

# Solution of a Differential Equation

**SUBJECT : (Mathematics)**

**CHAPTER NUMBER: 09**

**CHAPTER NAME : Differential equation**

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## Solution of a Differential Equation

An equation containing the dependent variable and independent variable and free from derivatives, which satisfies the differential equation is called the solution or primitive of the differential equation.

For example,  $y = e^x$  is a solution of the differential equation  $\frac{dy}{dx} - y = 0$ .

Also,  $y = 4e^x$  is a solution of the differential equation  $\frac{dy}{dx} - y = 0$ .

There are two types of solutions to a differential equation which are general solution and particular solution.

# Solution of a Differential Equation

## General Solution:

The solution which contains as many as arbitrary constants as the order of the differential equation is called the general solution of the differential equation.

For example,  $y = A \cos x + B \sin x$  is the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = 0.$$

# Solution of a Differential Equation

## Particular Solution

The solution which is obtained from the general solution of a differential equation by assigning particular values to the arbitrary constants is called a particular solution.

For example,  $y = 2 \cos x + 3 \sin x$  is a particular solution of the differential equation  $\frac{d^2y}{dx^2} + y = 0$ .

## Example

Verify that  $y = cx + 2c^2$  is a solution of the differential equation  $2\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$ .

## Example

Show that  $y = Ax + \frac{B}{x}$ ,  $x \neq 0$  is a solution of the differential equation  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$ .

## Example

State whether the following statements are true or false.

- a.  $x + y = \tan^{-1} y$  is a solution of the differential equation  $y^2 \frac{dy}{dx} + y^2 + 1 = 0$ .
- b.  $y = x$  is a particular solution of the differential equation  $\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$ .

## Example

Find the number of arbitrary constants in the particular solution of the differential equation of the third order.



## Assignment

Choose the correct answer from the given options.

1. The differential equation for  $y = A \cos \alpha x + B \sin \alpha x$ , where  $A$  and  $B$  are arbitrary constants is

a)  $\frac{d^2y}{dx^2} - \alpha^2 y = 0$

b)  $\frac{d^2y}{dx^2} + \alpha^2 y = 0$

c)  $\frac{d^2y}{dx^2} + \alpha y = 0$

d)  $\frac{d^2y}{dx^2} - \alpha y = 0$

2. If  $y = e^{-x}(A \cos x + B \sin x)$ , then  $y$  is a solution of

a)  $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 0$

b)  $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$

c)  $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0$

d)  $\frac{d^2y}{dx^2} + 2y = 0$

3. The number of solutions to  $\frac{dy}{dx} = \frac{y+1}{x-1}$  when  $y(1) = 2$  is

a) None

b) one

c) two

d) infinite

4.  $y = ae^{mx} + be^{-mx}$  satisfies which of the following differential equation?

a)  $\frac{dy}{dx} + my = 0$

b)  $\frac{dy}{dx} - my = 0$

c)  $\frac{d^2y}{dx^2} - m^2y = 0$

d)  $\frac{d^2y}{dx^2} + m^2y = 0$

**THANKING YOU**  
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