ODM EDUCATIONAL GROUP
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TANGENTS AND NORMALS

SUBJECT : MATHEMATICS
CHAPTER NUMBER:6
CHAPTER NAME :APPLICATION OF DERIVATIVES

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INTRODUCTION

In this topic we shall use the derivative of the function $y = f(x)$ to find the equation of tangent and normal to the curve at a given point.

Slope or Gradient of a line:

If a line makes an angle θ with the positive direction of x-axis in anticlockwise direction,

Then $tan\theta$ is called slope of the line.

Slope of the line perpendicular to x-axis is not defined.

Slope of the line parallel to x-axis is zero.

Equation of Tangent and Normal at a point to a curve:

Let $y = f(x)$ be a curve and $p(\alpha, \beta)$ be a point on it.

Then we know that slope of the tangent to the curve $y = f(x)$

at the point $p(\alpha, \beta)$ is given by $\left(\frac{dy}{dx}\right)_{at(\alpha,\beta)}$ $=f'(\alpha)$

Equation of tangent at a point:

As we know that the equation of a line passing through the point (α, β) having slope m is

$$y - \beta = m(x - \alpha)$$

Therefore the equation of the tangent to the curve $y = f(x)$ at $p(\alpha, \beta)$ is

$$y - \beta = \left(\frac{dy}{dx}\right)_{at(\alpha,\beta)} (x - \alpha)$$

$$y - \beta = f'(\alpha)(x - \alpha)$$

Equation of normal at a point:

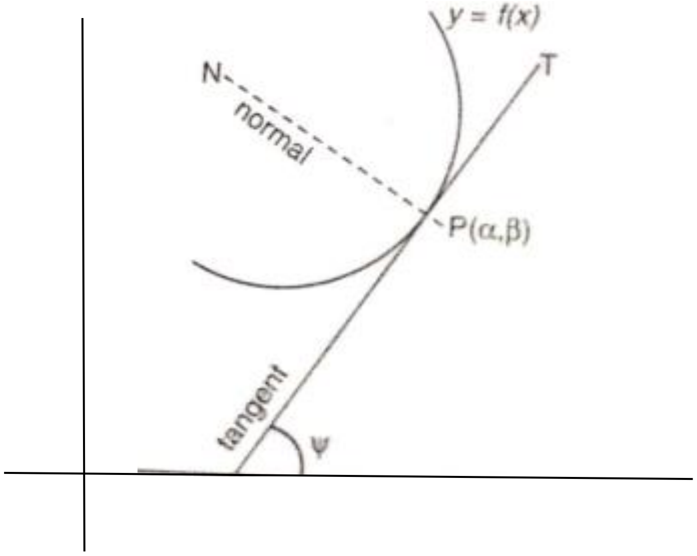
We know that normal to the curve at the point P in a line perpendicular to the tangent to the curve at P.

Hence, the slope of the normal to the curve $y = f(x)$ at P = $\frac{-1}{\left(\frac{dy}{dx}\right)_{at(\alpha,\beta)}} = \frac{1}{f\prime(x)}$

Equation of the normal to the curve $y = f(x)$ at $P(\alpha, \beta)$ is

$$(y - \beta) = \frac{-1}{\left(\frac{dy}{dx}\right)_{at(\alpha,\beta)}}(x - \alpha)$$

then, $$(y - \beta) = \frac{-1}{f\prime(\alpha)}(x - \alpha)$$



Working Rule:

Step I. Find $\frac{dy}{dx}$ from the equation of the given curve

Step II. If the equation of the tangent and normal at (α, β) is needed, find $\frac{dy}{dx}$ at(α, β) . This value of $\frac{dy}{dx}$ will be the slope of tangent at (α, β) .

slope of normal at (α, β) = $\frac{-1}{Slope\ of\ the\ tangent\ at(\alpha,\beta)}$

Step III. If the value of $\frac{dy}{dx}$ at (α, β) be m, the equation of normal and tangent at (α, β) will be

(a) $y - \beta = m(x - \alpha)$ (b) $y - \beta = \frac{-1}{m}(x - \alpha)$

Step IV. If $\frac{dy}{dx}$ at a point (α, β) is zero then the tangent is parallel to x-axis

If $\frac{dy}{dx}$ is at (α, β) is undefined then the tangent at (α, β) is parallel to y-axis and normal at (α, β) is parallel to x-axis. In this case equation of tangent at P will be $x = \alpha$ and that of normal will be $y = \beta$. .

Problem:1

Find the slope of the tangent to the curve $y^2 = x$ at the point $x = 1$.

Problem: 2

Find the equation of the tangent and normal to the curve $y = x^2 + 4x + 1$ at the point whose x-coordinate is 3.

Problem:3

Find the points on the curve $y = x^3 - x^2 - x + 3$, where the tangent is parallel to x-axis.

Problem:4

Find the point on the curve $y = (x-3)^2$, where tangent is parallel to the line joining $(4,1)\ and\ (3,0)$.

HOME ASSIGNMENT

Q1. Find the slope of tangent and normal to the curve $x^2 + 3y + y^2 = 5$ at (1,1)

Q2. The slope of the curve $2y^2 = ax^2 + b$ at (1, -1) is -1. Find a, b.

Q3. Find the sope of the normal to the curve $x = 1 - aSin\theta, \quad y = bCos^2\theta$ at $\theta = \frac{\pi}{2}$

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