Derivation of Bayes' theorem

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Partition of a set.

A family of sets A_1, A_2,…, A_n is said to form a partition of a set A if

1. $A_{1,} A_2, \ldots\ldots A_n$ are non-empty.

2. $A_i \cap A_j = \phi$ for $i \neq j$

3. $A = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n$

Definition of Bayes' Theorem:

If A_1, A_2, A_3…, A_n be n mutually exclusive and exhaustive events and A is an event that occurs together in conjunction with either A_i i.e. A_1, A_2,…, A_n from the partition of the sample space S and A be an event then.

$$P\left(\frac{A_k}{A}\right) = \frac{P(A_k) \times P\left(\frac{A}{A_k}\right)}{P(A_1) \times P\left(\frac{A}{A_1}\right) + P(A_2) \times P\left(\frac{A}{A_2}\right) + \dots\dots\dots\dots + P(A_n) \times P\left(\frac{A}{A_n}\right)}$$

Proof:-

Since $A_1, A_2, \ldots A_n$ form, a partition of S. Therefore

1. $A_1, A_2, \ldots A_n$ is non-empty sets.

2. $S = A_1 \cup A_2 \cup \ldots. \cup A_n$

Now

$$A = A \cap S = A \cap (A_1 \cup A_2 \cup \ldots. \cup A_n)$$
$$= (A \cap A_1) \cup (A \cap A_2) \cup \ldots. \cup (A \cap A_n) \quad \text{---------(1)}$$

Science $A_1, A_2, \ldots A_n$ are disjoint sets

$A \cap A_1, A \cap A_2, \ldots. A \cap A_n$ are also disjoint.

From (1), by addition theorem

$$P(A) = P(A \cap A_1) + P(A \cap A_2) + ... + P(A \cap A_n) \qquad \text{--------(2)}$$

Now

$$P\left(\frac{A_k}{A}\right) = \frac{P(A_k \cap A)}{P(A)}$$

$$= \frac{P(A_k \cap A)}{P(A_1 \cap A) + P(A_2 \cap A) + ----------- + P(A_n \cap A)}$$ From equation (2)

$$\Rightarrow P\left(\frac{A_k}{A}\right) = \frac{P(A_k) \times P\left(\frac{A}{A_k}\right)}{P(A_1) \times P\left(\frac{A}{A_1}\right) + P(A_2) \times P\left(\frac{A}{A_2}\right) + \text{..............} + P(A_n) \times P\left(\frac{A}{A_n}\right)}$$

Because $P(A \cap B) = P(B).P\left(A/B\right)$ Hence, Proved.

THANKING YOU

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