

Chapter- 3

Matrices

Concept, notation, order, equality, types of matrices

A matrix is a rectangular arrangement of numbers or functions arranged into a fixed number of rows and columns. The element of a matrix is always enclosed in the bracket [] or (). Matrices are represented by capital letters like A, B, C, etc.

A matrix having m rows and n columns is called a matrix order $m \times n$ (read as m by n matrix).

In general, a matrix of order $m \times n$ is written as.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

It can also be written in compact form as $A = (a_{ij})_{m \times n}$ represent an element of i th row and j th column.

Question – 1

Write all possible order of matrices having 24 elements

Solution:- $1 \times 24, 2 \times 12, 3 \times 8, 4 \times 6, 6 \times 4, 8 \times 3, 12 \times 2, 24 \times 1$

Question – 2

Write a matrix of order 2×3 where $a_{ij} = 2i - j$

Solution:- $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$

Types of Matrices:-

Row Matrix:- A matrix is said to be a row matrix if it has only one row.

Example:- $A = [2 \ 3 \ 5 \ 7]_{1 \times 4}$

Column Matrix:- A matrix having any number of rows but only one column is called a column matrix.

Example:- $A = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}_{5 \times 1}$

Rectangular Matrix:-

A matrix having m rows and n columns where $m \neq n$ is called a rectangular matrix.

$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 6 \end{bmatrix}_{2 \times 3}$

Square matrix:-

It is a matrix in which number of rows = number of columns $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3 \times 3}$

Diagonal Elements:-

The diagonal elements of a square matrix are the elements for which $i = j$. i.e the elements $a_{11}, a_{22}, a_{33}, \dots$. The line along which the diagonal elements lie is called the leading diagonal or principal diagonal.

Diagonal Matrix:-

It is a square matrix where diagonal elements are non-zero but the other elements are zero.

Example:- $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Scalar matrix:- It is a diagonal matrix where all the diagonal elements are equal

Example:- $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Equality of Matrices

Two matrices are said to be equal if their order is the same and their corresponding elements are equal.

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ then $a = 2, b = 3, c = 4, d = 5$

Question:-

If $\begin{bmatrix} x - y & 2x + z \\ 2x - y & 3z + w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ then find x, y, z and w

Solution:-

$$x - y = -1$$

$$2x + z = 5$$

$$2x - y = 0$$

$$3z + w = 13$$

Solving $x = 1, y = 2, z = 3, w = 4$

Addition, multiplication, and scalar multiplication.**Addition of Matrices:-**

Matrix addition is defined only when they are of the same order. The sum of matrices A and B is a matrix whose elements are obtained by adding the corresponding elements of A and B.

Example:-

$$\text{Let } A = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ 0 & 4 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 1 \\ -4 & 9 \end{bmatrix}, A - B = \begin{bmatrix} 1 & 5 \\ -4 & 1 \end{bmatrix}$$

Multiplication of a matrix by a scalar:-

If a scalar K is multiplied by a matrix A then all elements of matrix A are multiplied by constant K.

Example:-

$$\text{If } A = \begin{bmatrix} 3 & 4 \\ -5 & 1 \end{bmatrix} \text{ find } 5A$$

Solution:-

$$5A = \begin{bmatrix} 15 & 20 \\ -25 & 5 \end{bmatrix}$$

Example:-

$$\text{If } A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 3 & 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 0 & 1 \\ 0 & 0 & 4 \end{bmatrix} \text{ find } 3A + 4B$$

Solution:-

$$3A + 4B = \begin{bmatrix} 3 & 6 & 9 \\ 0 & -3 & 6 \\ 9 & 12 & 0 \end{bmatrix} + \begin{bmatrix} 8 & -4 & 0 \\ 2 & 0 & 4 \\ 0 & 0 & 16 \end{bmatrix} = \begin{bmatrix} 11 & 2 & 9 \\ 12 & -3 & 10 \\ 9 & 12 & 16 \end{bmatrix}$$

Simple properties of addition, multiplication, and scalar multiplication

Properties:-

Closure Law:- A matrix added with a matrix always gives a matrix. So Closure Law satisfies.

