Minors and Cofactors

SUBJECT : (Mathematics)
CHAPTER NUMBER: 04
CHAPTER NAME : Determinant

Minors and Cofactors

Minor:-

- If the row and column containing the element a_{11} (i.e, 1st row and 1st column) are removed, we get the second order determinant which is called the minor of element a_{11}.
- Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its ith row and jth column in which element a_{ij} lies.
- Minor of an element a_{ij} is denoted by M_{ij}
- Minor of an element of a determinant of order $n\,(n \geq 2)$ is a determinant of order $n-1$
- Eg. Find Minor 0 the element 6 in the determinant A given

Let $\Delta A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ since 6 lies in the second row and third column, its minor M_{23} is given by

$M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14 = -6$ (obtained by deleting R_2 and C_3 in Δ)

Cofactor:-

- If the minors are multiplied by the proper signs we get cofactors
- The cofactor of the element a_{ij} is $C_{ij} = (-1)^{i+j} M_{ij}$
- The signs to be multiplied are given by the rule $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$
- If elements of a row (or column) are multiplied with cofactors of any other row (or column) then their sum is zero.

$$\Delta = a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$$

$$= a_{11}(-1)^{1+1}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{12}(-1)^{1+2}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}(-1)^{1+3}\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$ (since R_1 and R_2 are identical)

E.g. find the cofactors of the element -2 in the given determinant A

Let $A = \begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$

Cofactor of -2 is $A_{12} = (-1)^{1+2} M_{12} = (-1)^3(4) = -4$

Question:

Find the minors and cofactors of elements of the matrix $A = [a_{ij}] = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$.

Answer:

$$M_{13} = \begin{vmatrix} 4 & -5 \\ 3 & 5 \end{vmatrix} = 20 + 15 = 35 \Rightarrow C_{13} = M_{13} = 35$$

$$M_{21} = \begin{vmatrix} 3 & -2 \\ 5 & 2 \end{vmatrix} = 6 + 10 = 16 \Rightarrow C_{21} = -M_{21} = -16$$

$$M_{11} = \begin{vmatrix} -5 & 6 \\ 5 & 2 \end{vmatrix} = -10 - 30 = -40 \Rightarrow C_{11} = M_{11} = -40$$

$$M_{12} = \begin{vmatrix} 4 & 6 \\ 3 & 2 \end{vmatrix} = 8 - 18 = -10 \Rightarrow C_{12} = -M_{12} = 10$$

$$M_{22} = \begin{vmatrix} 1 & -2 \\ 3 & 2 \end{vmatrix} = 2 + 6 = 8 \quad \Rightarrow C_{22} = M_{22} = 8$$

$$M_{23} = \begin{vmatrix} 1 & 3 \\ 3 & 5 \end{vmatrix} = 5 - 9 = -4 \quad \Rightarrow C_{23} = -M_{23} = 4$$

$$M_{31} = \begin{vmatrix} 3 & -2 \\ -5 & 6 \end{vmatrix} = 18 - 10 = 8 \quad \Rightarrow C_{31} = M_{31} = 8$$

$$M_{32} = \begin{vmatrix} 1 & -2 \\ 4 & 6 \end{vmatrix} = 6 + 8 = 14 \quad \Rightarrow C_{32} = = M_{32} = -14$$

$$M_{33} = \begin{vmatrix} 1 & 3 \\ 4 & -5 \end{vmatrix} = -5 - 12 = -17 \quad \Rightarrow C_{33} = M_{33} = -17$$

Question:

Find co-factor of Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

Answer:

$$C_{11} = \text{cofactor of } 1 = (-1)^{1+1} \begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} = 12 - 6 = 6;$$

$$C_{12} = \text{cofactor of } 2 = (-1)^{1+2} \begin{vmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = -(8-6) = -2;$$

$$C_{13} = \text{cofactor of } 3 = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} = 6 - 9 = -3;$$

$$C_{21} = \text{cofactor of } 2 = (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -(8-9) = 1;$$

$$C_{22} = \text{cofactor of } 3 = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} = 4 - 9 = -5;$$

$$C_{23} = \text{cofactor of } 2 = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = -(3-6) = 3;$$

$$C_{31} = \text{cofactor of } 3 = (-1)^{3+1} \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = 4 - 9 = -5;$$

$$C_{32} = \text{cofactor of } 3 = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = -(2-6) = 4;$$

and $C_{33} = \text{cofactor of } 4 = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1$

THANKING YOU

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