

Integration of the Different forms Using by Parts

SUBJECT : MATHEMATICS
CHAPTER NUMBER:7
CHAPTER NAME :INTEGRALS

CHANGING YOUR TOMORROW

Different forms Using By Parts

Integral of the type $\int e^x(f(x) + f'(x))dx = e^x \cdot f(x) + C$

Example

Evaluate the Integrals

$$(a) \int e^x (\sin x + \cos x) dx$$

$$(b) \int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx$$

Example

Evaluate the Integrals

$$(a) \int e^x \left(\frac{x^2 + 1}{(x + 1)^2} \right) dx$$

$$(b) \int \frac{x \cdot e^x}{(1 + x)^2} dx$$

Integrals of Some more Types Based on Integration By Parts Method

Here we will discuss some more special types of integrals which can be proved by using integration by parts and directly used to evaluate given integrals.

$$(a) \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log|x + \sqrt{x^2 - a^2}| + C$$

$$(b) \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log|x + \sqrt{x^2 + a^2}| + C$$

$$(c) \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

Example

Evaluate the Integrals

$$(a) \int \sqrt{1 - 4x^2} dx$$

$$(b) \int \sqrt{1 + \frac{x^2}{9}} dx$$

Another form of the Integral

Method to evaluate integrals of the form $\int \sqrt{ax^2 + bx + c} dx$

$$\text{Let } I = \int \sqrt{ax^2 + bx + c} dx$$

$$= \int \sqrt{t^2 \pm k^2} dt \text{ or } \int \sqrt{k^2 - t^2} dt \text{ (As discussed earlier)}$$

Then apply a suitable formula to integrate.

Example

Integrate

$$\int \sqrt{x^2 - 8x + 7} dx$$

Example

Integrate

$$\int \sqrt{1 + 3x - x^2} dx$$

Another form of the Integral

Method to Evaluate integrals of the form $\int (px + q)\sqrt{ax^2 + bx + c} dx$

To evaluate such integrals we firstly write $px + q = A \frac{d}{dx}(ax^2 + bx + c) + B$

$$= A(2ax + b) + B$$

Then find A and B by comparing the coefficient of like powers of x from both sides

Example

Evaluate $\int (x - 3)\sqrt{x^2 + 3x - 18} dx$

Assignments

1. Evaluate

a) $\int e^x (\tan x + \log(\sec x)) dx$

(b) $\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx$

c) $\int \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$

(d) $\int e^x \frac{(x-4)}{(x-2)^3} dx$

2. Answer the questions from Qno. 15 to 24 Exercise 7.6 and Exercise 7.7 from NCERT book.

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