Linear Differential Equation

SUBJECT : Mathematics
CHAPTER NUMBER: 09
CHAPTER NAME : Differential Equations

Linear Differential Equations

A differential equation is said to be linear if the dependent variable and its derivative occur only in the first degree and are not multiplied together.

The general form of a linear differential equation of the first order is

$$\frac{dy}{dx} + Py = Q$$

where P, Q are functions of x or constants.

Solution of Linear Differential Equation

The general solution to the linear differential equation $\frac{dy}{dx} + Py = Q$ is

$$ye^{\int P\,dx} = \int Qe^{\int P\,dx}\,dx + C\ ,$$

Here $e^{\int P\,dx}$ is called as Integrating factor(I.F.).

Thus the solution is $y(I.F.) = \int Q(I.F.)\,dx + C.$

Solution of Linear Differential Equation

Another form of linear differential equation is $\frac{dx}{dy} + P_1 x = Q_1$,

where P_1, Q_1 are functions of y only or constants.

The integrating factor in this case is $e^{\int P_1 dy}$ and its general solution is

$x(I.F.) = \int Q_1 (I.F.) dy + C,$ Here $I.F. = e^{\int P_1 dy}$

Solution of Linear Differential Equation

Steps involved to solve first order linear differential equation:

Step-1 Write the given differential equation in the form $\frac{dy}{dx} + Py = Q$ where P, Q are functions of x or constants.

Step-2 Find the integrating factor $(I.F.) = e^{\int P\,dx}$

Step-3 Write the solution of the given differential equation as $y(I.F.) = \int Q(I.F.)\,dx + C$.

In case, the first order linear differential equation is in the form $\frac{dx}{dy} + P_1 x = Q_1$, where P_1, Q_1 are functions of y only or constants. Then $I.F. = e^{\int P_1 dy}$ and the solution of the differential equation is given by x$(I.F.) = \int Q_1(I.F.)dy + C$.

Example

Find the integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$.

Example

Solve $\frac{dy}{dx}+\frac{4x}{x^2+1}y=-\frac{1}{(x^2+1)^3}$.

Example

Find the general solution of the differential equation $\frac{dy}{dx} - y = cosx$.

Example

Find the general solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$.

Example

Solve the differential equation $(x+2y^2)\frac{dy}{dx}=y$; given that when $x=2, y=1$.

Example

Find the equation of a curve passing through the point $(0, 1)$, if the slope of the tangent to the curve at any point (x, y) is equal to the sum of x coordinate and the product of the x coordinate and y coordinate of that point.

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