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Properties of Determinants

SUBJECT : (Mathematics)
CHAPTER NUMBER: 04
CHAPTER NAME : Determinant

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Properties of determinants

Helps in simplifying its evaluation by obtaining maximum number of zeros in a row or a column. These properties are true for determinates of any order.

Property – 1

The value of the determinant remains unchanged if its rows and columns are interchanged.

Verification:-

$$\text{Let } \Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Expanding along first row, we get.

$$\Delta = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$= a_1 (b_2 c_3 - b_3 c_2) - a_2 (b_1 c_3 - b_3 c_1) + a_3 (b_1 c_2 - b_2 c_1)$$

By interchanging the rows and columns of Δ, we get the determinant

$$\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Expanding Δ_1 along first column, we get.

$$\Delta_1 = a_1 (b_2 c_3 - c_2 b_3) - a_2 (b_1 c_3 - b_3 c_1) + a_3 (b_1 c_2 - b_2 c_1)$$

Hence $\Delta = \Delta_1$

Question:

Verify Property 1 for $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Answer:

Expanding the determinant along first row, we have

$$\Delta = 2\begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} + 5\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix}$$

$$= 2(0-20) + 3(-42-4) + 5(30-0)$$

$$= -40 - 138 + 150 = -28$$

By interchanging rows and columns, we get

$$\Delta_1 = \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix} \quad \text{(Expanding along first column)}$$

$$= 2\begin{vmatrix} 0 & 5 \\ 4 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 6 & 1 \\ 4 & -7 \end{vmatrix} + 5\begin{vmatrix} 6 & 1 \\ 0 & 5 \end{vmatrix}$$

$$= 2(0-20) + 3(-42-4) + 5(30-0)$$

$$= -40 - 138 + 150 = -28$$

Clearly $\Delta = \Delta_1$

Hence, Property 1 is verified.

Note:-

- It follows from above property that if A is a square matrix, then det (A) = det (A′), where A′ = transpose of A
- If $Ri = ith$ row and $Ci = ith$ column, then for interchange of row and columns, we will symbolically write $Ci \Leftrightarrow Ri$

Property – 2

It any two rows (or columns) of determinant are interchanged, then sign of determinant changes.

Verification:-

Let $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ Expanding along first row, we get $\Delta = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$

$$= a_1 (b_2c_3 - b_3c_2) - a_2 (b_1c_3 - b_3c_1) + a_3 (b_1c_2 - b_2c_1)$$

By interchanging the first and third rows of Δ we get the determinant

$$\Delta_1 = \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

Expanding along third row, we get

$$\Delta_1 = a_1(c_2b_3 - b_2c_3) - a_3(c_1b_3 - c_3b_1) + a_3(b_2c_1 - b_1c_2)$$

$$= -[a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)]$$, Hence $\Delta_1 = -\Delta$

Question

Verify Property 2 for $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Answer:

In previous example

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = -28$$

Interchanging rows R_2 and R_3 i.e., $R_2 \leftrightarrow R_3$, we have

$$\Delta_1 = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 5 & -7 \\ 6 & 0 & 4 \end{vmatrix}$$

Expanding the determinant Δ_1 along first row, we have

$$\Delta_1 = 2\begin{vmatrix} 5 & -7 \\ 0 & 4 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -7 \\ 6 & 4 \end{vmatrix} + 5\begin{vmatrix} 1 & 5 \\ 6 & 0 \end{vmatrix}$$

$$= 2(20-0) + 3(4+42) + 5(0-30)$$

$$= 40 + 138 - 150 = 28$$

Property – 3

If any two rows (or columns) of a determinant are identical (all corresponding elements are same) then value of determinate is zero.

Verification:-

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ in which two rows are identical i.e $a_1 = a_2, b_1 = b_2$ and $c_1 = c_2$

If we interchange the identical rows (or columns) of the determinant Δ, then Δ does not change.

However, by property – 2, it follows that Δ has changed its sign. Therefore $\Delta = -\Delta$ or $\Delta = 0$

Question:

$$\text{Evaluate } \Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$$

Answer:

Expanding along first row, we get

$$\Delta = 3\,(6-6) - 2\,(6-9) + 3\,(4-6)$$
$$= 0 - 2\,(-3) + 3\,(-2) = 6 - 6 = 0$$

Here R_1 and R_3 are identical..

