# Chapter- 1
# Relations and Functions

**Introduction:-**

**Relation from a set A to B:-**

Let A and B be two non-empty sets. Then a set R is said to be a relation from set A to set B if R is a subset of $A \times B$. i.e., if $R \subseteq A \times B$.

**Example:-**

Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Define $R = \{(a, b) : 2a = b , a \in A , b \in A \}$

Show that R is a relation from A to B. Also, find the number of possible relations from A to B.

**Solution:** We have,

$A \times B = \{(1, 2) , (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 1), (3, 3), (3, 4)\}$

Here, R = {(1, 2) , (2, 4)}.

Since, R ⊆ A × B, so R is a relation from A to B.

The number of possible relations from A to B is $2^9 = 512$.

**Relation on a set A:-** Let A be any non-empty set. Then a set R is said to be a relation on A if R is a subset of A × A . i.e., if R ⊆ A × A.

**Example:-**

Let A = {1, 2, 3} and define R = {(a, b) : 2a = b : a, b ∈ A}. Show that R is a relation on A. What is the possible number of relations on A.

**Solution:** We have

A × A = {(1, 1) , (1, 2), (1, 3), (2, 1), (2, 2) , (2, 3), (3, 1), (3, 2), (3, 3)}.

Here, R = {(1, 2)}. So, R is a relation on A .

The number of relations on A = $2^{3^2} = 512$.

## Types of Relations:-

1. **Empty or Void Relation:-** A relation R on the set A is called empty relation if no elements of A are related to any elements of A, i.e., if R = ∅.

**Example:-**

Let A = {1, 2, 3} and define R = {(a, b) : a - b = 12 } . Show that R is an empty relation on set A.

**Solution:** We have

A × A $= \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$.

Since R = {(a, b) : a - b =12 }, so ∅ ⊆ A × A.

Hence, R is an empty relation on set A.

2. **Universal Relation:-** A relation R on a set A is called universal relation if each element of A is related to every element of A. i.e. if R = A × A.

**Example:-**

Let A = {1, 2} and define R = {(a, b) : a + b > 0}. Show that R is a universal relation on set A.

**Solution:** We have, A × A $= \{(1,1),(1,2),(2,1),(2,2)\}$

Since R = {(a, b) : a + b > 0}, so R $= \{(1,1),(1,2),(2,1),(2,2)\}$ =A × A.

Hence, R is a universal relation on set A.

**Remark**:- Void and universal relations are called trivial relations.

3. **Identity Relation:-** A relation R on set A is called identity relation if every element of A is related to itself only. i.e., if R = {(a, a) : a ∈ A}.The identity relation on set A is denoted by $I_A$ .

**Example:-**

Let A = {1, 2, 3}, and the relation R defined by R = {(a, b) : a - b = 0; a, b ∈ A }. Show that R is an identity relation.

**Solution:** We have

A × A $= \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$.

Since R = {(a, b) : a - b = 0; a, b ∈ A }, so R $= \{(1,1),(2,2),(3,3)\} \subseteq$ A × A.

Hence, R is an identity relation on A.

**4. Reflexive Relation:-** A relation R on the set A is called reflexive relation if a R a for every a ∈ A . i.e., if (a, a) ∈ R for every a ∈ A.

**Example:-**

Let A = {1, 2, 3}. Define the relation $R_1$, $R_2$ on A as

(i) $R_1$ = {(1,1) , (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)}   (ii) $R_2$ = {(1, 2), (1, 3), (2, 3)}

Check whether $R_1$ and $R_2$ are reflexive or not.

**Solution:** (i) Since, (a, a) ∈ $R_1$, for every a ∈ A, so $R_1$ is a relation on set A.

$(ii)$ Since, $(1, 1)$ ∉$R_2$, so $R_2$ is not a reflexive relation on set A.

**Remarks:-**

- Identity and universal relations are reflexive, but empty relation is not reflexive.
- All reflexive relations are not identity relations.

5. **Symmetric Relation**:- A relation R on the set a is called symmetric relation if a R b implies b R a, for every a, b ∈ A.

**Example:-**

Let A = {1, 2, 3} define the relation $R_1$ and $R_2$ on A as

(i) $R_1 = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$ (ii) $R_2 = \{(1, 1), (2, 2), (1, 2), (2, 1), (3, 1)\}$

Check whether $R_1$, $R_2$, are symmetric or not.

