Homogeneous Differential Equation

SUBJECT : Mathematics
CHAPTER NUMBER: 09
CHAPTER NAME : Differential Equations

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Homogeneous Differential equation of the first order and first degree

Definition of Homogeneous Function

A function $f(x,y)$ is said to be a homogeneous function of degree n if $f(\lambda x, \square y) = \lambda^n f(x,y)$.

Example: Test the homogeneity of the function $f(x,y) = 3x^2 - 2y^2 + 7xy$

$$f(\lambda x, \square y) = 3(\lambda x)^2 - 2(\lambda y)^2 + 7(\lambda x)(\square y)$$

$$= \lambda^2(3x^2 - 2y^2 + 7xy) = \lambda^2 f(x,y)$$

Remember:

- If the sum of powers of x and y in each term is the same, then the function is homogeneous.
- If $f(x,y)$ is a homogeneous function of degree n, then we write

$$f(x,y) = x^n\left(\frac{y}{x}\right) = y^n\left(\frac{x}{y}\right)$$

Homogeneous Differential equation of the first order and first degree

Definition of Homogeneous Differential equation

A differential equation of the form $f(x,y)dx + g(x,y)dy = 0 \ldots\ldots(i)$

Is said to be homogeneous if $f(x,y)$ and $g(x,y)$ are homogeneous functions of the same degree.

Equation(i) can be written as $\frac{dy}{dx} = -\frac{f(x,y)}{g(x,y)} = F(x,y) \ldots\ldots(ii)$

Equation (ii) is a homogeneous differential equation if $F(x,y)$ is a homogeneous function of degree 0.

To solve equation (ii), put $y = vx$, where v is a function of x.

Example

Show that the differential equation $(x^2 - 2y^2)dx + 2xydy = 0$ is homogeneous and solve it.

Example

Show that the differential equation $x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.

Example

Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it.

Assignment

Show that the differential

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