Property – 4

It each element of a row (or a column) of a determinate is multiplied by a constant k, then its value gets multiplied by k.

Verification:-

$$\text{Let } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Let Δ_1 be the determinant obtained by multiplying the elements of the first row by k.

Then, $\Delta_1 = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$. Expanding along first row, we get.

$$\Delta_1 = ka_1(b_2c_3 - b_3c_2) - kb_1(a_2c_3 - c_2a_3) + kc_1(a_2b_3 - b_2a_3)$$

$$= k\left[a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)\right] = k\Delta$$

Hence $\begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

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Question:

$$\text{Evaluate} \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Answer:

$$\text{Note that} \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 6(17) & 6(3) & 6(6) \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6\begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0$$

Property – 5

If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

Verification:-

Take $\begin{vmatrix} a_1+\lambda_1 & a_2+\lambda_2 & a_3+\lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

LHS $= \begin{vmatrix} a_1+\lambda_1 & a_2+\lambda_2 & a_3+\lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Expanding the determinates along the first row, we get.

$$\Delta = (a_1+\lambda_1)(b_2c_3-c_2b_3)-(a_2+\lambda_2)(b_1c_3-b_3c_1)+(a_3+\lambda_3)(b_1c_2-b_2c_1)$$

$$= a_1(b_2c_3-c_2b_3)-a_2(b_1c_3-b_3c_1)+a_3(b_1c_2-b_2c_1)+\lambda_1(b_2c_3-c_2b_3)-\lambda_2(b_1c_3-b_3c_1)+\lambda_3(b_1c_2-b_2c_1)$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \text{RHS}$$

Question:

Show that $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0$

Answer:

We have $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix}$

$= 0 + 0 = 0$

Property – 6

If, to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same, i.e the value of determinant remain same if we apply the operation.

$R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Verification:-

$$\text{Let } \Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \text{ and } \Delta_1 = \begin{vmatrix} a_1 + kc_1 & a_2 + kc_2 & a_3 + kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Where Δ is obtained by the operation $R_1 \to R_1 + kR_3$. Here, we have multiplied the elements or the third row (R_3) by a constant k and added them to the corresponding elements of the first row (R_1).

Symbolically, we write this operation as $R_1 \to R_1 + kR_3$

$$\Delta_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} kc_1 & kc_2 & kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$= \Delta + o$ (since R_1 and R_3 are proportional)   Hence $\Delta = \Delta_1$

Note:

We can add or subtract any two row or column or all row or column of a determinant.

Property – 7

If each element of a row (or column) of a determinant is zero, then its value is zero

Examples

1) **Prove by using properties of determinant**

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0.$$

Answer:

$$A = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$$

Operating $C_1 \to C_1 + C_2$, we get

$$= \begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix} = 0.$$

$[\because C_1$ and C_3 are identical$]$

2) **Prove by using properties of determinant**

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0.$$

Answer:

$$\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$ Operating $R_1 \to R_1 + R_2 + R_3$, we get

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0.$$

3) **Prove by using properties of determinant**

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0.$$

Answer:

$$\Delta = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$$

Operating $C_1 \to C_1 + 9C_2$, we get

$$\begin{vmatrix} 65 & 7 & 65 \\ 75 & 8 & 75 \\ 86 & 9 & 86 \end{vmatrix} = 0.$$

[$\because$ C_1 and C_3 are identical]

4) **Prove by using properties of determinant**

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}.$$

Answer:

Let
$$\Delta = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

Operating $R_1 \to R_1 + R_2 + R_3$, we get

$$\Delta = \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

Operating $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Delta = 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$

Operating $R_1 \to R_1 + R_2 + R_3$

$$\Delta = 2\begin{vmatrix} a & p & x \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}.$$

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5) Prove by using properties of determinant

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2.$$

Answer:

$$\text{L.H.S.} = \Delta = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

Taking a, b and c common from R_1, R_2 and R_3 respectively.

$$\Delta = abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

Taking a, b and c common from C_1, C_2 and C_3 respectively, we get

$$\Delta = a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Operating $R_2 \to R_2 + R_1, R_3 \to R_3 + R_1$

$$\Delta = a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix}$$

Interchanging C_2 and C_3 we get

$$\Delta = (-1)\, a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

Since the determinant of a triangular matrix is product of its diagonal element.