**Solution:** $(i)$ Here $R_1 = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$

Since, $(a, b) \in R_1 \Rightarrow (b, a) \in R_1$, for every $a, b \in A$.

Hence, $R_1$ is a symmetric relation on set A.

$(ii)$ Since, $(3, 1) \in R_2$, but $(1, 3) \notin R_2$.

Hence, $R_2$ is not a symmetric relation on set $A$.

**Remarks:-**

- Identity and universal relation are symmetric

- Empty relation is also symmetric, as there is no situation in which $(a, b) \in R$.

**6. Transitive Relation**:- A relation R on the set A is called transitive relation if a R b and b R c implies a R c, for every a, b, c ∈ A, i.e., if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for every $a, b, c \in A$.

**Example:-**

Let A = {1, 2, 3}. Define $R_1$, $R_2$ on A as

(i) $R_1 = \{(1, 1), (1, 2), (2, 3)\}$ (ii) $R_2 = \{(1, 2), (1, 3)\}$

Check $R_1$ and $R_2$ are transitive or not.

**Solution:** (i) Since, $(1, 2) \in R_1$ and $(2, 3) \in R_1$ but $(1, 3) \notin R_1$, so $R_1$ is not a transitive relation on set A.

$(ii)$ Since there is no situation in which $(a, b) \in R_2$ and $(b, c) \in R_2$, so $R_2$ is a transitive relation on set A.

**Remarks:-**

- Identity and universal relations are transitive.
- If there is no situation in which (a, b) ∈ R and (b, c ) ∈ R, then the relation is transitive.

**7. Equivalence Relation:-** A relation R on a set A is called equivalence relation if R is reflexive, symmetric, and transitive.

**Equivalence Class: -** Let R be an equivalence relation on set A and let a ∈ A. Then we define the equivalence class of 'a' as

[a] = { b ∈ A : b is related to a } = {b ∈ A : (b, a) ∈ R }

**Example:-**

Let A = {1, 2, 3} . Define the relations $R_1$ on A as $R_1$ = {(1, 1), (1, 2), (2, 1), (2, 2)}

Check whether $R_1$ is an equivalence relation or not. If yes, then find the equivalence classes of all the elements of set A.

**Solution:** Since $(3, 3) \notin R_1$, so $R_1$ is not reflexive.

Hence, $R_1$ is not an equivalence relation.

**Example:-**

Prove that the relation R on Z, defined by $(a, b) \in R \Leftrightarrow a - b$ is divisible by n, $n \in Z$ is an equivalence relation on Z.

**Solution:**

Reflexive: For $a \in Z$, we have $a - a = 0 = 0 \times n$.

So, $(a, a) \in R$. Hence, R is reflexive.

Symmetric: Let $(a, b) \in R$, where $a, b \in Z$

$\Rightarrow a - b = n \times k$, where $k \in Z$

$\Rightarrow b - a = - n \times k = n(-k)$

So, $(b, a) \in R$. Hence, R is symmetric.

Transitive: Let (a, b) ∈ R and (b, c) ∈ R, where a, b, c ∈ Z.

⇒ a - b = n × k and b - c = n × m, where k, m ∈ Z

Adding, a - c = n ( k + m)

So, (a, c)∈ R, Hence, R is transitive.

Therefore, R is an equivalence relation.

**Example:-**

Write the smallest and largest equivalence relation on the set A = {1, 2, 3}.

**Solution:** The smallest equivalence relation on the set A is $I_A = \{(1,1),(2,2),(3,3)\}$.

The largest equivalence relation on set A is

$A \times A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

## MEMORY MAPS

A relation in set A is a subset of . We also write it as

Relation R on set A is a subset of , i.e

A relation R on set A is said to be an empty relation if no element of set A is related to any element of set A,

A relation R on set A is called a universal relation if each element of A is related to every element of A,

A relation R on set A is called an identity relation if each element of A is related to itself only.i.e, we

