Determinant of a square matrix (up to 3 x 3 matrices)

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- A determinant is defined as a (mapping) function from the set 0 square matrices to the set of real numbers
- Every square matrix A is associated with a number, called its determinant.
- Denoted by det (A) of |A| or Δ
- If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det(A)$
- Only square matrices have determinant
- The matrices which are not square do not have determinants
- Foe matrix $A, |A|$ is read as determinant of A and not modulus of A.

Types of Determinant:-

First Order Determinant:-

Let $A = [a]$ be the matrix of order 1, then determinant of A is defined to be equal to a. If $A = [a]$, then $\det(A) = |A| = a$.

Second order Determinant:- Let $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ matrix of order 2×2

$\det(A)=|A|=\Delta=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}=a_{11}a_{22}-a_{21}a_{12}$ Eg. Evaluate $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}=2(2)-4(-1)=4+4+8$

Question:

Evaluate $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

Answer:

$$\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}=x(x)-(x+1)(x-1)=x^2-(x^2-1)=x^2-x^2+1=1$$

Third Order Determinant:-

- Can be determined by expressing it in terms of second order determinants

- Let $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

The below method is explained for expansion around Row 1.

The value of the determinant, thus will be the sum of the product of element in line parallel to the diagonal minus the sum of the product of elements in line perpendicular to the line segment. Thus,

Then det $A = |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$$|A| = a_{11}\left(a_{22}a_{33} - a_{32}a_{23}\right) - a_{12}\left(a_{21}a_{33} - a_{31}a_{23}\right) + a_{13}\left(a_{21}a_{32} - a_{31}a_{22}\right)$$

The same procedure can be repeated for Row 2, Row 3, Column 1, Column 2 and column 3

Question:

Evaluate the determinant $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$.

Answer:

Note that in the third column, two entries are zero. So expanding along third column (C_3), we get

$$\Delta = 4\begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix}$$

$$= 4(-1-12) - 0 + 0 = -52$$

Question:

Find values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.

Answer:

We have $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

i.e. $3 - x^2 = 3 - 8$

i.e. $x^2 = 8$

Hence $x = \pm 2\sqrt{2}$

Note:-

- Expanding a determinant along any row or column gives same value.
- This method doesn't work for determinates of order greater than 3.
- For easier calculations, we shall expand the determinate along that row or column which contains maximum number of zeros

In general, if A = kB where A and B are square matrices of order n, then $|A| = k^n |B|$, when $n = 1, 2, 3$.

THANKING YOU

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