Integration of Trigonometric Functions

SUBJECT :MATHEMATICS
CHAPTER NUMBER:7
CHAPTER NAME :INTEGRALS

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: **1800 120 2316**
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Integrals in Different Forms

When the integral involves some trigonometric functions then some known trigonometric identities are used to evaluate integral easily.

Integrals of the form $\int sin^m x\, dx \; or \int cos^m x\, dx,\ m \in N$

To evaluate we express $sin^m x\ (or\ cos^m x)$ in terms of sines and cosines of multiples of x. For which we use the following trigonometrical identities.

(a) $sin^2 x = \frac{1-cos\ 2x}{2}$

(b) $sin^3 x = \frac{3\ sin\ x - sin\ 3x}{4}$

(c) $cos^2 x = \frac{1+cos\ 2x}{2}$

(d) $cos^3 x = \frac{cos\ 3x + 3\ cos\ x}{4}$

Example

Evaluate

(a) $\int sin^2 x \, dx$

$(b) \int cos^3 x \, dx$

$(c) \int cos^4 x \, dx$

Integrals in Different Forms

Integrals of the form $\int sin^m x \, . cos^n x \, dx, \;\; m, n \in N$

To evaluate the integrals of the form $\int sin^m x \, . cos^n x \, dx$ we may use the following algorithm

Integrals in Different Forms

Algorithm

Step – I:- Obtain the integral say $\int sin^m x \, . cos^n x \, dx$

Step – II:- Check the exponents of $sin\, x$ and $cos\, x$

Step – III:- If the exponent of $sin\, x$ is an odd positive integer put $cos\, x = t$

If the exponent of $cos\, x$ is an odd positive integer put $sin\, x = t$

If the exponents of $sin\, x$ and $cos\, x$ both are odd positive integers put either $sin\, x = t$ or $cos\, x = t$.

If the exponents of $sin\, x$ and $cos\, x$ both are positive even integers then express $sin^m x \, . cos^n x$ in terms of sines and cosines of multiples of x by using trigonometric results.

Example

Evaluate

(a) $\int sin^3 x \, . cos^4 x \, dx$

$(b) \int sin^2 x \, . cos^2 x \, dx$

Integrals in Different Forms

Integrals of the form $\int \sin p\,x.\cos q\,x\,dx$ or $\int \sin p\,x.\sin q\,x\,dx$ or $\int \cos p\,x.\cos qx\,dx$

To evaluate these types of integral firstly multiply and divide by 2 then use the following trigonometric identities.

$$2\sin A\,.\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\,.\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\,.\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\,.\sin B = \cos(A-B) - \cos(A+B)$$

Example

Evaluate

(a) $\int sin\,4\,x.cos\,3\,x\;dx$

$(b)\int \sin 4x\,.\sin 8x\,dx$

$(c)\int \cos 2x\,.\cos 4x\,.\cos 6x\,dx$

Assignment

1. Evaluate the integrals

(a) $\int \frac{1}{1+tanx} dx$

(b) $\int \frac{1}{sin^2 x\, cos^2 x} dx$

(c) $\int cos^4 2x \, \mathrm{dx}$

(d) $\int tan^3 2x\, sec2x\, dx$

(e) $\int cos^3 x\, dx$

(f) $\int sin2x\, sin4x\, sin6x\, dx$

2. Exercise 7.3 from NCERT book.

ODM EDUCATIONAL GROUP
Changing your Tomorrow

THANKING YOU

ODM EDUCATIONAL GROUP