**Reflexive Relation:**
A relation R in a set A is said to be reflexive, if , for every or we say aRa, for every .

**Symmetric Relation:**
A relation R in a set A is said to be symmetric, if , for all . We can also say , for every

**Transitive Relation:**
A relation R in a set A is said to be transitive, if and , for every . We can

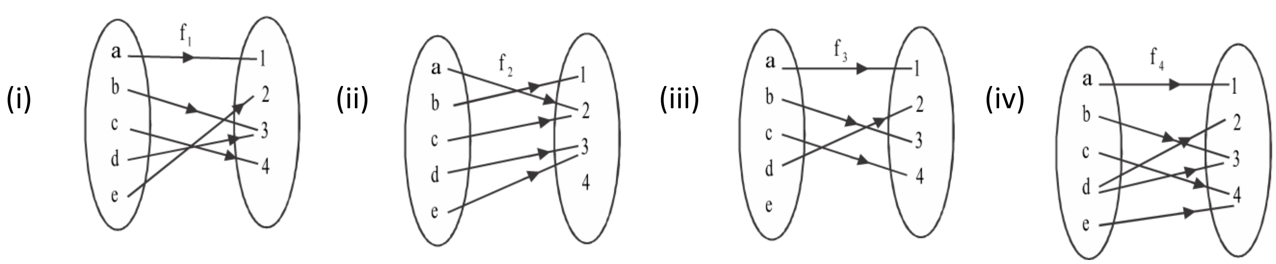
## Functions

### Introduction:

**Function from set A to set B:-** Let A and B be two non-empty sets, then a function $f$ from set A to set B is a rule (or map or correspondence) which associates each element of set A to exactly one element of set B. If $f$ is a function from set A to set B, then we denote it by $f : A \to B$.

**Example:-**

Check whether the maps in the following diagram are functions or not.




**Solution:** $(i)$ Every element in $A$ has exactly one image in $B$. So, $f_1$ is a function.

$(ii)$ Every element in $A$ has exactly one image in $B$. So, $f_2$ is a function.

$(iii)$ Element $e$ in $A$ does not have an image in $B$. So, $f_3$ is not a function.

$(iv)$ Element $d$ in $A$ does not have exactly one image in $B$. So, $f_4$ is not a function.

**Domain, Co-domain, and Range of a function:-**

Let $f : A \to B$ be function, then

(i) set A is called the domain of function $f$.

(ii) the set B is called the Co-domain of $f$.

(iii) the set of all images of elements of set A under $f$ is called range or image set of $A$ under $f$.

**Remarks:-**

- The range of A under $f$ is denoted by $f(A)$.
- If $f(a) = b$ then, $b$ is called an image of $a$ under $f$, and $a$ is called pre-image of $b$.
- The range is always a subset of the co-domain.

- If $n(A)=p, n(B)=q$, then the number of functions from A to B is $(q)^p$

**Types of Functions:-**

**1. One-one function or Injective function:-** A function $f:A\to B$ is said to be one-one if no two elements of A have the same image, i.e., if $a\neq b \Rightarrow f(a)\neq f(b)$ for all $a,b\in A$

or $f(a)=f(b)\Rightarrow a=b$ for all $a,b\in A$.

**Remarks:-**

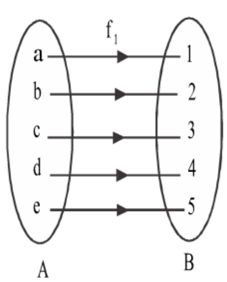
- If a function $f:A\to B$ is not one-one then it is called the many-one function.
- if a function $f:A\to B$ is one-one then $n(A)\leq n(B)$
- If $n(A)=p, n(B)=q$, then no of one-one function from A to B

$$= \begin{cases} 0, & \text{if} \quad p > q \\ {}^{q}P_{p} = \dfrac{q!}{(q-p)}, & \text{if} \quad p \le q \end{cases}$$

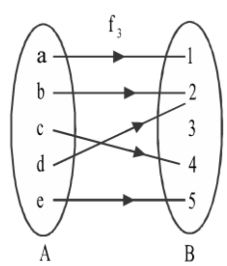
**Example:-**

Check whether the function in the diagrams is one-one or not.

(i)




(ii)




**Solution:** $(i)$ Every element in $A$ has a different image in $B$. So, $f_1$ is a one-one function.

$(ii)$ Elements $b$ and $d$ in $A$ have the same image 2 in $B$. So, $f_3$ is not a one-one function.

