Maxima and Minima

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 6
CHAPTER NAME : Application of Derivatives

First Derivative Test of Maxima and Minima

Local Maximum and minimum values of $y = f(x)$

Definition of local Maximum value:

A function $f(x)$ said to have a local maximum value at $x = a$, if $f(a)$ is greater than any other value that $f(x)$ can have in the some suitably small neighbourhood of $x = a$.

OR

A function is said to have local maximum value $f(a)$ at $x = a$ if $f(x)$ stop to increase at $x = a$ and begins to decrease as x increases beyond a.

Definition of Local Minimum value:

A function $f(x)$ said to have a local minimum value at $x = a$, if $f(a)$ is less than any other value that $f(x)$ can have in the some suitably small neighbourhood of $x = a$.

OR

A function is said to have local minimum value $f(a)$ at $x = a$ if $f(x)$ stop to decrease at $x = a$ and begins to increase as x increases beyond a.

First Derivative Test For Local Maxima Of a Function

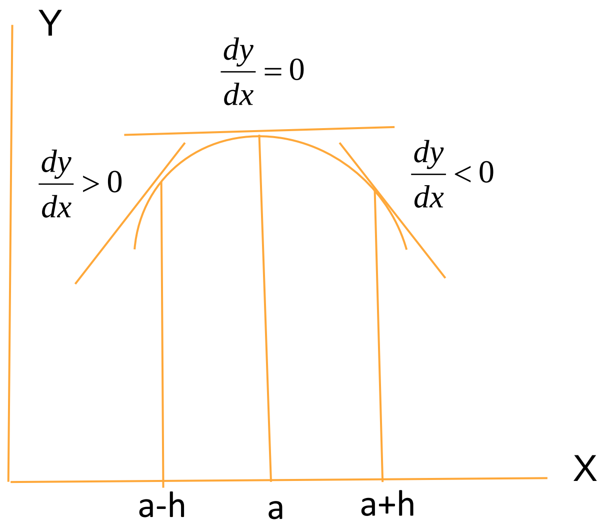
Function $f(x)$ has a local maximum value at $x = a$, if $f(x)$ stops to increase at $x = a$ and begins to decrease as x increases beyond 'a'.

Thus when x is slightly less than 'a', y increases and $\frac{dy}{dx}$ is $+ve$

When x is slightly greater than 'a' , y decreases and $\frac{dy}{dx}$ is $-ve$

Therefore $\frac{dy}{dx}$ changes its sign from $+ve$ to $-ve$ as x passes through the Value 'a'. But it can not change sign without passing through the value zero which must evidently be attained at $x = a$.

Hence, we have the following two conditions for $y = f(x)$ to have a A local Maximum value $f(a)$ at $x = a$.



(i) $\frac{dy}{dx} = 0$ at $x = a$.

(ii) $\frac{dy}{dx}$ changes its sign from $+ve$ to $-ve$ as x passes through the value a .

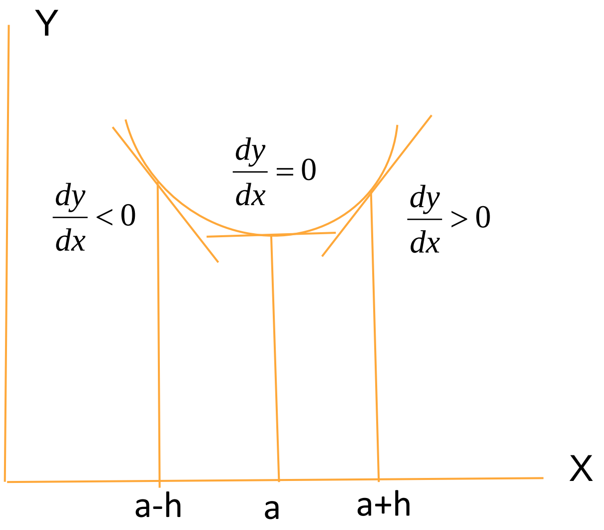
First Derivative Test For Local Minima Of a Function

Function $f(x)$ has a local minimum value at $x = a$, if $f(x)$ stops to decrease at $x = a$ and begins to increase as x increases beyond ‘a’.