Commutative Law:- $A + B = B + A$

Associative Law:- $A + (B + C) = (A + B) + C$

Existence of Additive Identity:-

A null matrix of the same as with the given matrix is the additive identity of the matrix. $A + O =$

$$O + A = A$$

Example:-

The additive identity of $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ is $O_{2 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Existence of additive inverse:-

A is the additive inverse of A

Question – 1

Find x and y if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Solution:- $x = y = 3$

Question – 2

If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ then find the matrix X, such that $2A + 3X = 5B$

Solution:-

$$X = \begin{bmatrix} -2 & -\frac{10}{3} \\ 4 & \frac{14}{3} \\ -\frac{31}{3} & -\frac{7}{3} \end{bmatrix}$$

Question – 3

Find the value of $x + y$ from matrix equation $2 \begin{bmatrix} x & 5 \\ 7 & y - 3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Solution:-

$$x + y = 2 + 9 = 11$$

Multiplication of Matrices

If A and B be any two matrices, then their product AB will be defined only when the number of columns in A is equal to the number of rows in B. if $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$ then their product $AB = C = [c_{ij}]$, will be a matrix of order $m \times p$, where, $(AB)_{ij} = c_{ij} = \sum_{r=1}^n a_{ir}b_{rj}$

Example -

$$\text{if } A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\text{Then } AB = \begin{bmatrix} 1.1 + 4.2 + 2.1 & 1.2 + 4.2 + 2.3 \\ 2.1 + 3.2 + 1.1 & 2.2 + 3.2 + 1.3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 11 & 16 \\ 9 & 13 \end{bmatrix}$$

Properties of Matrix Multiplication:-

If A, B, and C are three matrices such that their product is defined, then

- $AB \neq BA$ (Generally not commutative)
- $(AB)C = A(BC)$ (Associative Law)
- $IA = A = AI$ (I is an identity matrix for matrix multiplication)
- $A(B + C) = AB + AC$ (Distributive Law)
- If $AB = AC$ this does not imply that $B = C$ (Cancellation Law is not applicable)
- If $AB = 0$ It does not mean that $A = 0$ or $B = 0$ again product of two non-zero matrices may be zero matrices.

Note:-

- The multiplication of two diagonal matrices is again a diagonal matrix.
- The multiplication of two triangular matrices is again a triangular matrix
- The multiplication of two scalar matrices is also a scalar matrix.
- If A and B are two matrices of the same order, then
 - $(A + B)^2 = A^2 + B^2 + AB + BA$
 - $(A - B)^2 = A^2 + B^2 - AB - BA$
 - $(A - B)(A + B) = A^2 - B^2 + AB - BA$
 - $(A + B)(A - B) = A^2 - B^2 - AB + BA$
 - $A(-B) = (-A)B = -(AB)$

Positive Integral Powers of the matrix:-

The positive integral powers of matrix A are defined only when A is a square matrix. Also then

$$A^2 = A.AA^3 = A.A.A = A^2A$$

Also for any positive integers m, n

$$(a) A^m A^n = A^{m+n} \quad (b) (A^m)^n = A^{mn} = (A^n)^m \quad (c) I^n = I, I^m = I$$

$$(d) A^0 = I_n \text{ where A is a square matrix of order n.}$$

Example – 1

If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $A^2 - 4A - nI = 0$, then find the value of n.

Solution:-

$$A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}, 4A = \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix}, nI = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$$

$$\Rightarrow A^2 - 4A - nI$$

$$= \begin{bmatrix} 5-8-n & -4+4-0 \\ -4+4-0 & 5-8-n \end{bmatrix} = \begin{bmatrix} -3-n & 0 \\ 0 & -3-n \end{bmatrix}$$

$$\therefore A^2 - 4A - nI = 0$$

$$\Rightarrow \begin{bmatrix} -3-n & 0 \\ 0 & -3-n \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow -3 - n = 0 \Rightarrow n = -3$$

Example – 2

If $A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$ then find the element a_{21} of A^2

Solution:-

The element a_{21} is the product of the second row of A to the first column of A

$$\therefore a_{21} = [3 \quad -4] \begin{bmatrix} -1 \\ 3 \end{bmatrix} = -3 - 12 = -15$$

Transpose of a Matrix:-

The matrix obtained from a given matrix A by changing its rows into columns or columns into rows is called transpose of matrix A and is denoted by A^T or A' . From the definition, it is obvious that if the order of A is $m \times n$.

