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Matrices :(Concept, Order, Equality of Matrices)

SUBJECT : (Mathematics)
CHAPTER NUMBER: 03
CHAPTER NAME : Matrices

What we expect to learn?

- Students will learn about finding the order of a given matrix.
- Students will learn about different types of matrices.
- Students will learn about the equality of matrices.
- Students will learn about the multiplication of a matrix by a scalar.
- Students will learn about the transpose of a matrix.
- Students will learn the Addition of matrix and multiplication of matrices, properties, and applications.
- Students will learn about the Symmetric, skew-symmetric matrix.
- Students will Learn about the elementary operation of a matrix.
- Students will learn about the use of elementary operations in finding out the inverse of a matrix.
- Learn about invertible matrix.

Matrices

Concept, notation, order, equality, types of matrices

A matrix is a rectangular arrangement of numbers or functions arranged into a fixed number of rows and columns. The element of a matrix is always enclosed in the bracket [ ] or (). Matrices are represented by capital letters like A, B, C, etc.

A matrix having m rows and n columns is called a matrix order m x n (read as m by n matrix). In general, a matrix of order m x n is written as.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

It can also be written in compact form as a_{ij} represent an element of i th row and j th column.

Question – 1

Write all possible order of matrices having 24 elements

Solution:-

1 x 2, 2 x 12, 3 x 8, 4 x 6, 6 x 4, 8 x 3, 12 x 2, 24 x1

Question – 2

Write the order 2 x 3 where $a_{ij} = 2i - j$

Solution:-

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

Equality of Matrices:-

Two matrices are said to be equal if their order is the same and their corresponding elements are equal.

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ Then $a = 2, b = 3, c = 4, d = 5$

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Question:-

If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ then find x, y, z and w

Solution:-

$$x - y = -1$$
$$2x + z = 5$$
$$2x - y = 0$$
$$3z + w = 13$$

Solving $x = 1, y = 2, z = 3, w = 4$

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HOME ASSIGNMENT

Q1. For a 2 × 2 matrix, A = $[a_{ij}]$ whose elements are given by $a_{ij} = \frac{i}{j}$, write the value of a_{12}

Q2. Write the value of y if $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$.

Q3. write the value of $x - y + z$ from the equation $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Types of Matrices, Addition and its Properties

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Types of Matrices

Row Matrix:-

A matrix is said to be a row matrix if it has only one row.

Example: A = $[1 \quad 2 \quad 3]$

Having order 1 x 3

Column Matrix:-

A matrix having any number of rows but only one column is called a column matrix.

Example: $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Having order 3 x 1

Rectangular Matrix:-

A matrix having m rows and n columns where $m \neq n$ is called a rectangular matrix.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 7 & 6 \end{bmatrix}_{2\times 3}$$

Square matrix:-

It a matrix in which number of rows is equal to number of columns

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

Diagonal Elements:-

The diagonal elements of a square matrix are the elements for which i = j . i.e. the elements $a_{11}, a_{22,} a_{33} \dots\dots,$

The line along which the diagonal elements lie is called the leading diagonal or principal diagonal.

Diagonal Matrix:-

It is a square matrix where diagonal elements are non-zero but the other elements are zero.

Example:- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Scalar matrix:-

It is a diagonal matrix where all the diagonal elements are equal

Example:-

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$$

Identity matrix:-

It is a diagonal matrix where all diagonal elements are 1 and it is denoted by the symbol I

Example:- $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2} = I_2 \quad and \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3} = I_3$$

Addition of Matrices:-

Matrix addition is defined only when they are of the same order. The sum of matrices A and B is a matrix whose elements are obtained by adding the corresponding elements of A and B.

Example:

Let $A = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ 0 & 4 \end{bmatrix}$

$$A + B = \begin{bmatrix} 3 & 1 \\ -4 & 9 \end{bmatrix}, A - B = \begin{bmatrix} 1 & 5 \\ -4 & 1 \end{bmatrix}$$

Simple properties of addition

Properties:-

Closure Law:- A matrix added with a matrix always gives a matrix. So Closure Law satisfies.

