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APPLICATION OF INTEGRALS

SUBJECT : MATHEMATICS
CHAPTER NUMBER:8
CHAPTER NAME :Application of Integrals

Introduction

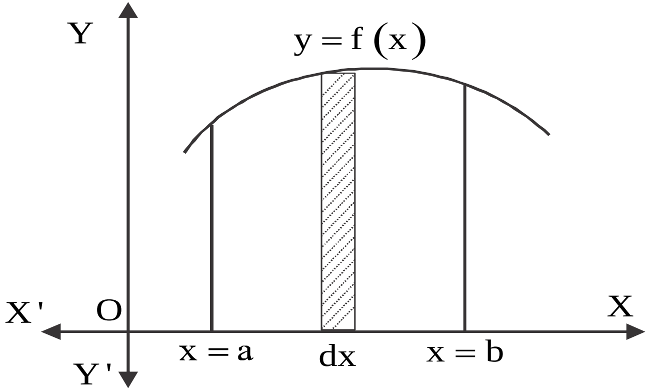
Integral has a large number of applications in science and engineering. Here we will study a specific application of integral to find the area under simple curves area between lines and area of circles, parabolas and ellipse. In geometry, we have learnt formulate to calculate areas of various geometrical figures such as triangles, rectangles, circles etc. But these formulae are in adequate to find the area enclosed by many curves. The concept of definite integral can be conveniently use to find the area enclosed by curves.

The Area Under Simple Curves

Area of the region bounded by x-axis

Let $f(x)$ be a continuous function defined on$[a, b]$. Then the area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ are given by.

Area = $\int_a^b y dx = \int_a^b f(x) dx$



Example

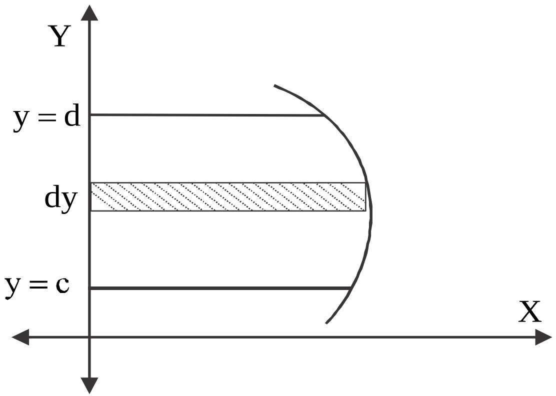
Find the area of the region bounded by the curve $y^2 = 4ax$ and the lines $x = 4, x = 9$ and the x-axis.

The Area Under Simple Curves

Area of Region bounded by Y −axis

The area bounded by the curve $x = f(y)$, the y-axis and the lines $y = c$ and $y = d$ are given by.

Area = $\int_c^d dA = \int_c^d x dy = \int_c^d f(y) dy$



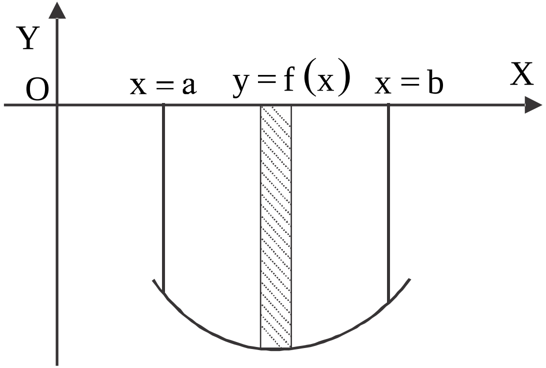
Example

Find the area of the region bounded by the curve $x^2 = y$ and the line $y = 4$.

Area of the Region when the Curve is Below the x-axis

If the curve $y = f(x)$ lies below the x-axis then the area bounded by the curve $y = f(x)$, x-axis, and the lines $x = a$ and $x = b$ come out to be negative. But the only numerical value of the area is taken into consideration. Thus if the area is negative then we take its absolute value.

i.e $\left|\int_a^b f(x)\,dx\right|$



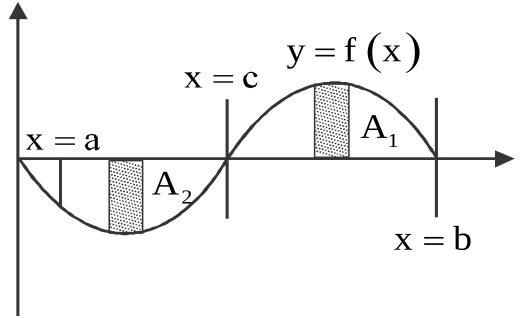
Area = $\left|\int_a^b y dx\right| = \left|\int_a^b f(x) dx\right|$

Area of the region when the curve is above and below the x-axis

Generally, some portions of the curve may be above the x-axis and some are below the x-axis which is shown in the figure given below.

Here, $A_1 > 0$ and $A_2 < 0$.

Therefore the area A bounded by the curve $y = f(x)$, x-axis and the lines $x = a$ and $x = b$ is given by $A = A_1 + |A_2| = \int_c^b f(x)dx + \left|\int_a^c f(x)dx\right|$



Example

Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates

$x = -1$ and $x = 1$.

Assignment

1. Find the area of the region bounded by the parabola $y^2 = 4ax$, its axis, and two ordinates $x = 4$ and $x = 9$ in the first quadrant.

2. Using integration find the area of the region bounded by the line $2x + y = 8$, the y-axis, and the lines $y = 2$ and $y = 4$

3. Find the area of the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ in the fourth quadrant.

4. Find the area bounded by the line $y = x$, the x-axis, and the lines $x = -1$ and $x = 2$.

THANKING YOU

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