Approximations

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 6
CHAPTER NAME : Applications of Derivative

Use of Derivative in Approximation

Differentials:-

Let $y = f(x)$ be a differential function of a function of x. and let Δx be small change in x.

and let the corresponding change in y be Δy, We define

(a) The differential of x, denote by dx is given by $dx \cong \Delta x$.

(b) The differential of y, denoted by dy, is given by $dy = f'(x)dx$

In case $dx = \Delta x$ is very small in comparison to x $then$ $dy \cong \Delta y$

$$\Delta y = \frac{dy}{dx}\Delta x.$$

Absolute Error:

The error Δx in x is called absolute error in x.

Relative Error:

If Δx is an error in x, then, $\frac{\Delta x}{x}$ is called the relative error in x.

Percentage Error:

If Δx is an error in x, then, $\frac{\Delta x}{x} \times 100$ is called the Relative error in x

Let $y = f(x)$ be a function of x and let Δx be a small change in x. Let corresponding change in y be, $\Delta y \; then$

$$y + \Delta y = f(x + \Delta x)$$

But, $\Delta y = \dfrac{dy}{dx} \Delta x = f'(x)\Delta x \quad$ (approximately)

$$f(x + \Delta x) = y + f'(x)\Delta x$$

$$f(x + \Delta x) = f(x) + f'(x) \times \Delta x \text{ (approximately)}$$

Example:- 1

Use differential to approximate $(25)^{\frac{1}{3}}$

Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$

Example:- 3

If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculate the volume.

Example:- 4

Find the approximate change in volume V of a cube of side x meters caused by increasing the side by 1%.

Q1. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

Q2. Find the approximate value of $(0.009)^{\frac{1}{3}}$

Q3. If the radius of the sphere is measured as 7m with an error of 0.02 m, then find the approximate error in calculating its volume.

THANKING YOU

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