Direction Cosines and Direction Ratios of a line

SUBJECT : MATHEMATICS
CHAPTER NUMBER:11
CHAPTER NAME :THREE DIMENTIONAL GEOMETRY

What we expect to learn?

- Students will be able to the concept of direction cosines of a line.

- Students will be able to learn the need to work with direction ratios of a line.

- Students will be able to learn how to find direction cosines of a line when the direction ratios are given.

- Students will able to find the equation of line in vector and cartesian forms under different conditions.

- Students will be able to learn how to find angle between two lines.

- Students will able to find shortest distance between two given lines and coplanarity of two lines.

- Students will able to find the equation of plane in vector and cartesian forms under different conditions.

- Students will be able to learn how to find angle between two planes.

- Students will be able to learn how to find distance of a point from a plane.

- Students will be able to learn in what condition a line will lie in a plane.

- Students will be able to learn to find the equation of a plane passing through the intersection of two given planes.

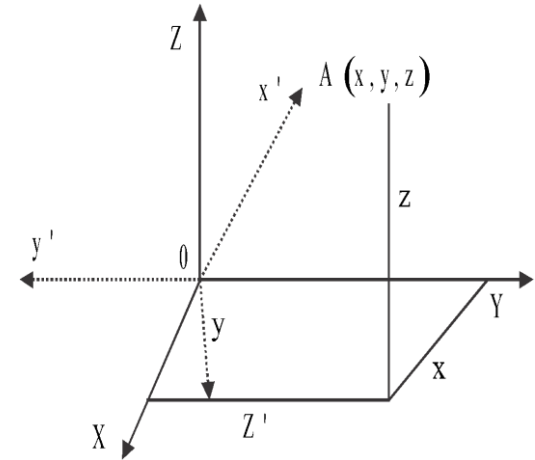Let A be any point in space such that

x=Perpendicular distance of A from YZ plane

y = Perpendicular distance of A from XZ plane

z = Perpendicular distance of A from XY plane

Then (x, y, z) are called co-ordinate of A we denote as $A(x, y, z)$

Direction Cosines(dcs) of a line

If a line makes angles α, β and γ with x, y, and z-axis respectively, then the angle α, β and γ are called direction angles of the line and cosines of these angles.

i.e $l = cos\ \alpha$, $m = cos\ \beta$ and $n = cos\ \gamma$ are called direction cosines.

➤ If l, m, n are direction cosines of a line, then $-l, -m, -n$ are also dcs of that line

➤ For any line, there are two sets of dcs.

➤ if l, m, n are dcs of a line that $l^2 + m^2 + n^2 = 1$

Direction Ratios(drs) of a line

Any three numbers which are parallel to dcs of a line are called direction ratio (drs) of that line

Let l, m, n be dcs of a line AB, then a, b, c are drs of AB

Where $a = \lambda l, b = \lambda m, c = \lambda n$, for some $\lambda \in R, \lambda \neq 0$

➤ Since λ being any real number so for any line, there are infinitely many sets of drs.

➤ By taking $\lambda = 1$, we get $a = l, b = m$, and $c = n$, for any line, a set of dcs is also a set of drs.

➤ The dcs of a line with drs as a, b, c are $l = \dfrac{a}{\pm\sqrt{a^2+b^2+c^2}}, m = \dfrac{b}{\pm\sqrt{a^2+b^2+c^2}}, n = \dfrac{c}{\pm\sqrt{a^2+b^2+c^2}}$

Direction Cosines and Direction Ratio of a line Through Two Given Points

The direction ratio of the line through two given points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$

is $x_2 - x_1, \ y_2 - y_1, \ z_2 - z_1$.

And direction cosines of the line through two given points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$

$$l = \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2 + (z_2 - z_1)^2}}, \ m = \frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2 + (z_2 - z_1)^2}}, \ n = \frac{z_2 - z_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2 + (z_2 - z_1)^2}}$$

Find the distance of the point $P(p, q, r)$ from the $x-$axis.

Can $\left(\dfrac{-1}{2\sqrt{3}}, \dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{3}}\right)$ be the direction cosines of any directed line ? Justify your answer.

What are the direction cosines of a line passing through the points $P(6, -7, -1)$ and $Q(2, -3, 1)$.

If a line makes, α, β, γ with the positive direction of co-ordinate axes then prove that

(a) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$ (b) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

Find the value of a for which points $(8, -7, a), (5,2,4)$ and $(6, -1,2)$ are collinear.

EXAMPLE

If a line makes angles 60^0 and 45^0 with positive direction of x and y axis respectively, then find the angle at which it is inclined to positive direction of z-axis.

Note- What is the direction cosines of x, y and z-axis ?

Assignments

1. What is the dcs of a line, if the drs of the line are $2, -1, -2$?

2. If a line makes angles $90^0, 135^0, 45^0$ with the positive direction of x, y, and z-axis

   respectively find its dcs.

3. If the dcs of a line is $\frac{1}{a}, \frac{1}{a}, \frac{1}{a}$, then find values of '$a$'.

4. Find the dcs of a line which makes equal angles with the co-ordinate axis

THANKING YOU
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