Integration as Inverse Process of Differentiation

SUBJECT :MATHEMATICS
CHAPTER NUMBER:7
CHAPTER NAME :INTEGRALS

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What we expect to learn?

- Students will be able understand the concept of indefinite integral as anti-derivative.
- Students will be able to know standard indefinite integrals and basic rules of indefinite integration .
- Students will be able to evaluate indefinite integrals by different methods.
- Students will be able to Use the Fundamental Theorem of Calculus to evaluate definite integrals.
- Students will be able to To understand how the limit of the sum of rectangles may be used to calculate the area bounded by a function.
- Students will able to understand the concept of definite integral and know the basic properties of definite integrals.
- Students will able to Use definite integrals to solve application problems.

Introduction

Integration is the inverse process of differentiation. In the differential calculus, we are given a function and we have to find the derivative or differential of this function, but in the integral calculus, we are to find a function whose differential is given. Thus, integration is a process which is the inverse of differentiation.

If a function 'f' is differentiable in an interval I, i.e its derivative f' exists at each point of I. Then functions that could have possibly given function as a derivative are called anti-derivative of the function. Further, the formula that gives all these anti-derivatives is called the indefinite integral of the function and such process of finding anti-derivatives is called integration.

The two forms of integral are indefinite and definite integral which together constitute integral calculus.

Integration as an Inverse Process of Differentiation

Let $F(x)$ and $f(x)$ be two functions connected such that $\frac{d}{dx}F(x) = f(x)$, then $F(x)$ is called integral $f(x)$ or indefinite integral or anti-derivatives.

If $\frac{d}{dx}F(x) = f(x)$ then for any constant C.

$\frac{d}{dx}[F(x) + C] = f(x)$. Thus $F(x) + C$ is an anti-derivatives of $f(x)$.

i.e $\int f(x)dx = F(x) + C$

Here C is an arbitrary constant known as the constant of integration.

$\int$ is the integral sign

$f(x)$ is the integrand, x is the variable of integration.

Example

Antiderivative (primitive or integral) of $f(x) = 2x$ is the function $F(x) = x^2$

Because $F'(x) = \frac{d}{dx}(x^2) = 2x = f(x)$

Which is written as $\int 2x \, dx = x^2 + C$

Some Standard Formulaes

(a) $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

(b) $\int dx = x + c$

(c) $\int \frac{1}{x} dx = log|x| + C$

(d) $\int sin\, x\, dx = -cos\, x + C$

(e) $\int cos\, x\, dx = sin\, x + C$

(f) $\int sec^2\, x\, dx = tan\, x + C$

(g) $\int cos\, e\, c^2 x dx = -cot\, x + C$

(h) $\int sec\, x\, .\, tan\, x\, dx = sec\, x + C$

(i) $\int cos\, e\, cx.\, cot\, x\, dx = -cos\, e\, cx + C$

(j) $\int a^x dx = \frac{a^x}{log\, a} + C, a > 0, a \neq 1$

(k) $\int e^x dx = e^x + C$

Some Standard Formulae

(l) $\int \frac{1}{\sqrt{1-x^2}} dx = sin^{-1} x + C \; or \; -cos^{-1} x + C$

(m) $\int \frac{1}{1+x^2} dx = tan^{-1} x + C \; or - tan^{-1} x + C$

(n$\int \frac{1}{x\sqrt{x^2-1}} dx = sec^{-1} x + C \; or - cos\,e\,c^{-1} x + C$

(o) $\int 0dx =$ C (constant)

Example

Evaluate the following integrals

I. $\int x^5 dx$

II. $\int \frac{1}{\sqrt{x}} dx$

III. $\int 5^x dx$

Algebra of Indefinite Integral

- The process of differentiation and integration are inverse of each other.

  i.e $\frac{d}{dx}\int f(x)dx = f(x)$ and $\int f^1(x)dx = f(x) + C$

- $\int\big(f(x) \pm g(x)\big)dx = \int f(x)dx \pm \int g(x)dx$

- $\int K.f(x)dx = K\int f(x)dx$, K is any constant

Note:- If more than one constant of integration is used while solving the integral, then at the end of the solution write only one constant of integration.

Example

Evaluate the following integrals

a) $\int(\sin x + \cos x)dx$

b) $\int(x + 1)dx$

c) $\int \frac{x^3-1}{x^2}dx$

d) $\int \frac{1}{1+\sin x}dx$

Integration by method of inspection

To find an anti-derivative of a given function by searching intuitively a function whose derivative is the given function.

Example:- Write an anti-derivative of $3x^2 + 4x^3$ by the method of inspection?

Solution:- $\frac{d}{dx}(x^3 + x^4) = 3x^2 + 4x^3$ $\therefore$ The anti-derivative of $3x^2 + 4x^3$ is $x^3 + x^4 + C$

Assignment

1. Evaluate the following integrals

$(a)\ \int \frac{2}{1+cos2x}$dx (b) $\int \frac{2}{1+cos2x}$dx

$(c)\ \int cot^2x\ dx$ $(d)\ \int e^{3\ logx}\ dx$

2. Find the anti derivative F of f defined by $f(x) = 4x^3 - 6$, where $F(0) = 3$.

3. Exercise 7.1 form NCERT book.

THANKING YOU

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