Example:- Transpose of Matrix $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}_{2 \times 3}$ is $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}_{3 \times 2}$

Properties of Transpose:-

(a) $(A^T)^T = A$ (b) $(A \pm B)^T = A^T \pm B^T$ (c) $(AB)^T = B^T A^T$

Non-commutative of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix

Non-Commutative of multiplication of matrices:-

Example:- If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ find AB, BA, show that $AB \neq BA$

Solution:- $AB = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}_{2 \times 3}$, $BA = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}_{3 \times 3}$

As $AB \neq BA$. So it is not commutative.

Zero Matrix as a product of two non-zero matrices.

Example:- $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$ $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

If the product of two matrices is a zero matrix, one of the matrices doesn't need to be a zero matrix.

Application of Matrices:-

Example – 1

Use matrix multiplication to divide Rs. 30, 000 into two parts such that the total annual interest at 9% on the 1st part and 11% on the second part amount Rs. 3060/-

Solution:- $A = [x \quad 30000 - x]$

$R = \begin{bmatrix} 0.09 \\ 0.11 \end{bmatrix}$ $AR = 3060$

$\Rightarrow [x \quad 30000 - x] \begin{bmatrix} 0.09 \\ 0.11 \end{bmatrix} = 3060$

$\Rightarrow x = 1200$

Two parts are 1200, 1800

Example – 2 Let $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I be the identity matrix of order 2.

Show that $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Solution:-

We have

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}$$

$$\text{And } I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix}$$

$$\therefore (I - A) = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\Rightarrow (I - A) = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} & -\frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \\ \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} & \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \end{bmatrix}$$

$$\Rightarrow (I - A) = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \frac{1-t^2}{1+t^2} & -\frac{2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix} \text{ where } t = \tan \frac{\alpha}{2}$$

$$\Rightarrow (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$\Rightarrow (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t(1+t^3)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{1+t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$\Rightarrow (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix} = I + A$$

Example – 3

If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI + bA)^n = a^n I + na^{n-1}bA$

Where I is a unit matrix of order 2 and n is a positive integer.

Solution:-

We shall prove the result by mathematical induction on n.

Step – 1, When n = 1, by the definition of integral powers of a matrix, we have

$$(aI + bA)^1 = aI + bA = a^1I + 1a^0bA = a^1I + 1a^{1-1}bA$$

So, the result is true for n = 1

Step – 2, Let the result be true for n = m. Then

$(aI + bA)^m = a^mI + ma^{m-1}bA$. Now we shall show that the result is true for $n = m + 1$

i.e. $(aI + bA)^{m+1} = a^{m+1}I + (m + 1)a^m bA$

By the definition of integral powers of a matrix, we have

$$(aI + bA)^{m+1} = (aI + bA)^m(aI + bA)$$

$$\Rightarrow (aI + bA)^{m+1} = (a^mI + ma^{m-1}bA)(aI + bA)$$

$$\Rightarrow (aI + bA)^{m+1} = (a^mI)(aI) + (a^mI)(bA) + (ma^{m-1}bA)(aI) + (ma^{m-1}bA)(bA)$$

$$\Rightarrow (aI + bA)^{m+1} = (a^m a)(I.I) + a^m b(IA) + ma^m b(AI) + ma^{m-1}b^2(AA)$$

$$\Rightarrow (aI + bA)^{m+1} = a^{m+1}I + a^m bA + ma^m bA + ma^{m-1}b^2A^2 \quad [\because IA = AI = A, I.I = I]$$

$$\Rightarrow (aI + bA)^{m+1} = a^{m+1}I + (ma^m b + a^m b)A + ma^{m-1}b^2A^2$$

$$\Rightarrow (aI + bA)^{m+1} = a^{m+1}I + (m + 1)a^m bA + ma^{m-1}b^2A^2 \quad [\because A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow A^2 =$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}]$$

$$\Rightarrow (aI + bA)^{m+1} = a^{m+1}I + (m + 1)a^m bA$$

This shows that the result is true for $n = m + 1$, whenever it is true for $n = m$. Hence, by the principle of mathematical induction, the result is valid for any positive integer n.