Commutative Law:- $A + B = B + A$

Associative Law:- $A + (B + C) = (A + B) + C$

Existence of Additive Identity:-

A null matrix of the same order with the given matrix is the additive identity of the matrix.

$$A + O = O + A = A$$

Example:-

The additive identity of $A_{2\times 3} = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ is $0_{2\times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Existence of additive inverse:-

- A is the additive inverse of A

Example:

If A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, then its additive inverse is $-A = \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \end{bmatrix}$

HOME ASSIGNMENT

Q1. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix}, B = \begin{bmatrix} 8 & 2 \\ 4 & 3 \end{bmatrix}$, then find the value of $A + B,\ B + A and\ A - B$

Q2. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 4 \\ 4 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} -2 & 3 & 8 \\ 7 & 3 & -5 \\ 2 & 6 & 3 \end{bmatrix}$ $and\ C = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 1 \\ -3 & 2 & 5 \end{bmatrix}$ then compute

$$A + (B - C) = (A + B) - C$$

Q3. Compute $\begin{bmatrix} Cos^2x & Sin^2x \\ Sin^2x & Cos^2x \end{bmatrix} + \begin{bmatrix} Sin^2x & Cos^2x \\ Cos^2x & Sin^2x \end{bmatrix}$

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Scalar multiplication and Multiplication of matrices

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Multiplication of a matrix by a scalar:-

If a scalar K is multiplied by a matrix A then all elements of matrix A are multiplied by constant K.

Question:

If $A = \begin{bmatrix} 3 & 4 \\ -5 & 1 \end{bmatrix}$ Then Find 5A

Solution:

$$5A = \begin{bmatrix} 15 & 20 \\ -25 & 5 \end{bmatrix}$$

Question:

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 3 & 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 0 & 1 \\ 0 & 0 & 4 \end{bmatrix}$ Then find 3A + 4B

Solution:

$$3A + 4B = \begin{bmatrix} 3 & 6 & 9 \\ 0 & -3 & 6 \\ 9 & 12 & 0 \end{bmatrix} + \begin{bmatrix} 8 & -4 & 0 \\ 2 & 0 & 4 \\ 0 & 0 & 16 \end{bmatrix} = \begin{bmatrix} 11 & 2 & 9 \\ 12 & -3 & 10 \\ 9 & 12 & 16 \end{bmatrix}$$

Multiplication of Matrices

If A and B be any two matrices, then their product AB will be defined only when the number of columns in A is equal to the number of rows in B. if $A = [a_{ij}]_{m\times n}$ *and* $B = [b_{ij}]_{n\times p}$ Then their product $AB = C = [c_{ij}]_{m\times n}$

Example:-

If $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Then $AB = \begin{bmatrix} 1.1+4.2+2.1 & 1.2+4.2+2.3 \\ 2.1+3.2+1.1 & 2.2+3.2+1.3 \end{bmatrix}$

$$AB = \begin{bmatrix} 11 & 16 \\ 9 & 13 \end{bmatrix}$$

Question – 1

Find x and y if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Question – 2

If $A=\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}, B=\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}$ Find the matrix x, such that $2A+3X=5B$

Question – 3

Find the value of x + y from the matrix equation $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Question - 4

Compute $\begin{bmatrix} 2 & 1 \\ 4 & 7 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 4 \end{bmatrix}$

Question – 5

If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $A^2 - 4A - nI = 0$ then find the value of n.

Question – 6

If $A = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$ then find the element a_{21} of A^2

Question – 6

Find the value of x : $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

Question – 7

If $F(x) = \begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$, Show that $F(x)F(y) = F(x+y)$.

HOME ASSIGNMENT

Q1. If $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$, then find the value of $x + y$.

Q2. Simplify $Cos\theta\begin{bmatrix}Cos\theta & Sin\theta\\-Sin\theta & Cos\theta\end{bmatrix}+Sin\theta\begin{bmatrix}Sin\theta & -Cos\theta\\Cos\theta & Sin\theta\end{bmatrix}$

Q3. If $\begin{bmatrix}9 & -1 & 4\\-2 & 1 & 3\end{bmatrix}=A+2\begin{bmatrix}1 & 2 & -1\\0 & 4 & 9\end{bmatrix}$, then find the value of matrix A.

Q4. Write the order of the product of matrix $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}2 & 3 & 4\end{bmatrix}$.

Simple Properties of Multiplication

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Properties of Matrix Multiplication:-

If A, B, and C are three matrices such that their product is defined, then

$\Rightarrow \quad AB \neq BA \ \ (Generally\ not\ commutative)$

$\Rightarrow \quad (AB)C = A(BC) \ \ (Associative\ Law)$

$\Rightarrow \quad IA = A = AI\ (I\ is\ the\ Identity\ Matrix\ of\ Matrix\ Multiplication)$

$\Rightarrow \quad A(B+C) = AB + AC \ \ (Distributive\ Law)$

$\Rightarrow \quad If\ \ AB = AC\ this\ not\ imply\ that\ B = C\ (cancellation\ Law\ is\ not\ applicable)$

$\Rightarrow \quad If\ \ AB = O\ \ It\ does\ not\ mean\ that\ A = O\ or\ B = O\ \ again\ product\ of\ two\ non-zero\ matrices$
$may\ be\ \ zero\ matrix$