## 2. Onto function or Surjective function:-

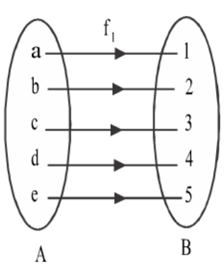
A function $f : A \to B$ is said to be onto if, for each $b \in B$, there exists $a \in A$ such that $f(a) = b$, we say that $a$ is pre-image of $b$. In other words, $f$ is onto if Range of $f =$ Co-domain of $f$, i.e., if every element in B has a preimage in A.

**Remarks:-**

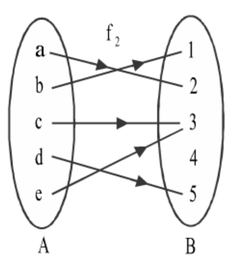
- If a function $f : A \to B$ is not onto then it is called into function.
- If a function $f : A \to B$ is onto then $n(A) \geq n(B)$
- Let A be any finite set such that $n(A) = p$ then, the number of onto functions from A to A is $p!$.

**Example:-** Check whether functions in the following diagram are onto:

(i)




(ii)




**Solution:** $(i)$ Since, every element in $B$ has preimage in $A$, so, $f_1$ is onto function.

$(ii)$ Since, $4 \in B$ does not have pre-image in $A$, so, $f_2$ is not onto function.

### 3. Bijective Function:-

A function $f: A \to B$ is said to be bijective if it is both one-one and onto.

**Remarks:**

- If $f: A \to B$ is a bijection, then $n(A) = n(B)$.
- Let $A$ and $B$ be two non-empty finite sets such that $n(A) = p$ and $n(B) = q$. Then,

Number of bijective functions from  to

**Example:-**

Classify the following function as one– one, onto, or bijection:

$f: N \to N$ defined by $f(x) = x^2 + 1$.

**Solution:** <u>One – one</u>: Let $x_1, x_2 \in N$ be any two elements.

Then, $f(x_1) = f(x_2) \Rightarrow x_1^2 + 1 = x_2^2 + 1$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$

So, $f$ is one – one.

<u>Onto:</u> Let $y \in N$ be any element.

Then, $f(x) = y \Rightarrow x^2 + 1 = y$

$$\Rightarrow x = \sqrt{y - 1}$$

For $y = 1 \in N$, we have $x \notin N$.

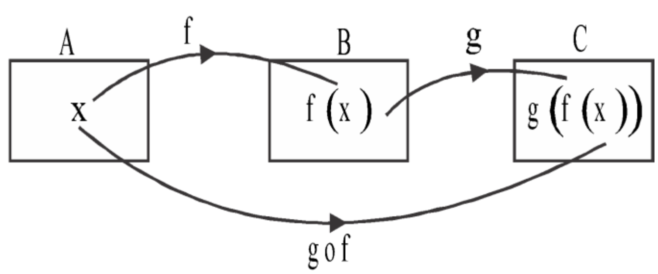
So, $f$ is not onto.

Hence, $f$ is not a bijection.

**Composition of Functions:-**

The composition of two functions is a chain process in which the output of the first function becomes the input of the 2nd function. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions.

For every $x \in A$, there is exactly one element $f(x) \in B$. For $f(x) \in B$, there is exactly one element $g(f(x)) \in C$. This result is a new function from A to C as shown in the figure.




**Definition:** Let and be any two functions. Then the composition of f and g is a function defined as .

**Remarks:-**

- The composition $gof$ exists if the range of $f \subseteq$ domain of g.
- The composition $fog$ exists if the range of $g \subseteq$ domain of f.
- It may be possible $gof$ exists but $fog$ does not exist
- gof and fog may or may not be equal.

**Example:** If $f : R \to R$ and $g : R \to R$ is given by $f(x) = \cos x$ and $g(x) = 5x^2$. Find gof and fog show that $fog \neq gof$.

**Solution:** $gof(x) = g(f(x)) = g(cosx) = 5\, cos^2 x$

and $fog(x) = f(g(x)) = f(5x^2) = cos\, cos\, (5x^2)$

**Properties of the composition of Functions:-**

1. Composition of functions is not necessarily commutative. Let $f: A \longrightarrow B$ and $g: B \longrightarrow C$, then $fog \neq gof$.

