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Integration Using Trigonometric Substitutions

SUBJECT : MATHEMATICS
CHAPTER NUMBER: 7
CHAPTER NAME : INTEGRALS

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Integrals of Some Particular functions

Here some standard formulae and the methods to solve some standard integrals with the help of these formulae.

(a) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} log \left|\frac{x-a}{x+a}\right| + C$

(b) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} log \left|\frac{a+x}{a-x}\right| + C$

(c) $\int \frac{dx}{x^2+a^2} = \frac{1}{a} tan^{-1}\left(\frac{x}{a}\right) + C$

(d) $\int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x + \sqrt{x^2-a^2}\right| + C$

(e) $\int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x + \sqrt{x^2+a^2}\right| + C$

(f) $\int \frac{dx}{\sqrt{a^2-x^2}} = sin^{-1}\left(\frac{x}{a}\right) + C$

Example

Evaluate

(a) $\int \frac{dx}{x^2 - 16}$

(b) $\int \frac{dx}{\sqrt{9 - 25x^2}}$

Integrals in Different Forms

Integral of the form $\int \frac{1}{ax^2+bx+c}dx \quad or \quad \int \frac{dx}{\sqrt{ax^2+bx+c}}$

We write $ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left\{\left(x + \frac{b}{2a}\right)^2 + \left(\frac{c}{a} - \frac{b^2}{4a^2}\right)\right\}$

$= a(t^2 \pm k^2)$, here $t = x + \frac{b}{2a}$, $\frac{c}{a} - \frac{b^2}{4a^2} = \pm k^2$

Thus $\int \frac{1}{ax^2+bx+c}dx = \frac{1}{a}\int \frac{1}{t^2 \pm k^2}dt$ and $\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}}\int \frac{1}{\sqrt{t^2 \pm k^2}}dt$ which can be

integrated by using suitable formulae.

Example

Evaluate

$$(a)\ \int \frac{1}{3x^2 + 13x - 10}\,dx$$

$$(b)\ \int \frac{1}{\sqrt{3 - x + x^2}}\,dx$$

Integrals in Different Forms

Integral of the type $\int \frac{px+q}{ax^2+bx+c}dx$ *and* $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

To evaluate such integrals we first write the numerator as

$$px + q = A\left\{\frac{d}{dx}(ax^2 + bx + c)\right\} + B$$

$$\Rightarrow px + q = A(2ax + b) + B$$

Then find A and B by comparing the coefficients of like powers of x from both sides.

Now the given integral reduced to one of the known forms which can be integrated easily.

Example

Evaluate the following integrals

$$(a)\int \frac{x+2}{2x^2+6x+5}dx$$

$$(\mathrm{b})\int \frac{x+1}{\sqrt{2x^2+x-3}}dx$$

Assignments

1. Evaluate the integrals

(a) $\int \frac{1}{4x^2+12x+5}\,dx$

(b) $\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$

$(c)\int \frac{1-3x}{3x^2+4x+2}\,dx$

(d) $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$

2. Exercise 7.4 from NCERT book.

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