Example – 4

If A is a square matrix such that $A^2 = A$ shows that $(I + A)^3 = 7A + I$

Solution:-Using matrix multiplication, we obtain

$$(I + A)^2 = (I + A)(I + A)$$

$$= I(I + A) + A(I + A)$$

$$= I^2 + IA + AI + A^2$$

$$= I + A + A + A^2$$

$$= I + 2A + A^2$$

$$= I + 2A + A$$

$$= I + 3A$$

$$\therefore (I + A)^3 = (I + A)^2(I + A)$$

$$\Rightarrow (I + A)^3 = (I + 3A)(I + A)$$

$$= I(I + A) + 3A(I + A)$$

$$= I^2 + IA + 3(AI) + 3(AA)$$

$$= I + A + 3A + 3A^2$$

$$= I + A + 3A + 3A.$$

$$= I + 7A$$

Symmetric and Skew symmetric matrices.

Symmetric Matrix:- A square matrix $A = [a_{ij}]$ is called a symmetric matrix if $a_{ij} = a_{ji}$ for all i, j , or $A^T = A$.

Example:-
$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

Note:-

- Every unit matrix and square zero matrix are symmetric matrices.
- The maximum number of different elements in a symmetric matrix is $\frac{n(n+1)}{2}$

Skew-Symmetric Matrix:- A square matrix $A = [a_{ij}]$ is called the skew-symmetric matrix. If $a_{ij} = -a_{ji}$ for all, i, j or $A^T = -A$

Example:-
$$\begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix}$$

Note:-

- All principal diagonal elements of the skew-symmetric matrix are always zero because for any diagonal element $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$
- The diagonal elements of a skew-symmetric matrix are always 0.

Proof:

If $A = [a_{ij}]$ be a skew-symmetric matrix. Then,

$$a_{ij} = -a_{ji} \text{ for all } i, j$$

$$\Rightarrow a_{ij} = -a_{ij} \text{ for all values of } i$$

$$\Rightarrow 2a_{ij} = 0$$

$$\Rightarrow a_{ii} = 0 \text{ for all values of } i$$

$$\Rightarrow a_{11} = a_{22} = a_{33} = \dots = a_{nn} = 0.$$

Example:-

1. If the matrix $A = \begin{bmatrix} 0 & a & 5 \\ 3 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric. Find the value of a, b, c

Answer:

For a skew-symmetric $A = [a_{ij}]$, we have

$$a_{ij} = -a_{ji} \text{ for all } i \neq j \text{ and } a_{ii} = 0 \text{ for all } i$$

Thus, if $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric, then $A_{22} = 0, A_{12} = -A_{21}$ and $A_{31} = -A_{13}$

$$\Rightarrow b = 0, a = -2 \text{ and } c = -3$$

2. Let A be a square matrix then prove that

- $A + A'$ is a symmetric matrix

Prove:

Let $P = A + A^T$, Then,

$$P^T = (A + A^T)^T = A^T + (A^T)^T \quad \left[\because (A+B)^T = A^T + B^T \right]$$

$$\Rightarrow P^T = A^T + A \quad \left[\because (A^T)^T = A \right]$$

$$\Rightarrow P^T = A + A^T = P \quad \text{[By commutativity of matrix addition]}$$

$\therefore P$ is a symmetric matrix.

- $A - A'$ is a skew-symmetric matrix

Prove:

Let $Q = A - A^T$. Then,

$$Q^T = (A - A^T)^T = A^T - (A^T)^T \quad \left[\because (A+B)^T = A^T + B^T \right]$$

$$\Rightarrow Q^T = A^T - A \quad \left[\because (A^T)^T = A \right]$$

$$\Rightarrow Q^T = -(A - A^T) = -Q$$

$\Rightarrow Q$ is skew-symmetric

- AA^T and $A^T A$ are symmetric matrices

Prove:

$$(AA^T)^T = (A^T)^T A^T \quad \text{[By reversal law]}$$

$$\Rightarrow (AA^T)^T = AA^T \quad \left[\because (A^T)^T = A \right]$$

$\Rightarrow AA^T$ is symmetric

Similarly, it can be proved that $A^T A$ is symmetric

3. If A and B are symmetric matrices then show that AB is symmetric if $AB = BA$

Prove:

AB is symmetric

$$\Leftrightarrow (AB)^T = AB \quad \left[\because (AB)^T = B^T A^T \right]$$

$$\Leftrightarrow B^T A^T = AB \quad \left[\because A \text{ and } B \text{ are symmetric matrices } \therefore A^T = A, B^T = B \right]$$

$$\Leftrightarrow BA = AB$$

4. Let A and B be symmetric matrices of the same order then show that

- $A + B$ is a symmetric matrix. *Changing your Tomorrow* 

Prove:

We have,

$$(A + B)^T = A^T + B^T = A + B \quad \left[\because A^T = A, B^T = B \right]$$

$A + B$ is symmetric

- $AB - BA$ is a skew-symmetric matrix

Prove:

$$(AB - BA)^T = (AB)^T - (BA)^T$$

$$(AB - BA)^T = B^T A^T - A^T B^T \quad \text{[By reversal law]}$$

$$(AB - BA)^T = BA - AB \quad [\because B^T = B, A^T = A]$$

$$(AB - BA)^T = -(AB - BA)$$

$AB - BA$ is a symmetric matrix

➤ $AB + BA$ is a symmetric matrix

Prove:

$$(AB + BA)^T = (AB)^T + (BA)^T$$

$$= B^T A^T + A^T B^T$$

[By reversal law]

$$= BA + AB$$

$[\because A^T = A, B^T = B]$

$$= AB + BA$$

$AB + BA$ is symmetric matrix

Theorem – 1

Every square matrix can be uniquely expressed as the sum of a symmetric and skew-symmetric matrix.

Proof

Let A be a square matrix then $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = P + Q$

Now $P^T = \frac{1}{2}(A + A^T)^T = \frac{1}{2}(A^T + A) = P$

$\Rightarrow P^T = P$, so P is a symmetric matrix.

$$Q^T = \left(\frac{1}{2}(A - A^T)\right)^T = \frac{1}{2}(A^T - (A^T)^T)$$

$$= \frac{1}{2}(A^T - A)$$

$$= -\frac{1}{2}(A - A^T) = -Q$$

$$\Rightarrow Q^T = -Q$$

So Q is a skew-symmetric matrix.

Question:-

Express the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -13 & 4 & 1 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

Answer:

$$\text{Here, } B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

$$\text{Let } P = \frac{1}{2}(B + B') = \frac{1}{2} \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix}$$

$$\text{Now } P' = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} = P$$

Thus $P = \frac{1}{2}(B + B')$ is a symmetric matrix

$$\text{Also, let } Q = \frac{1}{2}(B - B') = \frac{1}{2} \begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

$$\text{Then, } Q' = \begin{bmatrix} 0 & \frac{1}{2} & \frac{5}{2} \\ \frac{-1}{2} & 0 & -3 \\ \frac{-5}{2} & 3 & 0 \end{bmatrix} = -Q$$

Thus $Q = \frac{1}{2}(B - B')$ is a skew-symmetric matrix.

$$\text{Now } P + Q = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = B$$

Thus, B is represented as the sum of a symmetric and a skew-symmetric matrix.

Concept of elementary row and column operations.**Elementary Transformations or elementary operations of a matrix:-**

The following three operations applied on the rows (columns) of a matrix are called elementary row (column) transformations.

- Interchange of any two rows (columns) denoted by $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$
- Multiplying all elements of a row (column) of a matrix by a non-zero scalar denoted by $R_i \rightarrow kR_i$ or $C_i = kC_i$
- Adding to the elements of a row (column), the corresponding elements of any other row (column) multiplied by any scalar K, denoted by $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$

Method of finding the inverse of a matrix by Elementary transformation:-

Let A be a non-singular matrix of order n. Then A can be reduced to the identity matrix I_n by a finite sequence of elementary transformation only. As we have discussed every elementary row transformation of a matrix is equivalent to pre-multiplication by the corresponding elementary matrix. Therefore there exist elementary matrices E_1, E_2, \dots, E_k such that $(E_k E_{k-1} \dots E_2 E_1)A = I_n$

$$\Rightarrow (E_k E_{k-1} \dots E_2 E_1)AA^{-1} = I_n A^{-1} \text{ (post multiplying by } A^{-1}\text{)}$$

$$\Rightarrow (E_k E_{k-1} \dots E_2 E_1)I_n = A^{-1} \quad (\because I_n A^{-1} = A^{-1} \text{ and } AA^{-1} = I_n)$$

$$\Rightarrow A^{-1} = (E_k E_{k-1} \dots E_2 E_1)I_n$$

Algorithm for finding the inverse of a non-singular matrix by elementary row transformations:-

Let A be a non-singular matrix of order n

Step – I:- Write $A = I_n A$

Step – II:- Perform a sequence of elementary row operations successively on the LHS and the pre-factor I_n on the RHS till we obtain the result $I_n = BA$

Step – III:- Write $A^{-1} = B$.

