Solution of Differential Equations by Method of Separation of Variables

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Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: **1800 120 2316**
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Solution of Differential equations by method of separation of variables

A general differential equation of first order and first degree is of the form $f\left(x, y, \frac{dy}{dx}\right) = 0$. --$(i)$

The general solution of (i) represents the equation of the family of curves in one arbitrary constant.

In this section, we shall discuss several techniques of obtaining solutions to the following type of differential equations.

Solution of Differential equations by method of separation of variables

Differential Equations of the type $\frac{dy}{dx} = f(x)$

We have $\frac{dy}{dx} = f(x) \Leftrightarrow dy = f(x)dx$

Integrating both sides, we have $\int dy = \int f(x)\, dx$

$\Rightarrow y = \int f(x)\, dx + C$, which is the general solution.

Example

Solve the following differential equations.

1. $\frac{dy}{dx} = \frac{x}{x^2+1}$

2. $\frac{dy}{dx} = xlogx$

3. $x\frac{dy}{dx} + 1 = 0;\ y(1) = 0.$

Solution of Differential equations by method of separation of variables

Differential Equations of the type $\frac{dy}{dx} = f(y)$

We have $\frac{dy}{dx} = f(y) \Leftrightarrow dx = f(y)dy$

Integrating both sides, we have $\int dx = \int \frac{1}{f(y)}\, dy$

$\Rightarrow x = \int \frac{1}{f(y)}\, dy + C$, which is a general solution.

Example

Solve the following differential equations.

a) $\frac{dy}{dx} = sin^2 y$

b) $\frac{dy}{dx} + 2y^2 = 0, y(1) = 1$

Solution of Differential equations by method of separation of variables

Differential Equations in the variable separable form:

A differential equation is said to have separable variables if it is of the form $f(x)dx = g(y)dy$.

Such types of equations can be solved by integrating both sides.

The general solution is given by $\int f(x)dx = \int g(y)dy + C$, where C is an arbitrary constant.

Remember:

There is no need of introducing arbitrary constants of integration on both sides as they can be combined to give just one arbitrary constant.

Example

Solve the following differential equations.

i. $2(y+3) - xy\frac{dy}{dx} = 0$

ii. $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

iii. $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}\,dy = 0$

iv. $(1+y^2)\tan^{-1}x\,dx + 2y(1+x^2)dy = 0$

Example

Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

Assignments

1. Find the solution of the differential equation $e^x tan y\, dx + (1 - e^x) sec^2 y\, dy = 0$.

2. Find the equation of a curve passing through the point $(-2, 3)$, given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

3. In a bank, principal increases continuously at the rate of $r\%$ per year. Find the value of r if $Rs. 100$ double itself in 10 years. (use $log_e 2 = 0.6931$)

THANKING YOU

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