body is starting from rest:

As if the body starts from rest hence at t=0 , v =0

Using the condition in equation (i) we get ;

Equation (ii) gives expression for velocity at any instant

$$a = \frac{dv}{dt} \Rightarrow dv = adt \Rightarrow \int dv = \int adt \Rightarrow v = \int (2t-4)dt$$
$$\Rightarrow v = 2\int tdt - 4\int dt = t^2 - 4t + C \quad \text{......}$$
$$0 = 0^2 - 4\times 0 + C \Rightarrow C = 0$$
$$\Rightarrow v = t^2 - 4t \quad \text{......}$$

Solution

(b) the expression for position at any instant if body is starting from x=2m

As if the body starts from x=2m, hence at t=0, x=2m

Using the condition in equation (i) we get;

Using the value of C in equation (i) we get;

This equation is the expression for position at any instant ……

$$v=\frac{dx}{dt}\Rightarrow dx=vdt\Rightarrow\int dx=\int vdt\Rightarrow x=\int(t^2-4t)dt$$

$$\Rightarrow x=\int t^2dt-4\int tdt=\frac{t^3}{3}-2t^2+C$$

$$2=\frac{0^3}{3}-2\times 0^2+C\Rightarrow C=2$$

$$x=\frac{t^3}{3}-2t^2+2$$

SOME NUMERICALS :

1. Velocity at any instant is ; $v = 4 - 2t$
   Find (i) Position at t = 2s if the body starts from origin .
   (ii) displacement in the interval 2s to 4s .
   (iii) Average velocity in the interval 2s to 4s

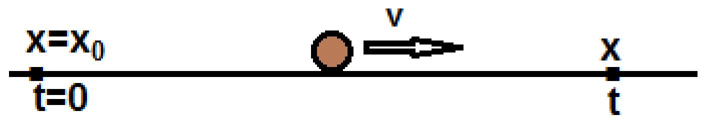
2. Acceleration at any instant is ; $a = 2t - 4$
   Find (i) the expression for velocity at any instant t if body is starting from rest .
   (ii) the expression for position at any instant if body is starting from x=2m.

3. Acceleration at any instant is ; $a = t - 4$
   Find (i)the velocity change in 2s to 4 s .
   (ii) average acceleration in 2s to 4s

4. Velocity at any instant is ; $v = 4 - t^2$
   What are the displacement and distance covered in 0 to 3s ?
5. Acceleration at at any instant is ; $a = 4t - 8$
   Find the average acceleration in the interval 0 to 4s .

UNIFORM MOTION

- Velocity (v) is constant. i.e. body must be travelling in one direction with constant speed.
- So distance = magnitude of displacement and average and instantaneous velocity and speed are of the same magnitude



- So a = 0

As

$$v = \frac{dx}{dt} \Rightarrow dx = vdt$$

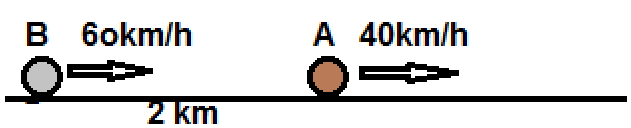
Integrating both sides within interval 0 to t we have

$$\int_{x_0}^{x} dx = \int_0^t vdt = v\int_0^t dt$$
$$\Rightarrow [x]_{x_0}^{x} = v[t]_0^t$$
$$\Rightarrow x - x_0 = v(t - 0)$$
$$\Rightarrow s = vt$$

NUMERICAL

1. Two cars A and B travelling along a straight road with constant speeds 40 km/h and 60km/h respectively with B following A at a distance 2km. Calculate;
   (i) The time after which B will cross A.

   (ii) The on-road distances covered by A and B during this.



Solution:

During crossing;

$s_A = (40km/h)t$

$s_B = (60km/h)t$

Now; $s_B - s_A = (20km/h)t$

$\Rightarrow 2km = (20km/h)t \Rightarrow t = (1/10)h$

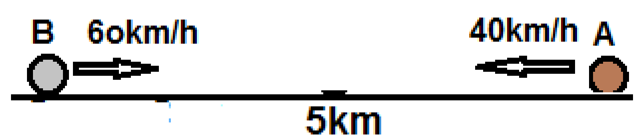
On-road distances

$s_A = (40km/h)t = (40km/h) \times (1/10)h = 4km$

$s_B = (60km/h)t = (60km/h) \times (1/10)h = 6km$

2. Two cars A and B approaching each other along a straight road with constant speeds 40 km/h and 60km/h respectively. At t=0, the distance between them is 5km. Calculate;
   (i) The time after which B will cross A.