The following steps will be helpful to find the inverse of a square matrix of order 3 by using elementary row transformations.

Step – I:- Introduce unity at the intersection of the first row and first column either by interchanging two rows or by adding a constant multiple of elements of some other row to the first row.

Step – II:- After introducing unity at (1, 1) place introduce zeros at all other places in the first column.

Step – III:- Introduce unity at the intersection 2nd row and 2nd column with the help of the 2nd and 3rd row.

Step – IV:- Introduce zeros at all other places in the second column except at the intersection of the 2nd and 2nd columns.

Step – V:- Introduce unity at the intersection of the 3rd row and third column.

Step – VI:- Finally introduce zeros at all other places in the third column except at the intersection of the third row and third column.

Example:- 1

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ using elementary row transformation.

Solution:-

$$A = IA$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad \text{by } R_2 \rightarrow R_2 - 2R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \quad \text{by } R_1 \rightarrow R_1 - 3R_2$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

Problem

Using elementary transformation find the inverse of the following matrices.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Answer:

To use elementary row operations we may write $A = IA$

$$\text{Or } \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A, \text{ then } \begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \quad (\text{applying } R_2 \rightarrow R_2 - 2R_1)$$

$$\text{Or } \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} A \quad \text{(applying } R_2 \rightarrow -\frac{1}{5}R_2)$$

$$\text{Or } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} A \quad \text{(applying } R_2 \rightarrow R_1 - 2R_2)$$

$$\text{Thus } A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix}$$

Homework:-

Using elementary transformation find the inverse of the following matrices.

(a) $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

Invertible matrices and proof of the uniqueness of inverse if it exists

The uniqueness of Inverse:-

The inverse of a square matrix, if it exists, is unique

Let $A = (a_{ij})_{n \times n}$ be any square matrix

If possible, A has two inverses B and C

$$\Rightarrow AB = I = BA \dots\dots\dots(1)$$

$$\& AC = I = CA \dots\dots\dots(2)$$

$$\text{Now, } B = BI = B(AC) = (BA)C = IC = C$$

Example – 1

Find the inverse of the matrix if exists

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$$

Solution:-

Let $A = IA$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad R_1 \Leftrightarrow R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \quad R_3 \rightarrow R_3 - 3R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A \quad R_1 \rightarrow R_1 - 2R_2$$

$$R_3 \rightarrow R_3 + 5R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} A \quad \text{by } R_1 \rightarrow R_1 + R_3$$

$$R_2 \rightarrow R_1 - 2R_3$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$$

Problem - 1

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary row transformation.

Answer:

$$A = IA$$

$$\text{Or, } \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad (\text{Applying } R_2 \rightarrow R_2 + R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} A \quad (\text{Applying } R_2 \rightarrow R_2 + 2R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -4 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A \quad (\text{Applying } R_1 \rightarrow R_1 + (-2)R_2, R_3 \rightarrow R_3 + 2R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A \quad (\text{Applying } R_1 \rightarrow R_1 + 2R_3)$$

Hence, $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Problem – 2

$A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$ find A^{-1} using elementary row transformation.

Answer:

$A = IA$

Or, $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & -1 & -1 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ (Applying $R_1 \rightarrow R_1 - R_2$)

$\Rightarrow \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ -3 & 3 & 1 \end{bmatrix} A$ (Applying $R_2 \rightarrow R_2 + (-2)R_1$ & $R_3 \rightarrow R_3 + (-3)R_1$)

$\Rightarrow \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 1/2 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3/2 & 0 \\ -3 & 3 & 1 \end{bmatrix} A$ (Applying $R_2 \rightarrow R_2 (1/2)$)

$\Rightarrow \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 0 \\ -1 & 3/2 & 0 \\ -5 & 6 & 1 \end{bmatrix} A$ (Applying $R_1 \rightarrow R_1 + R_2$ and $R_3 \rightarrow R_3 + 2R_2$)