Note:-

- The multiplication of two diagonal matrices is again a diagonal matrix.
- The multiplication of two triangular matrices is again a triangular matrix
- The multiplication of two scalar matrices is also a scalar matrix.
- If A and B are two matrices of the same order, then

$$(A+B)^2 = A^2 + B^2 + AB + BA$$

$$(A-B)^2 = A^2 + B^2 - AB - BA$$

$$(A-B)(A+B) = A^2 - B^2 + AB - BA$$

$$(A+B)(A-B) = A^2 - B^2 - AB + BA$$

$$A(-B) = (-A)B = -(AB)$$

Positive Integral Powers of the matrix:-

The positive integral powers of matrix A are defined only when A is a square matrix. Also then

$$A^2 = A.A, \qquad A^3 = A.A.A. = A^2 A$$

Also for any positive integers m, n

$a)\quad A^m A^n = A^{m+n}$

$b)\quad (A^m)^n = A^{mn} = (A^n)^m$

$c)\quad I^n = I \text{ and } I^m = I$

$d)\quad A^0 = I_n$ where A is a square matrix of order n.

Non-Commutative of multiplication of matrices:-

Example:-

If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$, find AB, BA, Show that $AB \neq BA$

Example

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ then verify associative law of multiplication.

Zero Matrix as a product of two non zero matrices.

Example:-

A = $\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$, then $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

If the product of two matrices is a zero matrix, one of the matrices doesn't need to be a zero matrix.

Applications of Matrix Multiplication:-

Example – 1

Use matrix multiplication to divide Rupees 30, 000 in two parts such that the total annual interest at 9% on the 1^{st} part and 11% on the second part amount Rupees 3060/-

Example – 2

Let $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I be the identity matrix of order 2. Show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Example – 3

A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, Show that $A^2 - 5\mathrm{A} + 7\mathrm{I} = 0$

Example – 4

If A is a square matrix such that $A^2 = A$ show that $(I+A)^3 = 7A + I$

HOME ASSIGNMENT

Q1. If A = $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Prove that $AB \neq BA$

Q2. If a matrix A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k.

Q3. Find the value of x from the matrix equation

$$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix}=\begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Q4. If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then prove that $A^2 - 3A + 2I$

Transpose of a matrix and its Properties

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Transpose of a Matrix:-

The matrix obtained from a given matrix A by changing its rows into columns or columns into rows is called transpose of matrix A and is dented by A^T or A'. From the definition, it is obvious that if the order of A is $m \times n$

Example:- Transpose of Matrix $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}_{2\times3}$ is $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}_{3\times2}$

Properties of Transpose:-

a) $\left(A^T\right)^T = A$

b) $\left(A \pm B\right)^T = A^T \pm B^T$

c) $\left(AB\right)^T = B^T A^T$

Example:

If A = $\begin{bmatrix}2\\3\\4\end{bmatrix}$ and B = $\begin{bmatrix}-2 & 4 & 3\end{bmatrix}$. Prove that $(AB)^T = B^T A^T$

Symmetric Matrix

A square matrix $A = [a_{ij}]$ is called symmetric matrix if $a_{ij}=a_{ji}$ for all i and j Or $\boldsymbol{A} = \boldsymbol{A^T}$

Example:- $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

NOTE

- Every unit matrix and square zero matrix are symmetric matrices.
- Maximum number of different elements in a symmetric matrix is $\frac{n\,(n+1)}{2}$

Skew-Symmetric Matrix

A square matrix $A = [a_{ij}]$ is called Skew-symmetric matrix if $a_{ij}=-a_{ji}$ for all i and j Or $\boldsymbol{A} = -\boldsymbol{A}^{\boldsymbol{T}}$

Example:- $\begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix}$

NOTE

- All principal diagonal elements of the skew-symmetric matrix are always zero because for any diagonal element $a_{ij}=-a_{ji}$ so $a_{ij}= 0$
- The diagonal elements of a skew-symmetric matrix are always 0.

Proof:

Let $A = \left[a_{ij} \right]$ be the skew-skew symmetric matrix. Then,

$a_{ij} = -a_{ji}$ for all i, j

$\Rightarrow$ $a_{ii} = -a_{ii}$ for all values of i = j

$\Rightarrow$ $2a_{ii} = 0$

$\Rightarrow$ $a_{ii} = 0$

$\Rightarrow$ $a_{11} = a_{22} = a_{33} = ... = a_{nn} = 0$

Example:-

If the matrix $A = \begin{bmatrix} 0 & a & 5 \\ 3 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric. Find the value of a, b, c

Example

Let A be a square matrix then prove that $A + A'$ is a symmetric matrix

Example

If A and B are symmetric matrices then show that AB is symmetric if AB = BA

Theorem

Every square matrix can be uniquely expressed as the sum of a symmetric and skew-symmetric matrix.