   (ii) The on-road distances covered by A and B during this.



Solution:

During crossing

$s_A = (40km/h)t$

$s_B = (60km/h)t$

Now; $s_B + s_A = (100km/h)t$

$\Rightarrow 5km = (100km/h)t \Rightarrow t = (1/20)h$

On-road distances;

$s_A = (40km/h)t = (40km/h) \times (1/20)h = 2km$

$s_B = (60km/h)t = (60km/h) \times (1/20)h = 3km$

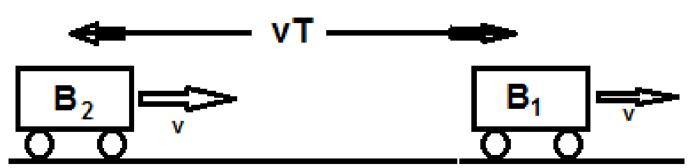
3. Two towns A and B are connected by regular bus service with a bus leaving in either direction every T mins. A man cycling with a speed of 20 km/hr in the direction of A to B notices that a bus goes past him every 18mins in the direction of his motion, and every 6mins in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?

Solution:

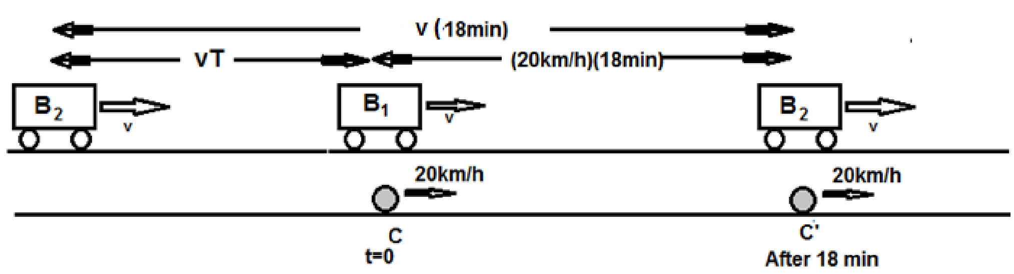
Given that, T= time gap between two successive buses

Let v = speed of each bus

So the distance between two consecutive buses in any direction = vT



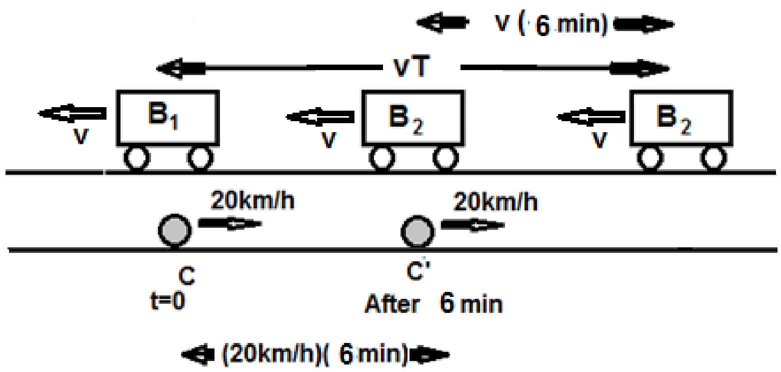
For buses crossing the cyclist in its direction of motion.



From figure;

$$vT + (20km/h)(18\,min) = v(18\,min)$$
$$\Rightarrow v(18\,min - T) = (20km/h)(18\,min)\,..........(i)$$

For buses crossing the cyclist in its direction of motion



$$vT-(20km/h)(6\,min)=v(6\,min)$$

$$\Rightarrow v(T-6\,min)=(20km/h)(6\,min)\,.........(ii)$$

Dividing equation (i) by equation (ii) we get

$$\frac{v(18\,min-T)}{v(T-6\,min)}=\frac{(20km/h)(18\,min)}{(20km/h)(6\,min)}$$

$$\Rightarrow \frac{(18\,min-T)}{(T-6\,min)}=3$$

$$\Rightarrow 18\,min-T=3T-18min$$

$$\Rightarrow 4T=36min$$

$$\Rightarrow T=9min$$

Using the value of T in equation (i) we get

$$v(18\,min-9\,min)=(20km/h)(18\,min)$$

$$\Rightarrow v=40km/h$$

Kinematic Equations of Uniformly Accelerated Motion:

Consider an object moving with uniform acceleration 'a' along a straight line. Let 'u' be the initial velocity (i.e., at t = 0) and $'v'$ its velocity after time t, called the final velocity. Let s be the displacement of the body in time t.