$\Rightarrow \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 0 \\ -1 & 3/2 & 0 \\ -5/4 & 3/2 & 1/4 \end{bmatrix} A$ (Applying $R_3 \rightarrow 1/4 R_3$)

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5/8 & 5/4 & 1/8 \\ -3/8 & 3/4 & -1/8 \\ -5/4 & 3/2 & 1/4 \end{bmatrix} A$ (Applying $R_2 \rightarrow R_1 + (1/2)R_3$ & $R_2 \rightarrow R_2 + (1/2)R_3$)

Hence, $A^{-1} = \begin{bmatrix} -5/8 & 5/4 & 1/8 \\ -3/8 & 3/4 & -1/8 \\ -5/4 & 3/2 & 1/4 \end{bmatrix}$

Home Work: -

Find A^{-1} , of the following matrices using elementary row transformation.

$$(a) A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

$$\text{Answer: } -\frac{1}{30} \begin{bmatrix} -2 & 4 & -10 \\ 44 & -7 & -5 \\ -5 & -5 & 5 \end{bmatrix}$$

$$(b) A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

$$\text{Answer: } \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix}$$

$$(c) A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

$$\text{Answer: } \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$$

Word problems on matrices

Example – 1

A factory produces three stickers P, Q and R printed with the messages ‘SAVE TREES’. ‘SAVE ELECTRICITY’ and ‘SAVE FUEL’ respectively. These stickers are sold in two markets, M_1 and M_2 . Annual sale (in units) is indicated below.

	P	Q	R
M_1	2000	4000	10000
M_2	5000	7000	3000

Represent the above information in the form of a 2×3 matrix. What do the entry in the second row and first column represent?

Solution:- We can represent the given information in the form of a 2×3 matrix given by

$$A = \begin{matrix} & P & Q & R \\ M_1 & \begin{bmatrix} 2000 & 4000 & 10000 \\ 5000 & 7000 & 3000 \end{bmatrix} \\ M_2 & \end{matrix}$$

Entry in the second row and first column represents the annual sale of sticker P in market M_2 .

Example – 2

Two Dealers P and Q sell three items, viz., notebooks, pencils, and erasers. In April, Dealer P sells 250 notebooks, 400 pencils, and 300 erasers, whereas dealer Q sells 350 notebooks, 500 pencils, and 470 erasers. In f May. Dealer P sells 380 notebooks, 440 pencils, and 360 erasers, whereas the dealer Q sells 450 notebooks, 520 pencils, and 420 erasers. Using matrices, represent

- The combined sale of two months for each Dealer in each item?
- The change in sales from April to May for each Dealer in each item?

Solution:-

We can represent the given sale information for April as the following matrix

$$A = \begin{matrix} & \textit{Notebook} & \textit{Pencil} & \textit{Eraser} \\ \begin{matrix} P \\ Q \end{matrix} & \begin{bmatrix} 250 & 400 & 300 \\ 350 & 500 & 470 \end{bmatrix} \end{matrix}$$

We can represent the given sale information for May as the following matrix.

$$B = \begin{matrix} & \textit{Notebook} & \textit{Pencil} & \textit{Eraser} \\ \begin{matrix} P \\ Q \end{matrix} & \begin{bmatrix} 380 & 440 & 360 \\ 450 & 520 & 420 \end{bmatrix} \end{matrix}$$

(i) The matrix of combined sales of two months for each dealer in each item can be obtained by adding the two matrices, which are given by.

$$\begin{aligned} A + B &= \begin{bmatrix} 250 & 400 & 300 \\ 350 & 500 & 470 \end{bmatrix} + \begin{bmatrix} 380 & 440 & 360 \\ 450 & 520 & 420 \end{bmatrix} \\ &= \begin{bmatrix} 250+380 & 400+440 & 300+360 \\ 350+450 & 500+520 & 470+420 \end{bmatrix} = \begin{bmatrix} 630 & 840 & 660 \\ 800 & 1020 & 890 \end{bmatrix} \end{aligned}$$

Hence, the combined sale of two months for each dealer in each item has the following form

$$A = \begin{matrix} & \textit{Notebook} & \textit{Pencil} & \textit{Eraser} \\ \begin{matrix} P \\ Q \end{matrix} & \begin{bmatrix} 630 & 840 & 660 \\ 800 & 1020 & 890 \end{bmatrix} \end{matrix}$$