2. Composition of functions is associative. Let $f : A \to B, g : B \to C$ and $h : C \to D$ then $(hog)of = ho(gof)$

3. Let $f : A \to B$ and $g : B \to C$ be two functions.

   (i) If both are one-one then gof is one-one

   (ii) If both are onto then gof is onto.

4. Let $f : A \to B$ and $g : B \to C$ be two functions such that $gof : A \to C$

(i) If gof is onto, then g is onto.

(ii) If gof is one-one then f is one-one.

(iii) If gof is onto and g is one-one then f is onto.

(iv) If gof is one-one and f is onto then g is one-one.

**Example:**

Let $f: R \to R$ be signum function as $f(x)=\begin{cases} 1 & \text{if} \quad x>0 \\ 0 & \text{if} \quad x=0 \\ -1 & \text{if} \quad x<0 \end{cases}$ and $g: R \to R$, be the greatest integer function given by $g(x)=[x]$. Do fog and gof coincide in $(0,1]$?

**Solution:-**

Let $x \in (0,1)$ be any element

$\text{fog}(x) = f(g(x)) = f([x])$

$= f(0) \text{ as } x \in (0,1) = 0$

Also $(\text{gof})(x) = g(f(x)) = g(1) = [1] = 1$ as $x \in (0,1)$

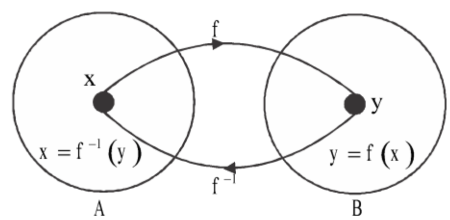
$\therefore (\text{fog})(x) \neq (\text{gof})(x)$ for every $x \in (0,1)$ ; so fog and gof does not coincide in $(0,1]$

**The inverse of a Function:-**

Let f be a one-one and on-to function from A to B. Let $y$ be an arbitrary element of B. Then f being onto, there exists an element $x \in A$ such that $f(x) = y$, Also f being one-one this x must be unique.

Thus for each $y \in B$, there exists a unique element $x \in A$ such that $f(x) = y$. So we may define a function denoted by $f^{-1}$ as $f^{-1} : B \to A$. Such that $f^{-1}(y) = x \Leftrightarrow f(x) = y$.

The function $f^{-1}$ is called the inverse of f.




**Definition (2)**

Another definition of the inverse function. Let be one-one and onto function, then the function such that and , where are identity functions on A and B respectively, is called the

**Remarks:-**

function, then is a function which associates to each y of B, a unique inverse of function f.

- If the inverse of a function $f$ exists then $f$ is called an invertible function.
- A function $f$ is invertible if and only if $f$ is one-one and onto.

- The two definitions of the Inverse function given above are equivalent.
- The domain of $f^{-1}$ = Range of f and range of $f^{-1}$ = domain of f.
- $\left(f^{-1}of\right)(x) = x, \forall x \in$ the domain of f i.e $f^{-1}of$ is an identity function.
- $\left(f^{-1}\right)^{-1} = f$
- If f is one-one and onto then $f^{-1}$ is also one-one and onto.

**Working Rule to find Inverse of a Function:-**

Let defined by

Step – I:- Prove that f is one-one i.e take and show that

Step – II:- Prove that f is onto i.e for any , there exists

Step – III:- Find x in terms of y from let

**Example -1**

Consider $f : R \to R$ given by $f(x) = 4x + 3$. Show that f is invertible, find the inverse of f.

**Solution:** Given $f: R \to R$ defined by $f(x) = 4x + 3$.

**One – one:** Let $x_1, x_2 \in R$ be any two elements.

Then, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3$

$\Rightarrow x_1 = x_2$

So, $f$ is one – one.

**Onto:** Let $y \in R$ be any element.

Then, $f(x) = y \Rightarrow 4x + 3 = y$

$\Rightarrow x = \frac{y-3}{4}$

For every $y \in R$, we have $x \in R$. So, $f$ is onto.

Thus, $f$ is a bijection and hence invertible.