First Equation: Velocity – Time Relation:

The acceleration is defined as the time rate of change of velocity, i.e.,

$$a = \frac{dv}{dt}$$

i.e., $dv = adt$

Integrating both sides $\int dv = \int adt$

i.e., $v = at + C_1$

where C_1 is a constant of integration.

As the initial velocity of the object is u, i.e, at t = 0, $v = u$

$\therefore$ $u = a \times 0 + C_1 \Rightarrow C_1 = u$

Substituting this value in (A), we get

$$V = at + u = u + at$$

This is the first equation of motion.

Second Equation: Displacement time Relation: The velocity of an object is defined as the rate of change of displacement, i.e.,

$$v = \frac{ds}{dt}$$

Or $ds = vdt$

But from (1), $v = u + at$

∴

$$= udt + atdt$$

Integrating we get

$$\int ds = \int udt + \int atdt + C_2$$

Where C_2 is constant of integration.

Or $$s = ut + a.\frac{t^2}{2} + C_2$$

Att = 0, s = 0, therefore, equation (B) gives

$$c_2 = 0$$

Hence $$s = ut + \frac{1}{2}at^2$$

This is the second equation of motion,

Third Equation: Velocity-Displacement

Relation: The first equation is:

$$v = u + at$$

Squaring we get $$v^2 = (u + at)^2$$

$$= u^2 = 2uat + a^2t^2$$

$$= u^2 = 2a\left(ut + \frac{1}{2}at^2\right)$$

Using equation (2), i.e., $ut + \frac{1}{2}at^2 = s$, we get

$$v^2 = u^2 + 2as$$

This is the third equation of motion.

Displacement in the nth second of motion: Let $s_{n-1} and s_n$ be the displacements of the object in (n-1) second and n second respectively. Then displacement of the object in nth second will be

$$s_n^{th} = s_n - s_{n-1}$$

From the second equation of motion, $s_n = un + \frac{1}{2}an^2$

And $s_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$

$\therefore$ $s_n^{th} = s_{n-}s_{n-1}$

$$= un + \frac{1}{2}an^2 - \left\{u(n-1) + \frac{1}{2}a(n-1)^2\right\}$$

$$= un + \frac{1}{2}an^2 - \left\{un - u + \frac{1}{2}a(n^2 - 2n + 1)\right\}$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + \frac{1}{2}a(2n-1)$$

i.e., $s_n^{th} = u + \frac{1}{2}a(2n-1)$ ....(4)

This equation gives displacement in nth second for an object moving with uniform acceleration.

1. A particle starting from rest move along a straight path with uniform acceleration. Find

(i) the ratio between the distances covered in 1s, 2s, 3s, ……. so on.

(ii) the ratio between the distances covered in 1^{st} sec, 2^{nd} sec, 3^{rd} sec, ……. so on.

(iii) the ratio between the distances covered in 1^{st} 1sec, next 2s, next 3s, ……. so on

Solution:

(i) u=0, a=const.

Distance in 1s; $s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}a.1^2 = \frac{a}{2}$

Distance in 2s; $s_2 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}a.2^2 = \frac{a}{2}.4$

Distance in 3s; $s_3 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}a.3^2 = \frac{a}{2}.9$

So the ratio between the distances covered in 1s, 2s, 3s, ……. so on

$$= s_1 : s_2 : s_3 : ... = \frac{a}{2} : \frac{a}{2}(4) : \frac{a}{2}(9) : ...$$

$$= 1 : 4 : 9 : .... = 1^2 : 2^2 : 3^2 : ....$$

(ii) u=0, a=const.

Distance in 1st sec; $s_{1st} = u + \frac{a}{2}(2n-1) = 0 + \frac{a}{2}(2\times1-1) = \frac{a}{2}$

Distance in 2nd sec; $s_{2nd} = u + \frac{a}{2}(2n-1) = 0 + \frac{a}{2}(2\times2-1) = \frac{a}{2}.3$

Distance in 3rd sec; $s_{3rd} = u + \frac{a}{2}(2n-1) = 0 + \frac{a}{2}(2\times3-1) = \frac{a}{2}.5$

So the ratio between the distances covered in 1s, 2s, 3s, ……. so on

$$= s_{1st} : s_{2nd} : s_{3rd} : ... = \frac{a}{2} : \frac{a}{2}(3) : \frac{a}{2}(5) : ...$$

$$= 1 : 3 : 5 : .... = \text{Ratio of odd numbers}$$

(iii) u=0, a=const.

