Consistency Inconsistency, Number of solutions of system of linear equations by examples

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Consistency Inconsistency, Number of solutions of system of linear equations by examples

Application of Determinants and matrices:-

- Used for solving the system of linear equations in two or three variables and for checking the consistency of the system of linear equations.
- Consistent system:- a system of equations is said to be consistent if its solution (one or more) exists.

Inconsistent system:- A system of equations is said to be inconsistent if its solution does not exist.

Solution of system of linear equations using inverse of a matrix:-

Let the system of Equations be as below:-

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Then, the system of equations can be written as, $AX = B$ i.e

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Case – I

If A is a non-singular matrix, then its inverse exists,

$AX = B$

$A^{-1}(AX) = A^{-1}B$ (premultiplying by A^{-1})

$(A^{-1}A)X = A^{-1}B$ (by associative property)

$1X = A^{-1}B \qquad X = A^{-1}B$

This matrix equation provides unique solution for the given system of equations as inverse of a matrix is unique. This method of solving system of equations is known as matrix method.

Case – II

If A is a singular matrix, then $|A| = 0$

In this case, we calculate (adj A) B.

If (adj A) $B \neq O$ (O being zero matrix), then solution does not exist and the system of equation s called inconsistent.

If (adj A) B = O, then system may be either consistent or inconsistent according as the system have either infinitely any solutions or no solution.

Summary:-

For a square matrix a in matrix equation AX = B

$|A| \neq O$, there exists unique solution

$|A| = O$ and (adj A) $B \neq O$, then there exists no solution

$|A| = O$ and (adj A) B = O, then system may or may not be consistent

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