Shortest Distance Between Two lines

SUBJECT : MATHEMATICS
CHAPTER NUMBER:11
CHAPTER NAME :THREE DIMENTIONAL GEOMETRY

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Shortest Distance between two lines

Consider two lines L_1 and L_2 whose vector equations are given by $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$

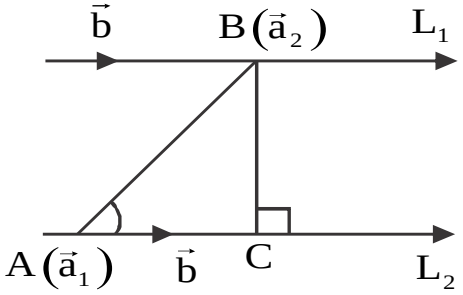
Let $\vec{a}_1$ be the p.v of point A on L_1 and $\vec{a}_2$ be the p.v of point B on L_2. Then the following cases arise

Case – I (L_1 and L_2 are intersecting)

In this case, the distance between L_1 and L_2 is zero

Shortest Distance between two lines

Case – II (L_1 and L_2 are parallel lines)



In this case, L_1 and L_2 are coplanar and $\vec{b}_1 = \vec{b}_2 = \vec{b}$

The shortest distance between two lines L_1 and L_2 is

$$\frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

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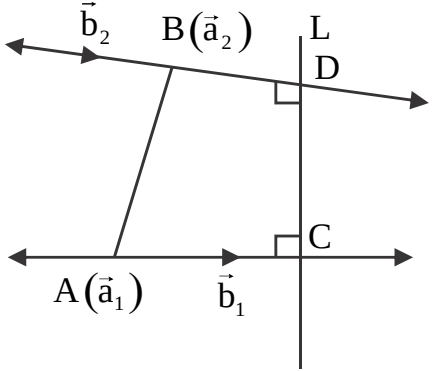
Case – III (L_1 and L_2 are neither intersecting nor parallel)

In this case, L_1 and L_2 are non-coplanar and $\vec{b}_1 \neq \vec{b}_2$

The shortest distance between L_1 and L_2 is $\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$



EXAMPLE

Find the shortest distance between two lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ and $\vec{r} = (2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + \mu(4\hat{\imath} + 6\hat{\jmath} + 8\hat{k})$

EXAMPLE

Find the shortest distance between two lines $\vec{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{r} = \left(2\hat{\imath} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{j} + 4\hat{k}\right)$. Write the nature of the lines.

EXAMPLE

Show that the two lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$

intersect. Also find their point of intersection.

EXAMPLE

Find the image of the point $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Assignments

1. Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3}$ intersect each other and find their point of intersection

2. Find the equation of the line passing through (2, -1, 3) and perpendicular to the lines.

$\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} - 2\hat{\jmath} + \hat{k}) and \vec{r} = (2\hat{\imath} - \hat{\jmath} - 3\hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

3. Question no 14 to 17 from Exercise 11.2 from NCERT.

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