Distance in 1st 1s; $s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}a.1^2 = \frac{a}{2}$

Distance in next 2s = Distance covered in 3s – distance covered in 1s

$$= s_3 - s_1 = \left(0 + \frac{1}{2}a.3^2\right) - \left(0 + \frac{1}{2}a.1^2\right) = \frac{a}{2}.9 - \frac{a}{2}.1 = \frac{a}{2}.8$$

Distance in next 3s = distance in 6s – the distance in 3s

$$= s_6 - s_3 = \left(0 + \frac{1}{2}a.6^2\right) - \left(0 + \frac{1}{2}a.3^2\right) = \frac{a}{2}.36 - \frac{a}{2}.9 = \frac{a}{2}.27$$

So the ratio between the distances covered in 1st 1s, next 2s, next 3s, ……. so on

$$= s_1 : (s_3 - s_1) : (s_6 - s_3) : \ldots = \frac{a}{2} : \frac{a}{2}(8) : \frac{a}{2}(27) : \ldots$$
$$= 1 : 8 : 27 : \ldots. = 1^3 : 2^3 : 3^3 : \ldots.$$

2. Two trains A and B of length 400m each are moving on a parallel track with a uniform speed of 72 km/h in the same direction with A ahead of B. The driver of B decides to overtake A and accelerates by. If after 50 sec the guard of B just brushes past the driver of A, what was the original distance between them?

Solution: For A;

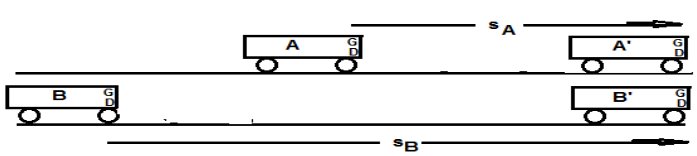
$$s_A = 72km/h \times 50s = 20m/s \times 50s = 1000m$$

For B; $s_B = 72km/h \times 50s + \frac{1}{2} \times 1m/s^2 \times (50s)^2 = 20m/s \times 50s + \frac{1}{2} \times 2500m$

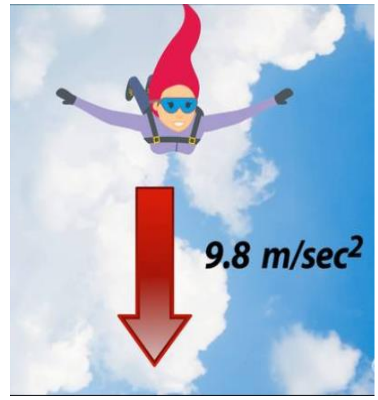
$$\Rightarrow s_B = 1000m + 1250m = 2250m$$

The initial gap between A and B is; $s_B - s_A = 2250m - 1000m$

$$\Rightarrow s_B - s_A = 1250m$$



CONCEPT OF FREE FALL

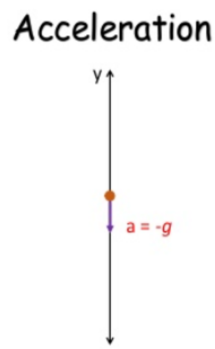


An object released near the surface of the Earth is accelerated downward under the influence of the force of gravity. The magnitude of the acceleration due to gravity is represented by g. If air resistance is neglected, the object is said to be in free fall.

If the height through which the object falls is small compared to the earth's radius,g can be taken to be constant, equal to 9.8 m/s^2. Freefall is thus a case of motion with uniform acceleration.

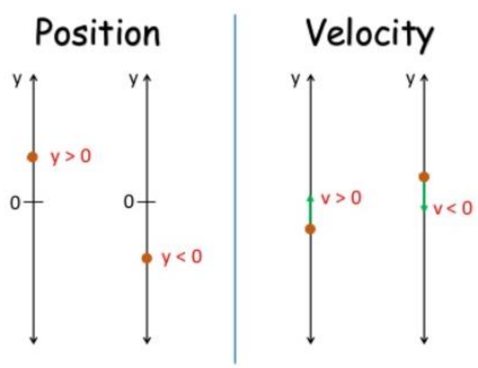
SIGN CONVENTION

We assume that the motion is in y-direction, more correctly in –y-direction because we choose upward direction as positive. Since the acceleration due to gravity is always downward, it is in the negative direction and we have

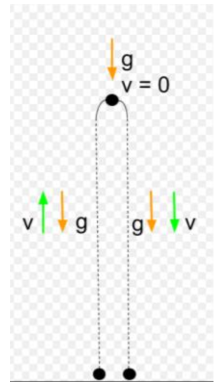


a = – g = – 9.8 m/ s^2

Similarly for Position and Velocity



Suppose a body is thrown vertically upwards. After some time it returns to the ground level.



