Fundamental Theorem of Integral Calculus

SUBJECT : MATHEMATICS
CHAPTER NUMBER:7
CHAPTER NAME :INTEGRALS

Fundamental Theorem of Integral Calculus

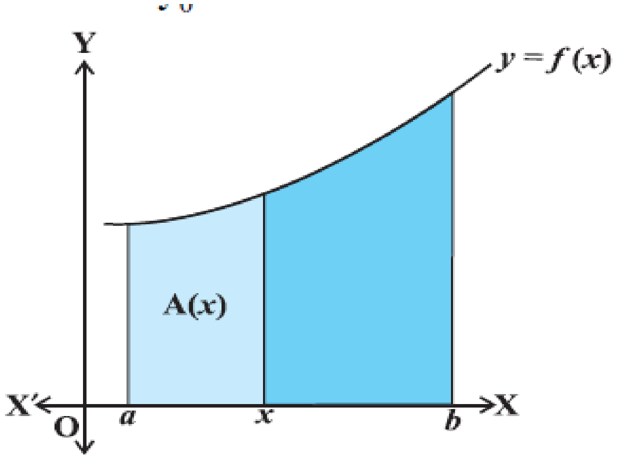
The fundamental theorem of integral calculus is a connection between indefinite integral and the definite integral.

The **First Fundamental theorem of integral calculus**

Let 'f' be a continuous function defined on the closed interval [a, b], then

$$\frac{d}{dx}\int_a^x f(t)dt = f(x), \qquad \text{for all } x \in [a, b]$$

Here $\int_a^x f(t)dt$ represents the area of the light shaded region



Fundamental Theorem of Integral Calculus

Second Fundamental Theorem of Integral Calculus

Let 'f' be a continuous function defined on the closed interval $[a, b]$ and F be an anti-derivative of 'f' then,

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

Note:-

1. This theorem tells us that $\int_a^b f(x)dx =$ (value of the anti derivative of f at upper limit)-

(value of the anti derivative of f at lower limit)

2. This theorem is very useful, because it gives us a method of calculating the definite integral

more easily, without calculating the limit of a sum.

3. There is no need to keep integration constant C because

$$\int_a^b f(x)dx = [F(x) + C]_a^b = (F(b) + C) - (F(a) + C) = F(b) - F(a)$$

Example

Evaluate $\int_{1}^{\sqrt{3}} \frac{1}{1+x^2}\,dx$

Example

Evaluate

$$(a)\ \int_0^1 (4x^3 + 3x^2 - 2x + 1)\, dx$$

$$(b)\ \int_0^{\pi/4} tan^2x\, dx$$

Evaluation of Definite Integrals by Substitution

There are several methods for finding the indefinite integral. One of the important methods for finding the indefinite integral is the method of substitution

To evaluate $\int_a^b f(x)dx$ by substitution, we use the following steps

Working Rule

Step – 1 Substitute some part of integral as another variable (say t) such that its differentiation exists in the integral so that given integral reduces to a known form.

Step – 2 Change the upper and lower limits corresponding to the new variable.

Step – 3 Integrate the new integral w.r.t the new variable.

Step – 4 Find the difference of the values of the answer obtained in step – III at new upper and lower limits.

Example

Evaluate

$$\int_{0}^{\pi/2} \frac{\cos x}{(1+\sin x)^2}\,dx$$

Example

Evaluate

$$\int_{-1}^{1} 5x^4\sqrt{x^5+1}\,dx$$

Assignments

1. Evaluate

(a) $\int_0^1 (4x^3 + 3x^2 - 2x + 1)\, dx$

(b) $\int_0^1 \frac{\tan^{-1} x}{1 + x^2} dx$

(c) $\int_0^{\pi/4} tan^2 x\, dx$

(d) $\int_1^3 \frac{1}{x(1 + \log x)} dx$

(e) $\int_0^2 x\sqrt{x + 2} dx$

(f) $\int_0^{2/3} \frac{1}{9x^2+4} dx$

2. Answer all the questions from exercise 7.9 and 7.10 NCERT book.

THANKING YOU

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