So, $f^{-1}: R \to R$ exists and we have $f^{-1}(y) = \frac{y-3}{4}$ [ $\because f(x) = y \Leftrightarrow x = f^{-1}(y)$ ]

Hence, the inverse of $f$ is given by $f^{-1}(x) = \frac{x-3}{4}$.

**Properties of Invertible Functions:-**

(1) If $f : X \to Y$ $g : Y \to Z$ are two invertible functions. Then gof is also invertible with $(gof)^{-1} = f^{-1} o\, g^{-1}$.

(2) If $f : X \to Y$ is invertible, then its inverse is unique.

(3) If $f : X \to Y$ is invertible then $f^{-1} o f = I_X$ and $f o f^{-1} = I_Y$

(4) Let $f : X \to Y$ and $g : Y \to X$ be two functions such that $gof = I_X$ and $fog = I_Y$ then f and g are bijections and $g = f^{-1}$.

**Example:**

If $A=\{a,b,c,d\}$ and the function $f=\{(a,b),(b,d),(c,a),(d,c)\}$. Write $f^{-1}$.

**Solution:** $f^{-1}=\{(b,a),(d,b),(a,c),(c,d)\}$.

**Example:**

If $f(x)=\dfrac{4x+3}{6x-4}, x\neq\dfrac{2}{3}$ show that $fof(x)=x$ for all $x\neq\dfrac{2}{3}$. What is the inverse of $f$?

**Solution:** Given $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$.

Now, $fof(x)=f(f(x)=f\left(\frac{4x+3}{6x-4}\right)=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{34x}{34}=x$.

$\Rightarrow (fof)(x)=x$, for all $x\neq\frac{2}{3}$.

Since, $(fof)(x)=x=I(x)$, for all $x\neq\frac{2}{3}$

So, $f^{-1} = f \Rightarrow f^{-1}(x) = f(x)$, for all $x \neq \frac{2}{3}$

$\Rightarrow f^{-1}(x) = \frac{4x+3}{6x-4}$, for all $x \neq \frac{2}{3}$

Hence, the inverse of $f$ is given by $f^{-1}(x) = \frac{4x+3}{6x-4}$, for all $x \neq \frac{2}{3}$.

**Example:**

Show that the modulus function $f : R \to R$, given by $f(x) = |x|$ is neither one-one nor onto.

**Solution:-**

**For one-one** $f(3) = |3| = 3$ $\quad\quad f(-3) = |-3| = 3$

As $f(3) = f(-3)$ but $3 \neq -3$ so f is not one-one

**For onto** $\text{Range } f = R^{+} U \{0\}$ Co-dom of f = R

As Range $f \neq$ co-dom f so f is not onto

**Example:**

Give an example of a function

(i) Which is one-one but not onto

(ii) Which is not one-one but onto

(iii) Which is neither one-one nor onto.

**Solution:-**

(i) Let $A = \{1, 2\}, B = \{4, 5, 6\}$ and let $f = \{(1, 4), (2, 5)\}$. Since every element of A has different images in B so f is one-one. Also, the element $6 \in B$ that does not have a pre-image is A. So f is not onto

(ii) Let $A = \{1, 2, 3\}, B = \{4, 5\}$ and $g = \{(2, 4), (1, 4), (3, 5)\}$ Since $1, 2 \in A$ have the same image 4 is B. So, g is not one-one. Also, every element of B has a pre-image is A, so g is onto

(iii) $A=\{1,2,3\}, B=\{4,5\}$ and $h=\{(1,4),(2,4),(3,4)\}$. Since elements $1,2,3\in A$ have the same image 4 in B. So h is not one-one. Also, the element $5\in B$ does not have a pre-image in A so h is not onto.

**Example:**

If the function $f:R\to R$ is defined by $f(x)=2x-3$ and $g:R\to R$, $g(x)=x^3+5$. Then find fog and show that fog is invertible. Also find $(fog)^{-1}$, Hence find $(fog)^{-1}(9)$.

