Word problems on Matrices

SUBJECT : (Mathematics)
CHAPTER NUMBER: 04
CHAPTER NAME : Determinant

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Example:1

On her birthday seema decided to donate some money to children of an orphanage home. If there were 8 children less everyone would have got 10 rupees more. How ever there were 16 children more every would have got 10 rupees less. Using matrix method , find the number of children and the amount distributed by Seema.

Answer:

Let the number of children be ₹ x and the amount donated by Seema to each child be ₹ y.

$\therefore$ From question

$$(x-8)(y+10) = xy \text{ and } (x+16)(y-10) = xy$$

$$\Rightarrow \quad xy + 10x - 8y - 80 = xy \text{ and } xy - 10x + 16y - 160 = xy$$

$$\Rightarrow \quad 5x - 4y = 40 \qquad \ldots(i)$$

$$\text{and} \quad 5x - 8y = -80 \qquad \ldots(ii)$$

Equation (i) and (ii) may be written in matrix form as

$$AX = B \quad \Rightarrow \quad X = A^{-1}B \qquad \ldots(iii)$$

Where $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$

Now, $|A| = -40 + 20 = -20$

$$Adj\, A = \begin{bmatrix} -8 & -5 \\ 4 & 5 \end{bmatrix}^T = \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$

$$\therefore \quad A^{-1} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$

Putting A^{-1} in (iii), we get

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}\begin{bmatrix} 40 \\ -80 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20}\begin{bmatrix} -320 & -320 \\ -200 & -400 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20}\begin{bmatrix} -640 \\ -600 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$$

$\Rightarrow \quad x = 32,\ y = 30$

Example-

Two Trusts A & B receive rupees 70000 and rupees 65000 respectively from central govt for award prize to persons of a District in three fields Agriculture, Education and Social service. Trust A awarded 10, 5 and 15 persons in the field of Agriculture, , Education and Social service respectively while Trust B awarded 15,10 & 5 persons respectively. If all three prizes together amount rupees 6000, then find the amount of prize by matrix method.

Answer:

Let x, y, z be amount of prize to be awarded in the field of Agriculture, Education and Social service respectively.

Then situation can be written in matrix form as AX = B

$$\text{Where } A=\begin{bmatrix}10 & 5 & 15\\ 15 & 10 & 5\\ 1 & 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}, B=\begin{bmatrix}70000\\ 55000\\ 6000\end{bmatrix}$$

$$\text{Now, } |A|=\begin{vmatrix}10 & 5 & 15\\ 15 & 10 & 5\\ 1 & 1 & 1\end{vmatrix}$$

$$= 10(10-5) - 5(15-5) + (15-10) = 75 \neq 0$$

Hence, A^{-1} exists and system have unique solution.

Also,

$$C_{11} = (-1)^{1+1}\begin{vmatrix} 10 & 5 \\ 1 & 1 \end{vmatrix} = (10-5) = 5; \qquad C_{12} = (-1)^{1+2}\begin{vmatrix} 15 & 5 \\ 1 & 1 \end{vmatrix} = -(15-5) = -10$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 15 & 10 \\ 1 & 1 \end{vmatrix} = (15-10) = 5; \qquad C_{21} = (-1)^{2+1}\begin{vmatrix} 5 & 15 \\ 1 & 1 \end{vmatrix} = -(5-15) = 10$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 10 & 15 \\ 1 & 1 \end{vmatrix} = (10-15) = -5; \qquad C_{23} = (-1)^{2+3}\begin{vmatrix} 10 & 5 \\ 1 & 1 \end{vmatrix} = -(10-5) = -5$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} 5 & 15 \\ 10 & 5 \end{vmatrix} = (25-150) = -125; \qquad C_{32} = (-1)^{3+2}\begin{vmatrix} 10 & 15 \\ 15 & 5 \end{vmatrix} = -(50-225) = 175$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} 10 & 5 \\ 15 & 10 \end{vmatrix} = (100-75) = 25$$

$$Adj|A| = \begin{bmatrix} 5 & -10 & 5 \\ 10 & -5 & -5 \\ -125 & 175 & 25 \end{bmatrix}^T = \begin{bmatrix} 5 & 10 & -125 \\ -10 & -5 & 175 \\ 5 & -5 & 25 \end{bmatrix}$$

$$A^{-1} = \frac{adj\,(A)}{|A|} = \begin{bmatrix} \frac{5}{75} & \frac{10}{75} & \frac{-125}{75} \\ \frac{-10}{75} & \frac{-5}{75} & \frac{175}{75} \\ \frac{5}{75} & \frac{-5}{75} & \frac{25}{75} \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 1 & 2 & -25 \\ -2 & -1 & 35 \\ 1 & -1 & 5 \end{bmatrix}$$

Putting the value of X, A^{-1}, B in $X = A^{-1}B$, we get

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 1 & 2 & -25 \\ -2 & -1 & 35 \\ 1 & -1 & 5 \end{bmatrix}.\begin{bmatrix} 70000 \\ 55000 \\ 6000 \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 70000 + 110000 - 150000 \\ 740000 - 55000 + 210000 \\ 70000 + 55000 + 30000 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 30000 \\ 15000 \\ 45000 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 1000 \\ 3000 \end{bmatrix}$$

$\Rightarrow \quad x = 2000,\ y = 1000,\ z = 3000.$

Hence, prize in the field of agriculture = ₹ 2000
prize in the field of education = ₹ 1000
prize in the field of Social service = ₹ 3000

Example

A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for ₹ 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹ 70. Using matrix method, find cost of each variety of pen. *[CBSE (Central) 2016]*

Answer:

Let the cost of varieties of pens A, B and C be x, y, and z respectively.