$$= (-1)a^2b^2c^2(-1)\times(2)\times(2)$$

$$= 4a^2b^2c^2 = \text{R.H.S.}$$

Q6) Prove by using properties of determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$

Answer:

$$\text{Here LHS} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Operating $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$\text{LHS} = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3 - a^3 & c^3 - a^3 \end{vmatrix}$$

Taking $(b-a)$ and $(c-a)$ common from C_2 and C_3

$$= (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & b^2+ba+a^2 & c^2+ca+a^2 \end{vmatrix}$$

Operating $C_3 \to C_3 - C_2$, we get

$$\text{LHS} = (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^3 & b^2+ba+a^2 & c^2+ca-b^2-ba \end{vmatrix}$$

$$= (b-a)(c-a)(c^2+ca-b^2-ba)$$

$$= (b-a)(c-a)[c^2-b^2+a(c-b)]$$

$$= (b-a)(c-a)[(c-b)(c+b)+a(c-b)]$$

$$= (b-a)(c-a)(c-b)[c+b+a]$$

$$= (a-b)(b-c)(c-a)(a+b+c) = \text{RHS}.$$

Q7) Prove by using properties of determinant

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$$

Answer:

$$\Delta = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Operating $R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Taking $(5x + 4)$ common from R_1

$$\Delta = (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Operating $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\Delta = (5x+4)\begin{vmatrix} 1 & 0 & 0 \\ 2x & 4-x & 0 \\ 2x & 0 & 4-x \end{vmatrix}$$

Since determinant of triangular matrix is product of its diagonal elements.

$\therefore$ $$\Delta = (5x+4)(4-x)^2 = \text{RHS}$$

Q8) **Prove by using properties of determinant**

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3.$$

Answer:

$$\Delta = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$$

Operating $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} 2x+2y+2z & x & y \\ 2x+2y+2z & y+z+2x & y \\ 2x+2y+2z & x & z+x+2y \end{vmatrix}$$

Taking $2(x+y+z)$ from C_1

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

Operating $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$= 2(x+y+z)\begin{vmatrix} 1 & x & y \\ 0 & y+z+x & 0 \\ 0 & 0 & z+x+y \end{vmatrix}$$

Since determinant of a triangular matrix is product of its diagonal elements.

$\therefore$ $$\Delta = 2(x+y+z)(y+z+x)(z+x+y)$$

$$= 2(x+y+z)^3 = \text{RHS}$$

Q9) Prove by using properties of determinant

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

Answer:

Let

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Operate $C_1 \to C_1 - bC_3, C_2 \to C_2 + aC_3$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b+a^2b+b^3 & -a-a^3-ab^2 & 1-a^2-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$$

Taking $(1+a^2+b^2)$ common from C_1 and C_2, we get

$$\Delta = (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$

Operating $R_3 \to R_3 - bR_1$ we get,

$$\Delta = (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2-b^2+2b^2 \end{vmatrix}$$

Operating $R_3 \to R_3 + aR_2$, we get

$$\Delta = (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & 0 & 1+a^2+b^2 \end{vmatrix}$$

Since determinant of a triangular matrix is equal to product of its diagonal elements.

$\therefore \quad \Delta = (1+a^2+b^2)^3 = \text{RHS}$

Q10) Prove by using properties of determinant

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.$$

Answer:

Let $$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Multiply C_1, C_2 and C_3 by a, b and c respectively, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & ab^2 & ac^2 \\ a^2b & b(b^2+1) & bc^2 \\ a^2c & cb^2 & c(c^2+1) \end{vmatrix}$$

Taking a, b and c common from R_1, R_2 and R_3, we get

$$\Delta = \frac{abc}{abc}\begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$

Operating $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c^2 & b^2 & c^2+1 \end{vmatrix}$$

$$\Rightarrow \quad \Delta = (1+a^2+b^2+c^2)\begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$$

Operating $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\Delta = (1 + a^2 + b^2 + c^2)\begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Since the determinant of a triangular matrix is the product of its diagonal elements.

$$\Delta = 1 + a^2 + b^2 + c^2 = \text{R.H.S.}$$

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