# Equation of a line in Space

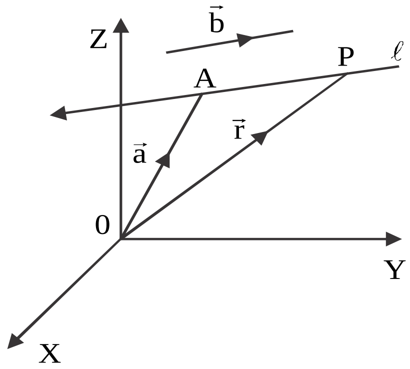
**SUBJECT : MATHEMATICS**
**CHAPTER NUMBER:11**
**CHAPTER NAME :THREE DIMENTIONAL GEOMETRY**

# Cartesian and Vector Equation of a line

**Vector equation** of a line through a given point where the position vector of the point is $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b},\ \lambda \in R$

Here the line $\ell$ passes through point A whose P.V is $\vec{a}$ and parallel to $\vec{b}$.

$\vec{r}$ be the P.V of an arbitrary point $P$ on the line




**Note:-**

If $\vec{b} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$, then $a, b, c$ are drs of the line and conversely if $a, b, c$ are drs of a line, then $\vec{b} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ will parallel to the line

# Cartesian Equation of line

The Cartesian equation of the line which passes through $(x_1, y_1, z_1)$ and $a, b, c$ as its drs is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

## EXAMPLE

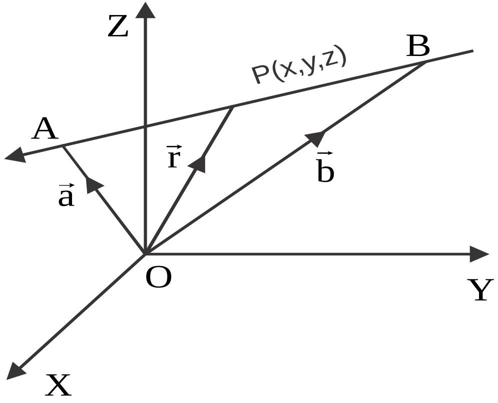
Find the vector and cartesian equation of the line passing through $(1, 2, 3)$ and having drs $2, 1, 0$.

# EXAMPLE

Find the equation of the line which passes through the point $(1, 2, 3)$ and is parallel to the vector $3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

# Equation of a line Passing Through Two Given Points

The **vector equation** of the line passing through the given points whose position vector as $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ .




In the figure $\vec{r}$ is the p.v of any point $P$ on line which passes through two points $A$ and B with p.v as $\vec{a}$ and $\vec{b}$ .

The **Cartesian equation** of the line passing through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_2}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Here we have $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}, \vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{b} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$

# EXAMPLE

Find the equation of the line through the points $(3, 4, -7)$ and $(1, -1, 6)$ in vector form as well as in the cartesian form.

# EXAMPLE

If the vector equation of a line is $\vec{r} = (2 + 3\lambda)\hat{\imath} - (1 + 5\lambda)\hat{\jmath} + 2(-3 + \lambda)\hat{k}$ , then reduce it to cartesian form.

# EXAMPLE

Find the direction ratio and the point through which the line passes $\frac{2x-1}{4} = \frac{3y-5}{2} = \frac{2-z}{3}$.

# Angle Between Two lines

Let $L_1$ and $L_2$ be two line having equations $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$. If $\theta$ be the angle between the two lines then, $cos\,\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

Here $\vec{b}_1 = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$ , $\vec{b}_2 = a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$ and $\theta$ is also the angle between $\vec{b}_1$ and $\vec{b}_2$

# Angle Between Two lines

- Two lines $L_1$ and $L_2$ are perpendicular if $\vec{b}_1.\vec{b}_2 = 0 \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$
- Two lines are parallel if $\vec{b} = \lambda\vec{b}$ $i.e \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- The angle $\theta$ between the two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is $cos\,\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$
- The angle $\theta$ between the two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

# EXAMPLE

Find the angle between the two lines $\vec{r} = \left(2\hat{\imath} - 5\hat{\jmath} + \hat{k}\right) + \lambda\left(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}\right)$

and $\vec{r} = \left(7\hat{\imath} - 6\hat{k}\right) + \mu\left(\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right)$ .

# EXAMPLE

Find the value $\lambda$ so that the lines $\frac{1-x}{3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{6-7}{7}$ are perpendicular to each other.

# Assignments

1. If the Cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write vector equation of the line

2. Show that the two lines given by $\vec{r} = (3\hat{\imath} + 8\hat{\jmath}) + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and

   $\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(-4i + 2j - 2k)$ are parallel to each other.

3. Question no 1 to 13 from Exercise 11.2 from NCERT.

# THANKING YOU

# ODM EDUCATIONAL GROUP