

METHODS OF INTEGRATION

SUBJECT :MATHEMATICS
CHAPTER NUMBER:7
CHAPTER NAME :INTEGRALS

CHANGING YOUR TOMORROW

Methods of Integration

Different methods for finding integral of a function are

- I. **Method of Substitution**
- II. **Integration by Parts**
- III. **Method of Partial Fraction**

Method of Substitution

In previous topic, we discussed the integration of those functions which are in standard forms. But integrals of certain functions cannot be obtained directly but they may be reduced to standard forms by proper substitution. The method of evaluating an integral by reducing it to standard form by a proper substitution is called integration by substitution.

Substitution method is used, when a suitable substitution of variable leads to simplification of integral. Try to substitute the variable whose derivative is present in the original integral and final integral must be written in terms of the original variable of integration.

Method of Substitution

Procedure

To evaluate an integral of the type $\int f(\phi(x))\phi'(x)dx$

we substitute $\phi(x) = t$ and $\phi'(x)dx = dt$

The above integral becomes $\int f(t)dt$

After evaluating this integral we substitute back the value of t .

Example

Evaluate the Integral

1. $\int 18x^2 \sqrt[4]{6x^3 + 5} dx$

2. $\int \frac{2x^3+1}{(x^4+2x)^3} dx$

Example

Evaluate the Integral

1. $\int \cot x \, dx$

2. $\int \operatorname{cosec} x \, dx$

3. $\int \tan^2 x \, dx$

Example

Evaluate the Integral

$$\int \frac{\cot(\log x)}{x} dx$$

Example

Evaluate the Integral

$$\int f(ax + b) dx$$

Ans: If $\int f(x)dx = g(x)$, then $\int f(ax + b)dx = \frac{1}{a}g(ax + b)+C$

Some Important deductions

$$(a) \int (ax + b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, n \neq -1$$

$$(b) \int \frac{1}{ax+b} dx = \frac{1}{a} \log|ax + b| + C$$

$$(c) \int a^{bx+c} dx = \frac{a^{bx+c}}{b \log a} + C, a > 0 \text{ and } a \neq 1$$

$$(d) \int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C$$

$$(e) \int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + C$$

$$(f) \int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + C$$

$$(g) \int \sec^2(ax + b) dx = \frac{1}{a} \tan(ax + b) + C$$

$$(h) \int \csc^2(ax + b) dx = -\frac{1}{a} \cot(ax + b) + C$$

Some Important deductions

$$(i) \int \sec(ax + b) \cdot \tan(ax + b) dx = \frac{1}{a} \sec(ax + b) + C$$

$$(j) \int \operatorname{cosec}(ax + b) \cdot \cot(ax + b) dx = -\frac{1}{a} \operatorname{cosec}(ax + b) + c$$

$$(k) \int \tan(ax + b) dx = \frac{1}{a} \log|\sec(ax + b)| + C$$

$$(l) \int \cot(ax + b) dx = \frac{1}{a} \log|\sin(ax + b)| + C$$

$$(m) \int \sec(ax + b) dx = \frac{1}{a} \log|\sec(ax + b) + \tan(ax + b)| + C$$

$$(n) \int \operatorname{cosec}(ax + b) dx = \frac{1}{a} \log|\operatorname{cosec}(ax + b) - \cot(ax + b)| + C$$

Examples

Evaluate the Integrals

1. $\int (2x - 3)^2 dx$

2. $\int \sqrt{3x + 2} dx$

3. $\int \frac{1}{2-3x} dx$

4. $\int e^{2x-3} dx$

5. $\int \cos e^{c^2(3x + 2)} dx$

6. $\int \sec^2(7 - 4x) dx$

7. $\int \sin(ax + b) \cdot \cos(ax + b) dx$

8. $\int 5^{3x+2} dx$

Assignments

1. Evaluate the integrals

$$(a) \int \frac{1}{\sqrt{5x-4}} dx$$

$$(b) \int \frac{\sin 4x}{\sin 2x} dx$$

$$(c) \int \tan^2(2x - 3) dx$$

$$(d) \int \frac{e^{\tan^{-1} x}}{1+x^2} dx$$

$$(e) \int \cos 6x \sqrt{1 + \sin 6x} dx$$

$$(f) \int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx$$

2. Exercise 7.2 from NCERT book.

THANKING YOU
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