

Area Under Simple Curves

SUBJECT : MATHEMATICS
CHAPTER NUMBER:8
CHAPTER NAME :Application of Integrals

CHANGING YOUR TOMORROW

Area of Symmetric Region

Sometimes the bounded region for which we have to calculate area is symmetrical about x -axis or y -axis or both x -axis and y -axis or origin. While determining the area of the symmetrical region, we have to check the symmetry of a curve is as given below.

If the equation of the curve contains only

- (i) Even powers of x , then it is symmetrical about the y -axis
- (ii) Even powers of y , then it is symmetrical about the x -axis
- (iii) The equation of the curve remains unchanged when x and y are replaced by $-x$ and $-y$ respectively then the curve is symmetrical in opposite quadrants

Area of the Region Bounded by a Curve and a Line

To find the area of the region bounded by a line and a circle, a line, and a parabola a line and an ellipse we use the following steps.

Step -I We draw the rough sketch of given curves and identify the region for which we have to find the area.

Step – II Find the point of intersection of the line and curve.

Step – III

(i) If the region is symmetrical then draw a vertical (or horizontal) strip and take suitable limits.

(ii) If the region is not symmetrical then draw a vertical strip (or horizontal strip) in the required area and take suitable limits.

(iii) To find the number of the vertical (horizontal) strip we generally draw perpendicular lines from the intersection points of the line and curve to the x –axis or y -axis.

For two vertical strips Area = $\int_{x=a}^c y_1 dx + \int_{x=c}^b y_2 dx$, where y_1 & y_2 represents the height of vertical stripes.

For two horizontal strips Area = $\int_{y=a}^c x_1 dy + \int_{y=c}^b x_2 dy$, where x_1 and x_2 represent the lengths of the horizontal strips.

Example

Find the area enclosed by the the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Example

Draw a rough sketch of the curve $y = |x - 2|$. Find the area under the curve and the line $x = 0$ and $x = 4$.

Example

Find the area of the region in the first quadrant enclosed by x -axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

Example

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

Example

The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a .

Assignment

01. Sketch the region $\{(x, 0): y = \sqrt{4 - x^2}\}$ and x -axis. Find the area of the region using integration
02. Using integration find the area endorsed by the circles $x^2 + y^2 = a^2$
03. Find the area of the circle $x^2 + y^2 - 2x = 0$
04. Find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$
05. Find the area of the parabola $y^2 = 4ax$ bounded by its latus rectum.

THANKING YOU
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