During **upward** motion:

v= +ve (direction upwards)

g=-ve (direction downwards)

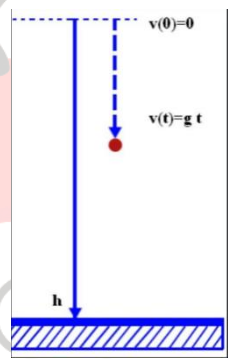
During **downward** motion

v = -ve (direction downwards)

g = -ve (direction downwards)

Suppose the object is released from rest

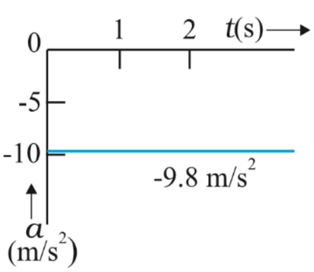
Therefore, v_0 = 0 and the equations of motion become:

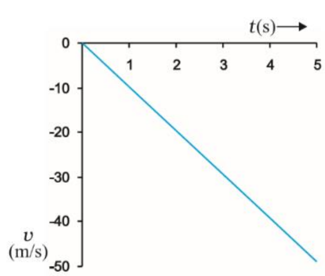


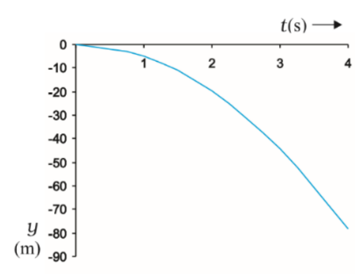
$$v = 0 - g\,t \qquad = -9.8\,t \quad \text{m s}^{-1}$$
$$y = 0 - \tfrac{1}{2}\,g\,t^2 \qquad = -4.9\,t^2 \quad \text{m}$$
$$v^2 = 0 - 2\,g\,y \qquad = -19.6\,y \quad \text{m}^2\,\text{s}^{-2}$$

The variation of acceleration, velocity, and distance, with time, have been plotted in

Figures given below.







Example: A ball is thrown vertically upwards with a velocity of **20 m/ s** from the top of a multi-storey building. The height of the point from where the ball is thrown is 25.0 m from the ground.

(a) How high will the ball rise? and

(b) how long will it be before the ball hits the ground? Take **g = 10 m/** s^2.

Answer (a) Let us take the y-axis in the vertically upward direction with zero at the ground, as shown in Fig. 3.13.

Now $v_o = +20 \text{ m s}^{-1}$,

$a = -g = -10 \text{ m s}^{-2}$,

$v = 0 \text{ m s}^{-1}$

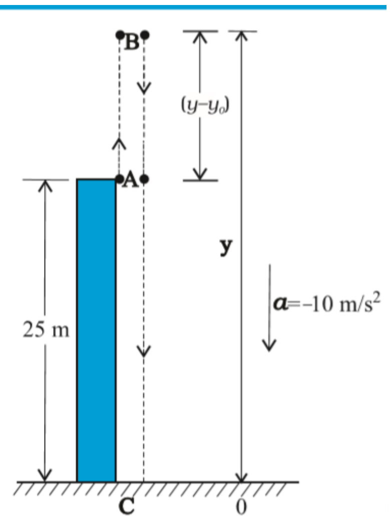
If the ball rises to height y from the point of launch, then using the equation

$$v^2 = v_0^2 + 2a(y - y_0)$$

we get

$$0 = (20)^2 + 2(-10)(y - y_0)$$

Solving, we get, $(y - y_0) = 20$ m.



The total time taken can also be calculated by noting the coordinates of initial and final positions of the ball with respect to the origin chosen and using equation

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

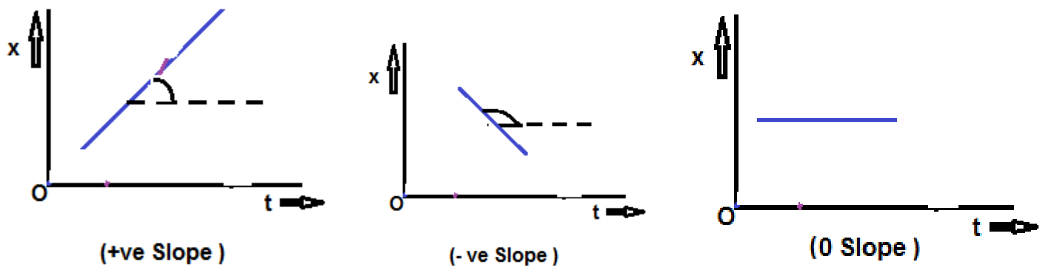
$y_0 = 25$ m $\quad y = 0$ m

$v_o = 20 \text{ m s}^{-1}$, $\quad a = -10 \text{m s}^{-2}$, $\quad t = ?$

Left to students……………………

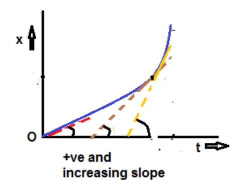
Concept of slope:

The slope of a line is defined as the tangent of a line with a positive direction of the x-axis.

