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DEFINITE INTEGRAL AS LIMIT OF SUM

SUBJECT : MATHEMATICS
CHAPTER NUMBER:7
CHAPTER NAME :INTEGRALS

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Definite Integral

Introduction

In chapter-6 we used the tangent and rate of change problems to introduce the derivative, which is the central idea in differential calculus. In the same way, this chapter starts with area, which is used to formulate the idea of definite integral that is the basic concept of integral calculus. The Fundamental Theorem of Calculus relates the connection between Integral calculus and Differential calculus . We will see in this chapter that is greatly simplifies the solution of many problems.

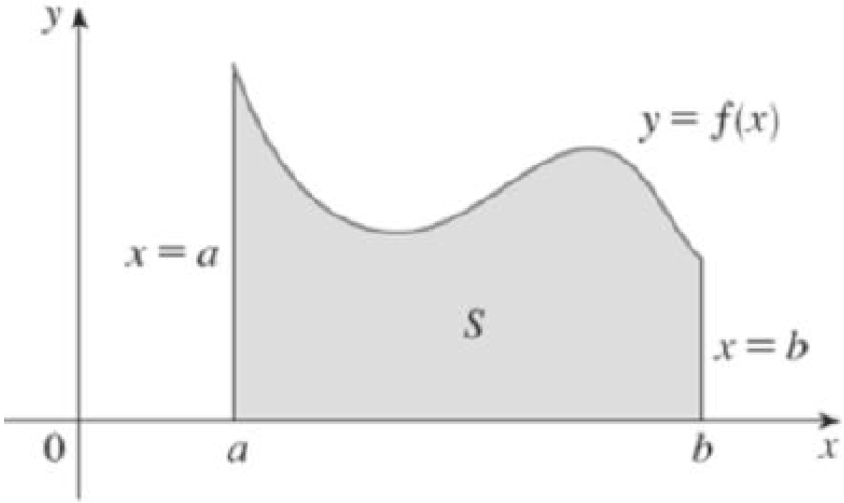
Definite Integral

An integral of the form of $\int_a^b f(x)dx$ is known as definite integral, 'a' and 'b' are called the lower and upper limits of a definite integral.

The definite integral $\int_a^b f(x)dx$ has a unique value, which represents the area bounded by the curve $y = f(x)$, the ordinates $x = a, x = b$ and the x-axis. The value of the definite integral can be evaluated either as the limit of a sum or if it has an anti-derivative F, then its value is the difference between the values of F at the endpoints i.e $\mathrm{F(b)} - \mathrm{F(a)}$

Definite Integral

Graphical representation of Definite integral



Definite Integral

Definite Integral as the limit of a sum

Suppose $f(x)$ be a continuous function in $[a, b]$ divide interval $[a, b]$ into n equal subintervals each of length h so that $h = \frac{b-a}{n}$

Then $\int_a^b f(x)dx = \lim_{h \to 0} h[f(a) + f(a+h) + \ldots + f(a+(n-1)h)]$

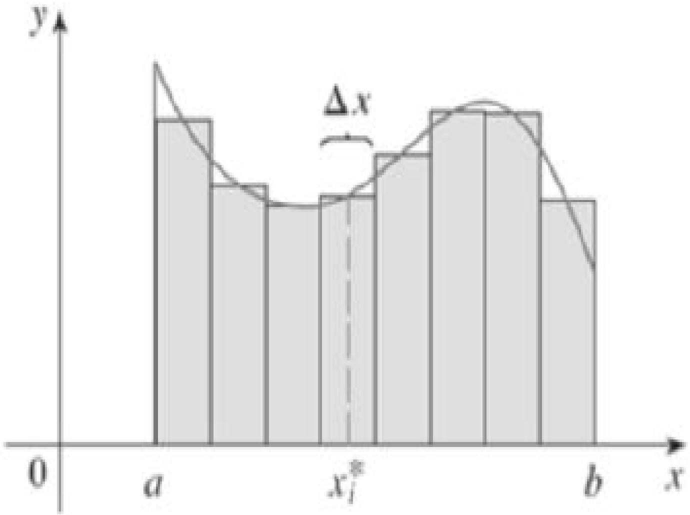
The above expression also written as

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a) + f(a+h) + \ldots.. + f(a+(n-1)h)]$$

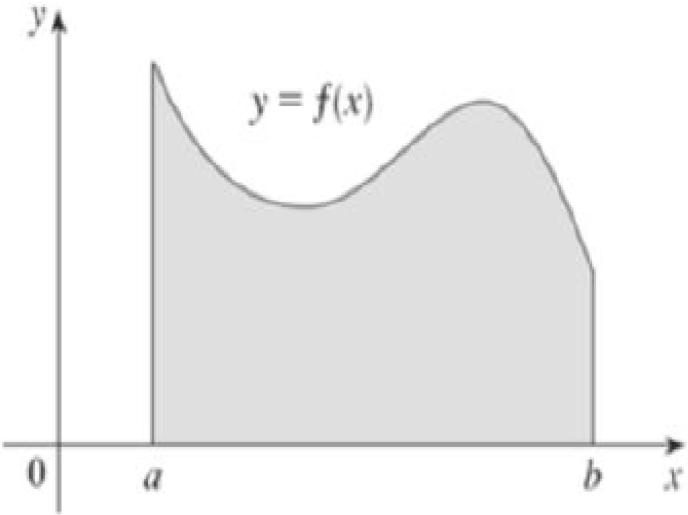
Where $h = \frac{b-a}{n} \to 0 \; as \; n \to \infty$

Definite Integral

Comparison between limit as a sum and area under the curve



Sum of areas of rectangles



Area under the curve $y = f(x)$ from a to b

Definite Integral

Another form of Definite Integral as a limit of Sum

If $f(x)$ be a continuous function in $[a, b]$ divide interval $[a, b]$ into n equal subintervals each of length h so that $h = \frac{b-a}{n}$

Then we got $\int_a^b f(x)dx = \lim_{h \to 0} h[f(a) + f(a+h) + \ldots + f(a+(n-1)h)]$

The above expression also written as

$$\int_a^b f(x)dx = \lim_{h \to 0} h[f(a+h) + f(a+2h) + \ldots + f(a+nh)]$$

Definite Integral

Working Rule

Suppose the given definite integral is $\int_a^b f(x)dx$. Then to find its value as the limit of a sum we use the following steps.

Step – I Compare the given integral with standard form and find the values of $f(x), a, b,$ and $nh = b - a$

Step – II Find the values of $f(x)$ at $x = a, a + h, a + 2h. \ldots., x = a + (n - 1)h$

Step – III Put the values obtained in the formula

$$\int_a^b f(x)dx = \lim_{h \to 0} h[f(a) + f(a + h) + f(a + 2h) + \ldots + f(a + (n - 1)h)]$$ and simplify it.

Step – IV While simplifying step – III we make a collection of constants $h, h^2, \ldots.$ etc and simplify it.

Step – V Finally, put the values of "nh" and simplify the limit to get the required result.

Example

Evaluate $\int_0^2 (x^2 + 1)dx$ as the limit of a sum.

Example

Evaluate $\int_0^2 e^x dx$ as the limit of a sum.

Assignments

1. Evaluate $\int_{1}^{3}(x^2 + 3x + e^x)\ dx$ as the limit of the sum.

2. Evaluate $\int_{-2}^{3}(3x^2 - 2x + 4)\ dx$ as the limit of a sum.

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