(ii) The matrix of change in sales from April to May for each dealer in each item can be obtained by taking the difference between the two matrices, which is given by

$$\begin{aligned} B - A &= \begin{bmatrix} 380 & 440 & 360 \\ 450 & 520 & 420 \end{bmatrix} - \begin{bmatrix} 250 & 400 & 300 \\ 350 & 500 & 470 \end{bmatrix} \\ &= \begin{bmatrix} 380-250 & 440-400 & 360-300 \\ 450-350 & 520-500 & 420-470 \end{bmatrix} = \begin{bmatrix} 130 & 40 & 60 \\ 100 & 20 & -50 \end{bmatrix} \end{aligned}$$

Hence, the change in sales from April to May for each dealer in each item has the following

form:

$$\begin{matrix} & \textit{Notebook} & \textit{Pencil} & \textit{Eraser} \\ \begin{matrix} P \\ Q \end{matrix} & \begin{bmatrix} 130 & 40 & 60 \\ 100 & 20 & -50 \end{bmatrix} \end{matrix}$$

Example – 3

A builder is constructing a three-story building. Each story is to be installed with 20 fans, 30 tube lights, and 25 LED bulbs. Represent the above information as a column matrix. Using scalar multiplication, find the total number of electrical items of each type in the building.

Solution:- We can represent the given information of electrical items in the following matrix.

$$A = \begin{matrix} \text{Fan} \\ \text{Tube light} \\ \text{LED light} \end{matrix} \begin{bmatrix} 20 \\ 30 \\ 25 \end{bmatrix}$$

The matrix of total electrical items of each type in the building can be obtained by multiplying the above matrix with the number of stories which is given by

$$3A = 3 \begin{bmatrix} 20 \\ 30 \\ 25 \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 75 \end{bmatrix}$$

Hence, the total number of fans, tube lights, and LED lights in the building are 60, 90, and 75 respectively.

Example – 4

A book store has 20 mathematics books, 15 physics books, and 12 chemistry books. Their selling prices are Rs. 300, Rs. 320, and Rs. 340 each respectively. Find the total amount the store will receive from selling all the items.

Solution:

We can represent the given price information as the following price matrix.

$$\begin{matrix} \text{Mathematics} \\ \text{Physics} \\ \text{Chemistry} \end{matrix} \begin{bmatrix} 300 \\ 320 \\ 340 \end{bmatrix}$$

We can represent the given sale information as the following Sale Matrix:

$$\begin{matrix} & \text{Mathematics} & \text{Physics} & \text{Chemistry} \\ \text{Book store} & \begin{bmatrix} 20 & 15 & 12 \end{bmatrix} \end{matrix}$$

Then, the amount received by the book store from selling all the items can be obtained by multiplying the sale matrix and price matrix, which is given by

$$\begin{bmatrix} 20 & 15 & 12 \end{bmatrix} \begin{bmatrix} 300 \\ 320 \\ 340 \end{bmatrix} = 6000 + 4800 + 4080 = 14880$$

Hence, the total amount is Rs. 14880

Example-5

Three schools A, B, and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand-made fans, mats, and plates from recycled material at a cost of Rs.25, Rs. 100 and Rs. 50 each. The number of articles sold is given below.

School/ Article	A	B	C
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also, find the total funds collected for the purpose.

Solution:

We can represent the given price information as the following price matrix.

Price

$$\begin{matrix} \text{Hand - fans} \\ \text{Mats} \\ \text{Plates} \end{matrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

We can represent the given sale information as the following sale matrix.

$$\begin{matrix} \text{School A} \\ \text{School B} \\ \text{School C} \end{matrix} \begin{bmatrix} \text{Hand - fans} & \text{Mats} & \text{Plates} \\ 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}$$

Then, the matrix of funds collected by the schools can be obtained by multiplying the sale matrix and price matrix, which is given by

$$\begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = \begin{bmatrix} 1000 + 5000 + 1000 \\ 625 + 4000 + 1500 \\ 875 + 5000 + 2000 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

Hence, the amount raised by school A school B, and school C are Rs. 7000, Rs. 6125 and 7875 respectively.

Also, the total funds collected = Rs.(7000 + 6125 + 7875) = Rs. 21000.