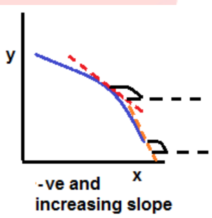


The slope of the tangent to x~t graph at any instant = velocity at that instant

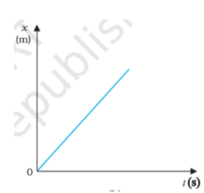
- If slope is + ve, then v = +ve i.e. body travels in +ve direction



- If slope is - ve, then v = -ve i.e. body travels in -ve direction



- If x~t graph is a straight line passing through the origin then the slope is constant.



- If the magnitude of the slope is increasing with time it is accelerated motion.
- If the magnitude of the slope is decreasing with time it is retarded motion.

GRAPH ANALYSIS

Position time graph

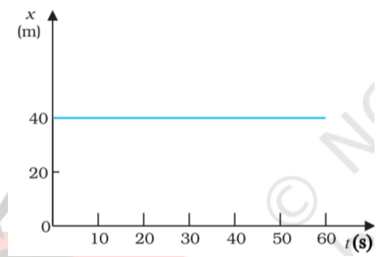
The motion of an object can be represented by a position-time graph.

Such a graph is a powerful tool to represent and analyse different aspects of the motion of an object.

For motion along a straight line, say X-axis, only x-coordinate varies with time and we have an

x-t graph.

Let us first consider the simple case in which an object is stationary e.g. a car standing still.



The position-time graph is a straight line parallel to the time axis, as shown in fig

If an object moving along the straight line covers equal distances in equal intervals of time, it is said to be in uniform motion along a straight line.

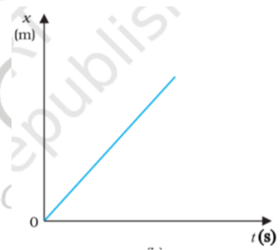
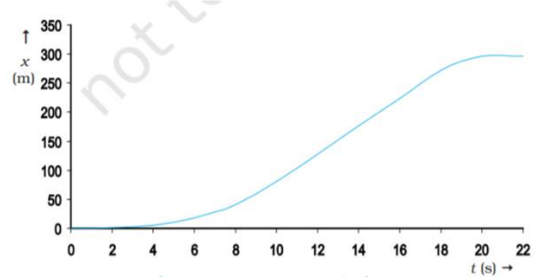


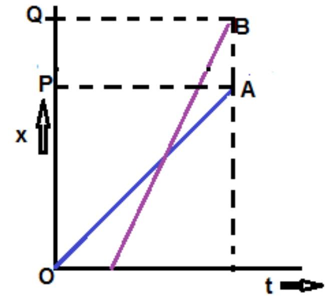
Fig. shows the position-time graph of such a motion

Now, let us consider the motion of a car that starts from rest at time t = 0 s from the origin O and picks up speed till t = 10 s and thereafter move with uniform speed till t = 18 s. Then the brakes are applied and the car stops at t = 20 s and x = 296 m. The position-time graph for this case is shown in Fig.



NCERT NUMERICAL

The x~t graphs for two children A and B returning from their school O to their homes P and Q respectively.



(a) (A/B) lives closer to the school than (B/A).

(b) (A/B) starts from the school earlier than (B/A).

(c) (A/B) walks faster than (B/A).

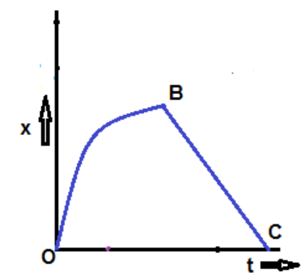
(d) A and B reach home at (same/different) time.

(e) (A/B) overtakes (B/A) on the road (once / twice).

Answer: Left for discussion

Numerical 2

The figure shows the x~t graphs for a particle travelling along the x-axis.



(a) What is the nature of the motion from O to B?

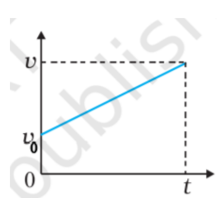
(b) What is the nature of the motion from B to C?

(c) Discuss the directions of motion of the particle?