**Solution:-**

Here $f:R\to R$ defined by $fog(x)=f(g(x))=f(x^3+5)=2(x^3+5)-3=2x^3+7$. Now to prove fog is invertible. One-one:- Let $x_1,x_2\in R$ and $(fog)(x_1)=(fog)(x_2)$

$\Rightarrow 2x_1^3 + 7 = 2x_2^3 + 7$

$\Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$

So fog is one-one Onto:- let $y \in R$ be any element then $fog(x) = y$

$\Rightarrow 2x^3 + 7 = y$

$\Rightarrow 2x^3 = y - 7 \Rightarrow x^3 = \frac{y-7}{2}$

$\Rightarrow x = \sqrt[3]{\frac{y-7}{2}}$ ………………………. (1)

For every, $y \in R$ we have $x \in R$ so fog is onto.

Thus, fog is an invertible function so $(fog)^{-1}: R \to R$ exists and from (1)

$(fog)^{-1}(y) = \sqrt[3]{\frac{y-7}{2}}; (fog)^{-1}(9) = \sqrt[3]{\frac{9-7}{2}} = 1$

**Example:**

If the function $f(x) = \sqrt{2x-3}$ is veritable, then find $f^{-1}$. Hence prove that $(fof^{-1})(x) = x$.

**Solution:-**

Given $f: R \to R$ defined by $f(x) = \sqrt{2x-3}$

One-one: Let $x_1, x_2 \in R$ and $f(x_1) = f(x_2)$

$\Rightarrow \sqrt{2x_1 - 3} = \sqrt{2x_2 - 3}$

$\Rightarrow 2x_1 - 3 = 2x_2 - 3$

$\Rightarrow x_1 = x_2$

So f is one-one

Onto:- Let $y \in R$ be any element then $f(x) = y$

$\Rightarrow \sqrt{2x-3} = y$

$\Rightarrow 2x - 3 = y^2$

$\Rightarrow x = \frac{y^2+3}{2}$ ..............................(1)

So f is onto. Thus f is on invertible function so $f^{-1} : R \to R$ exists and from (1) we have

$f^{-1}(y) = \frac{y^2+3}{2}$

The inverse of f is given by $f^{-1}(x)=\frac{x^2+3}{2}$

Now $\left(fof^{-1}\right)(x)=f\left(f^{-1}(x)\right)$

$$=f\left(\frac{x^2+3}{2}\right)=\sqrt{2\left(\frac{x^2+3}{2}\right)-3}$$

**Example:**

Consider $f:N\to N, g:N\to N$ and $h:N\to R$ define as $f(x)=2x,\ g(y)=3y+4$ and $f(x)=\sin x$ for all $x,y,z\in N$. Show that $ho(gof)=(hof)of$

**Solution:-**

Given $f:N\to N$, defined by $f(x)=2x; g:N\to N$ defined by $g(y)=3y+4$ and $h:N\to R, h(x)=\sin x$

Now $ho(gof):N \to R$ such that $\left[ho(gof)\right](x)=h\left[gof(x)\right]$

$=h\left(g\left(f(x)\right)\right)=h\left(g(2x)\right)=h\left[3(2x)+4\right]$

$=h(6x+4)=\sin(6x+4)$

Also $(hog)of:N \to R$ such that $\left[(hog)of\right](x)=(hog)\left(f(x)\right)$

$=(hog)(2x)=h\left(g(2x)\right)$

$=h\left[3(2x)+4\right]$

$=h(6x+4)=\sin(6x+4)$

Hence, $\left[ho(gof)\right](x)=\left[(hog)of\right](x);\forall x \in N$

## MEMORY MAPS

A function is said to be one-one (or injective), if the images of distance elements of A under the rule f are distinct in B. i.e for every
or we can also say that

**Onto (surjective) function:**
A function is said to be onto(or surjective), if every element of B is the image of some element of A under the rule f, i.e for every , there exists an element such that .
Note: A function is onto if and only

**One-one and onto (bijective) function:** A function is said to be one-one and onto (or bijective) if f is both one-one and onto.

**Composition of function:** Let and $g$ : B (range of f) be two functions. Then the composition of functions f and g is a function from A to C and is denoted by gof. We define gof as . For working, on element x first we apply f rule and whatever result is obtained in set B, we apply g rule on it to get the required result in set C.

**Invertible function**: A function is said to be invertible, if there exists a function such that . The function g is called the inverse of f and is denoted by .

**Note:-** For a function to be invertible, it must be one-one and onto, i.e. bijective.