Thus when x is slightly less than ‘a’, y decreases and $\frac{dy}{dx}$ is $-ve$

When x is slightly greater than ‘a’ , y increases and $\frac{dy}{dx}$ is $+ve$

Therefore $\frac{dy}{dx}$ changes its sign from $-ve$ to $+ve$ as x passes through the Value ‘a’. But it can not change sign without passing through the value zero which must evidently be attained at $x = a$.



Hence, we have the following two conditions for $y = f(x)$ to have a A local Minimum value $f(a)$ at $x = a$.

(i) $\frac{dy}{dx} = 0$ at $x = a$.

(ii) $\frac{dy}{dx}$ changes its sign from $-ve$ to $+ve$ as x passes through the value a .

Working Rules For First Derivative Test

Step-1

Find $\frac{dy}{dx}$ of the given function.

Step-2

let $\frac{dy}{dx} = 0$ for critical points (say $x = a, b, c......$) , (remember critical point is the point where derivative of the function is equal to zero.)

Step-3

(i) If $\frac{dy}{dx}$ changes its sign from $+ve$ to $-ve$ as x passes through the values '$a, b, c......$' then function attain the maximum value.

(ii) If $\frac{dy}{dx}$ changes its sign from $-ve$ to $+ve$ as x passes through the values '$a, b, c......$' then function attain the minimum value.

Problem: 1

Find all points of local maxima and local minima of the function given by $f(x) = x^3 - 3x + 3$

Problem: 2

Find all points of local maxima and local minima as well as corresponding local maximum and local minimum values for the function $f(x) = (x-1)^3(x+1)^2$

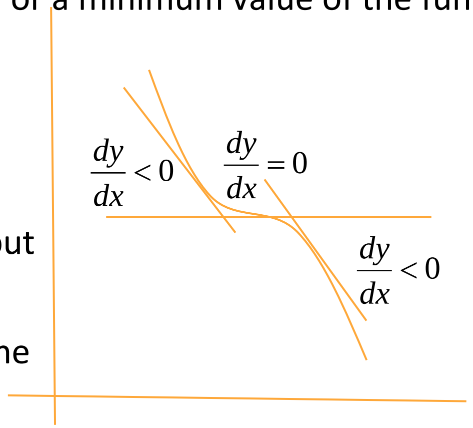
Points of Inflection

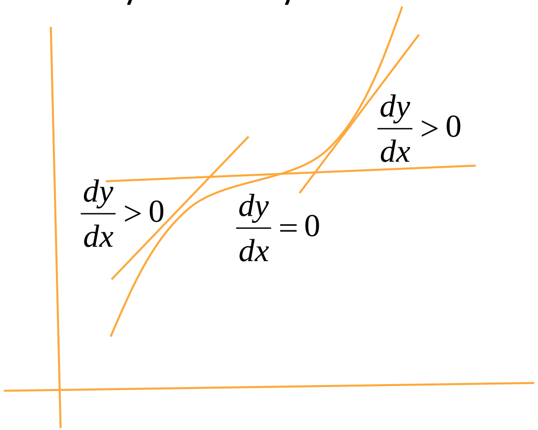
For the function y = f(x) to have maximum or minimum value at $x = a$, $\frac{dy}{dx} = 0$, at x = a. But if $\frac{dy}{dx} = 0$ at $x = a$, it is not essential that the function may have maximum or minimum values at $x = a$. For it may happens that $\frac{dy}{dx}$ does not change sign from $-ve\ to\ +ve\ as\ +ve\ to\ -ve$ as x passess through value a and cosequently, function may go on increasing or decreasing.

Thus the condition that $\frac{dy}{dx} = 0$ for a maximum or a minimum value of the function is only necessary and not sufficient. $y = f(x)$

DEFINITION

A point on the curve y = f(x) at which $\frac{dy}{dx} = 0$ but $\frac{dy}{dx}$ does not change sign as x passes through the point is called a point of inflexion.





Problem: 3

Find the point of inflexion of the function $f(x) = x^5$

HOME ASSIGNMENT

Q1. Find the local maximum and local minimum values of the function f given by

$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

Q2. Find the local maximum and local minimum values of the function $f(x) = x\sqrt{1-x}$,

$0 < x < 1$.

Q3. Prove that the function $f(x) = x^3 + x^2 + x + 1$ do not have maxima and minima.

THANKING YOU

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