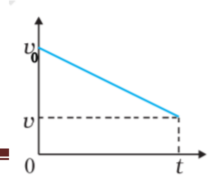
Answer: Left for discussion

Velocity-time graph

- The slope of a tangent to velocity ~ time graph at any instant = acceleration at that instant
- If slope is + ve, then a = +ve.

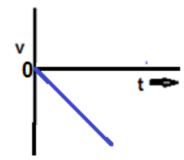


- If slope is - ve, then a = -ve.

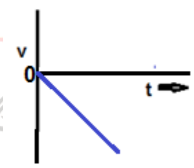


- If the graph is plotted in the +ve v- side, this means the body is moving in +ve direction.

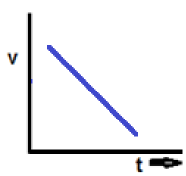
- If the graph is plotted in the -ve v- side, this means the body is moving in -ve direction.

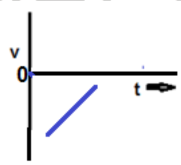


- If v=+ve and a = +ve or v = -ve and a = - ve then motion is accelerated .

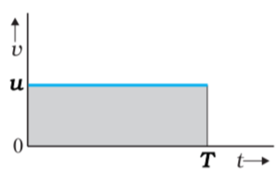


- If v = +ve and a = -ve or v = -ve and a = +ve then motion is retarded





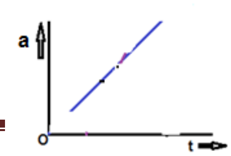
- The area below v ~ t graph and t-axis represent the displacement in the given interval.



Acceleration -time graph

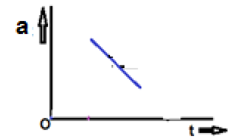
The slope of a tangent to a~t graph at any instant = Rate of change of acc. at that instant

- If the slope is + ve, then acceleration is increasing



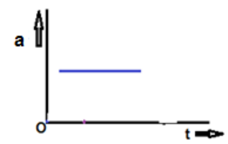
(Increasing acceleration)

- If the slope is - ve, then acceleration is decreasing



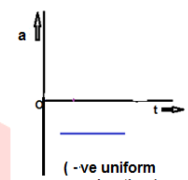
Decreasing acceleration

- If the graph is parallel to the time axis in the first quadrant then the acceleration is positive.



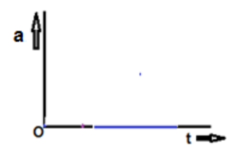
(+ve uniform acceleration)

- If the graph is parallel to the time axis in the fourth quadrant then the acceleration is negative.



(-ve uniform acceleration)

- This graph represents zero acceleration.



0 acceleration or uniform motion

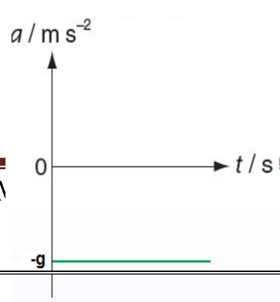
- The area below a ~ t graph represents the change in velocity in the interval.

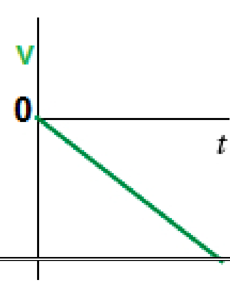
NUMERICAL

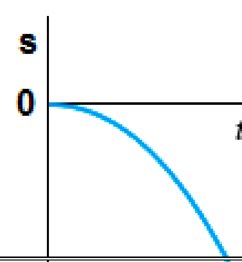
Draw a ~ t graph, velocity~time, speed~time, distance~time and displacement ~ time graphs for a particle just dropped from some height till it reaches the ground.

(upward direction: +ve, downward direction: -ve):

