

Conditional Probability

SUBJECT : (MATHEMATICS)
CHAPTER NUMBER: 13
CHAPTER NAME : PROBABILITY

CHANGING YOUR TOMORROW

What we expect to learn?

- Students will learn about conditional probability and its results.
- Students will learn about independent events and how to check for independent events.
- Students will learn about multiplication theorem of Probability and questions based on these.
- Students will learn about theorem of Total Probability and its extension to Bayes' Theorem followed by applications
- Students will learn about Probability Distribution terms related to these and its applications in different situations.
- Students will learn to find mean and variance of Random variables
- Students will learn about Bernoulli's trials and Binomial distributions conditions under which it is valid. Mean, Variance and Standard Deviation of Binomial Distribution.
- Students will Learn about Mean and Variance for the Binomial Distribution.

Conditional Probability

Let A and B be any two events, $B \neq \phi$ Then $P(A/B)$ denotes the conditional probability of occurrence of event A when B has already occurred.

Example:

Let a bag contain 2 red balls and 3 black balls. One ball is drawn from the bag and this ball is not replaced in the bag. The second ball is drawn from the bag.

Let B denotes the event of the occurrence of a red ball in the first draw and A denotes the event of the occurrence of a black ball in the second draw.

When a red ball has been drawn in the first draw, the number of balls left is 4 and out of these four balls one is a red ball and three are black balls.

$\therefore P(A/B) = \text{Probability of occurrence of a black ball in the second draw when a red ball has been drawn in the first draw} = \frac{3}{4}$

Definition:

Let E and F are two events associated with the sample space of a random experiment the conditional probability of the event E given that F has occurred i.e. $P(E/F)$ is given by

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} \quad \text{provided } P(F) \neq 0$$
$$= \frac{\text{no of elementary events favorable to } E \cap F}{\text{No of elementary events which are favourable to } F}$$

Properties of conditional probability

Let E and F be events of a sample space S of an experiment then.

1. $P\left(\frac{S}{F}\right) = P\left(\frac{F}{F}\right) = 1$

2. If A and B are any two events of a sample space S of F is an event of S such that $P(F) \neq 0$

then $P\left(\frac{A \cup B}{F}\right) = P\left(\frac{A}{F}\right) + P\left(\frac{B}{F}\right) - P\left(\frac{A \cap B}{F}\right)$

3. $P\left(\frac{E'}{F}\right) = 1 - P\left(\frac{E}{F}\right)$

Problem-1

If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B/A) = 0.4$ then find

- (i) $P(A \cap B)$ (ii) $P(A/B)$ (iii) $P(A \cup B)$

Problem-2

Determine $P(E/F)$ for following

A coin is tossed three times, where

E: head on third toss

F: heads on first two toss.

Problem-3

Assume that each born child is equally likely to be a boy or a girl. If a family has two children.

What is the conditional probability that both are girls given that?

- (i) The youngest is a girl
- (ii) At least one is a girl

Problem-4

An instructor has a question bank consisting of 300 easy True/False questions. 200 difficult True/False questions. 500 easy multiple choice questions and 400 difficult multiple-choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question Given that it is a multiple-choice question?

HOME ASSIGNMENT

Q1. A fair die is rolled. Consider the event $E = \{1,3,5\}$, $F = \{2,3\}$ and $G = \{2,3,4,5\}$ Find

(i) $P\left(\frac{E}{F}\right)$ and $P\left(\frac{F}{E}\right)$ (ii) $P\left(\frac{E}{G}\right)$ and $P\left(\frac{G}{E}\right)$

(iii) $P\left(\frac{E \cup F}{G}\right)$ and $P\left(\frac{E \cap F}{G}\right)$

Q2. Given that two numbers appearing on throw of two dice are different. Find the Probability of the event 'the sum of the numbers on the dice is 4'.

Q3. Consider the experiment of throwing a die, If a multiple of three comes up, throw the die again, and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given 'at least one die shows a 3'.

THANKING YOU

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