From question

$$x + y + z = 21$$

$$4x + 3y + 2z = 60$$

$$6x + 2y + 3z = 70$$

The given system of linear equation in matrix equation is as follows

$AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$\because \quad AX = B \quad \Rightarrow \quad X = A^{-1}B \qquad \ldots(i)$

Now $$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix} = 1(9-4) - 1(12-12) + 1(8-18) = 5 - 0 - 10 = -5 \neq 0$$

$A_{11} = (9-4) = 5 \qquad A_{21} = -(3-2) = -1 \qquad A_{31} = (2-3) = -1$

$A_{12} = -(12-12) = 0 \qquad A_{22} = (3-6) = -3 \qquad A_{32} = -(2-4) = 2$

$A_{13} = (8-18) = -10 \qquad A_{23} = -(2-6) = 4 \qquad A_{33} = (3-4) = -1$

$$\therefore \quad Adj\ A = \begin{bmatrix} 5 & 0 & -10 \\ -1 & -3 & 4 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.adj\ A = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

Now from (*i*) $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \Rightarrow x = 5, y = 8, z = 8$$

Example

In a survey of 20 richest persons of three residential society A, B, C it is found that in society A, 5 believe in honesty, 10 in hard work and 5 in unfair means while in B, 5 believe in honesty, 8 in hard work and 7 in unfair means and in C, 6 believe in honesty, 8 in hard work and 6 in unfair means. If the per day income of 20 richest persons of society A, B, C are ₹ 32,500, ₹ 30,500, ₹ 31,000 respectively, then find the per day income of each type of people by matrix method.

Answer:

Let x, y, z be the per day income of person believing in honesty, hard work and unfair means respectively. The given situation can be written in matrix form as $AX = B$

Where, $$A = \begin{bmatrix} 5 & 10 & 5 \\ 5 & 8 & 7 \\ 6 & 8 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 32500 \\ 30500 \\ 31000 \end{bmatrix}$$

Now, $$|A| = \begin{vmatrix} 5 & 10 & 5 \\ 5 & 8 & 7 \\ 6 & 8 & 6 \end{vmatrix} = 5(48 - 56) - 10(30 - 42) + 5(40 - 48) = 40 \neq 0$$

Hence, A^{-1} exists and system have unique solution.

Also, $C_{11} = (-1)^{1+1}\begin{vmatrix} 8 & 7 \\ 8 & 6 \end{vmatrix} = (48-56) = -8$, $C_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 7 \\ 6 & 6 \end{vmatrix} = -(30-42) = 12$

$C_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 8 \\ 6 & 8 \end{vmatrix} = (40-48) = -8$; $C_{21} = (-1)^{2+1}\begin{vmatrix} 10 & 5 \\ 8 & 6 \end{vmatrix} = -(60-40) = -20$

$C_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 5 \\ 6 & 6 \end{vmatrix} = (30-30) = 0$; $C_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 10 \\ 6 & 8 \end{vmatrix} = -(40-60) = 20$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 10 & 5 \\ 8 & 7 \end{vmatrix} = (70-40) = 30$; $C_{32} = (-1)^{3+2}\begin{vmatrix} 5 & 5 \\ 5 & 7 \end{vmatrix} = -(35-25) = -10$

$C_{33} = (-1)^{3+3}\begin{vmatrix} 5 & 10 \\ 5 & 8 \end{vmatrix} = (40-50) = -10$

$$Adj(A) = \begin{bmatrix} -8 & 12 & -8 \\ -20 & 0 & 20 \\ 30 & -10 & -10 \end{bmatrix}^T = \begin{bmatrix} -8 & -20 & 30 \\ 12 & 0 & -10 \\ -8 & 20 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{adj(A)}{|A|} = \begin{bmatrix} \frac{-8}{40} & \frac{-20}{40} & \frac{30}{40} \\ \frac{12}{40} & 0 & \frac{-10}{40} \\ \frac{-8}{40} & \frac{20}{40} & \frac{-10}{40} \end{bmatrix} = \begin{bmatrix} \frac{-1}{5} & \frac{-1}{2} & \frac{3}{4} \\ \frac{3}{10} & 0 & \frac{-1}{4} \\ \frac{-1}{5} & \frac{1}{2} & \frac{-1}{4} \end{bmatrix}$$

Putting the value of X, A^{-1}, B in $X = A^{-1}B$, we get

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-1}{5} & \frac{-1}{2} & \frac{3}{4} \\ \frac{3}{10} & 0 & \frac{-1}{4} \\ \frac{-1}{5} & \frac{1}{2} & \frac{-1}{4} \end{bmatrix} \cdot \begin{bmatrix} 32500 \\ 30500 \\ 31000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1500 \\ 2000 \\ 1000 \end{bmatrix}$$

$\Rightarrow \quad x = 1500, \; y = 2000, \; z = 1000$

Hence, per day income of person who believes in honesty = ₹ 1,500

Per day income of person who believes in hard work = ₹ 2,000

Per day income of person who believes in unfair means = ₹ 1,000

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