SOLUTION





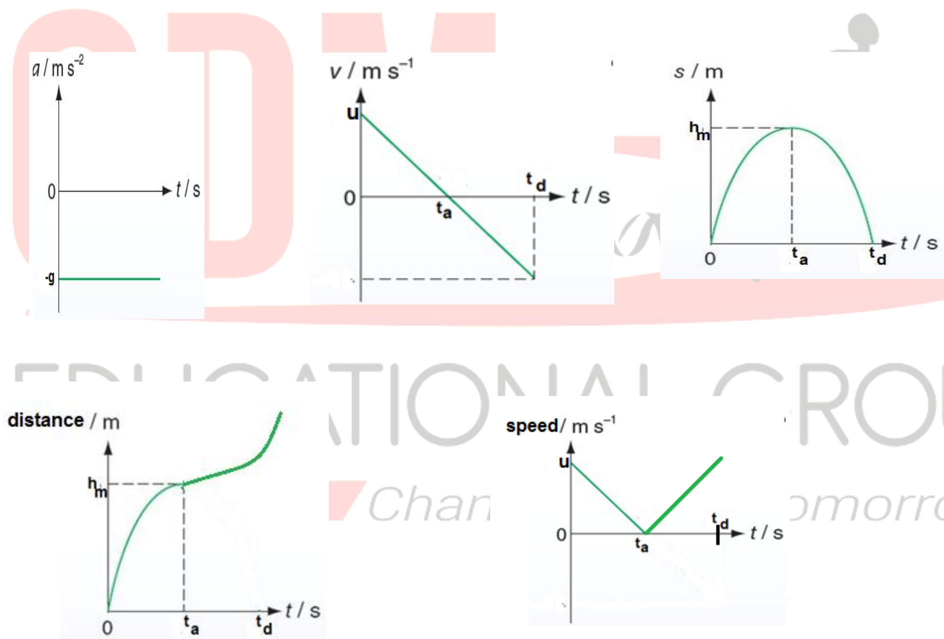


NUMERICAL

Draw a ~ t graph, velocity~time, speed~time, distance~time and displacement ~ time graphs for a particle thrown upward till it reaches the ground.

(Take upward direction: +ve, downward direction: -ve):

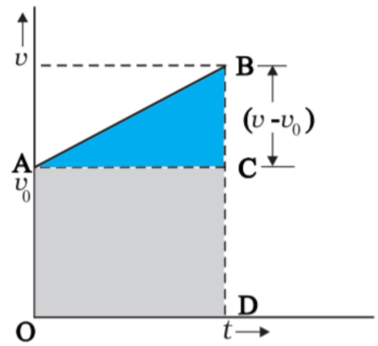
SOLUTION



KINEMATIC EQUATIONS OF UNIFORMLY ACCELERATED MOTION

(BY GRAPHICAL METHOD)

First equation of motion



Acceleration = $\frac{change\ in\ velocity}{time\ taken\ for\ change}$

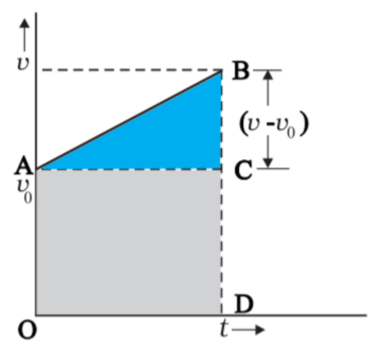
a =$\frac{BC}{AC}$

a= $\frac{v-v_0}{t}$

or

v= $v_0 + at$

The s**econd equation of motion**



The area of trapezium OABC gives the distance travelled.

Area = $\frac{1}{2} \times (OA + BD) \times OD$

= $\frac{1}{2} \times (v_0 + v) \times t$

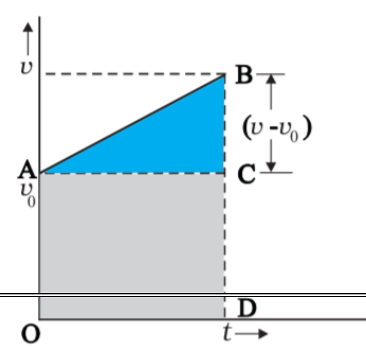
= $\frac{1}{2} \times (v_0 + v_0 + at) \times t$

= $v_0 + \frac{1}{2}at^2$

Distance covered = $v_0 + \frac{1}{2}at^2$

or

S= $v_0 + \frac{1}{2}at^2$



Third equation of motion

The area of trapezium OABC gives the distance travelled.

Area = $\frac{1}{2} \times (OA + BD) \times OD$

s = $\frac{1}{2} \times (v_0 + v) \times t$

$(v_0 + v)$ = $\frac{2s}{t}$..............................(1)

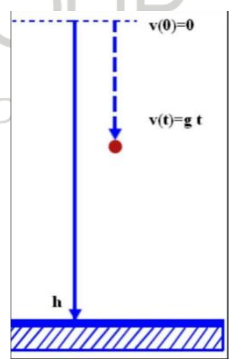
$(v - v_0)$ =at ....(2)

Multiplying above two equations we get

$v^2 - {v_0}^2 = 2as$

Suppose the object is released from rest

Therefore, v_0 = 0 and the equations of motion become:



v= $0 + at$

S= $0 + \frac{1}{2}at^2$

$v^2 - 0 = 2as$

The variation of acceleration, velocity, and distance, with time, have been plotted in

Figures given below.