Question:-

Express the matrix B = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

HOME ASSIGNMENT

Q1. If A be a square matrix then prove the following

(i) $A - A^T$ is a skew-symmetric matrix
(ii) AA^T and $A^T A$ are symmetric matrices

Q2. Let A and B are symmetric matrices of same order then show that

(i) A + B is a symmetric matrices
(ii) AB – BA is a skew-symmetric matrix
(iii) AB + BA is a symmetric matrix

Q3. Express the following matrices as the sum of symmetric and skew symmetric matrix.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Q4. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, B = [1 \quad 3 \quad -6]$, verify that $(AB)^T = B^T A^T$

Concept of elementary row and column operations.

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Concept of elementary row and column operations.

Elementary Transformations or elementary operations of a matrix: -

The following three operations applied on the rows (columns) of a matrix are called elementary row (column) transformation.

- Interchange of any two rows (columns) denoted by $R_i \Leftrightarrow R_j$ or $C_i \Leftrightarrow C_j$
- Multiplying all elements of a row (column) of a matrix by a non-zero scalar denoted by $R_i \rightarrow kR_j$ or $C_i = kC_j$
- Adding to the elements of a row (column), the corresponding elements of any other row (column) multiplied by any scalar K, dented by $R_i \rightarrow R_i + kR_j or C_i \rightarrow C_i + kC_j$

Method of finding the inverse of a matrix by Elementary transformation: -

Let A be a non-singular matrix of order n. Then A can be reduced to the identity matrix I_n by a finite sequence of elementary transformation only. As we have discussed every elementary row transformation of a matrix is equivalent to pre-multiplication by the corresponding elementary matrix. Therefore there exist elementary matrices E_1, E_2.....E_4 such that $(E_k E_{k-1} \ldots\ldots E_2 E_1)A = I_n$

$\Rightarrow (E_k E_{k-1} \ldots\ldots E_2 E_1)AA^{-1} = I_n A^{-1}$ (post multiplying by A^{-1})

$\Rightarrow (E_k E_{k-1} \ldots\ldots E_2 E_1)I_n = A^{-1}$ $\quad (\because I_n A^{-1} = A^{-1} and AA^{-1} = I_n)$

$$\Rightarrow A^{-1} = (E_k E_{k-1} \ldots\ldots E_2 E_1)I_n$$

Algorithm for finding the inverse of a non-singular matrix by elementary row transformations: -

Let A be a non-singular matrix of order n

Step – I:- Write $A = I_n A$

Step – II:- Perform a sequence of elementary row operations successively on the LHS and the pre-factor I_n on the RHS till we obtain the result $I_n = BA$

Step – III:- Write $A^{-1} = B$.

The following steps will be helpful to find the inverse of a square matrix of order 3 by using Elementary row transformations.

Step – I: - Introduce unity at the intersection of the first row and first column either by interchanging two rows or by adding a constant multiple of elements of some other row to the first row.

Step – II: - After introducing unity at (1, 1) place introduce zeros at all other places in the first column.

Step –III: - Introduce unity at the intersection 2^{nd} row and 2^{nd} column with the help of the 2^{nd} and 3^{rd} row.

Step – IV: - Introduce zeros at all other places in the second column except at the intersection of 2nd and 2nd Column.

Step – V: - Introduce unity at the intersection of 3rd row and third column.

Step – VI: - Finally introduce zeros at all other places in the third column except at the intersection of the third row and third column.

Example - 1

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ using elementary row transformation.

Example-2

Using elementary transformation find the inverse of following matrices.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

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Q1. Using elementary transformations, find the inverse of each of the matrices, if it exists.

(i) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

Invertible matrices and proof of the uniqueness of inverse if it exists

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Invertible matrices and proof of the uniqueness of inverse if it exists

The inverse of a square matrix, if it exists, is unique

Let $A = (a_{ij})_{n\times n}$ be any square matrix

If possible, A has two inverses B and C

$\Rightarrow AB = I = BA$……………………….(1)

$\& AC = I = CA$………………………..(2)

Now, $B = BI = B(AC) = (BA)C = IC = C$

Example - 1

Find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$

Example -2

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary row transformation.

Example -3

$A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$ find A^{-1} using elementary row transformation.

HOME ASSIGNMENT

Find A^{-1}, of the following matrices using elementary row transformation.

(a) $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix}$

Answer:- $-\frac{1}{30}\begin{bmatrix} -2 & 4 & -10 \\ 44 & -7 & -5 \\ -5 & -5 & 5 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$

Answer :- $\begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Answer:- $\begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$

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