Angle Between Two Planes & Angle between a Line and a Plane

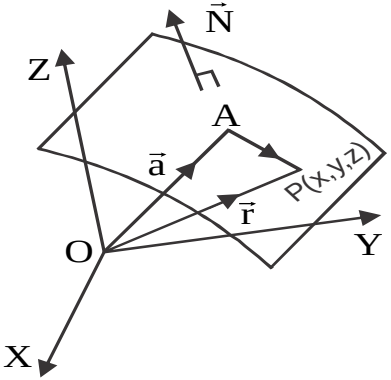
SUBJECT : MATHEMATICS
CHAPTER NUMBER:11
CHAPTER NAME :THREE DIMENTIONAL GEOMETRY

Website: www.odmegroup.org
Email: info@odmps.org
Toll Free: **1800 120 2316**
Sishu Vihar, Infocity Road, Patia, Bhubaneswar- 751024

Equation of a Plane Perpendicular to a Given Vector and Passing Through a Given Point

Vector equation of the plane passing through a given point whose position vector $\vec{a}$ and perpendicular to $\vec{N}$ is $(\vec{r} - \vec{a}).\vec{N} = 0$

Here $\vec{r}$ is the p.v. of any point P on a plane and A is a point on the plane whose p.v is $\vec{a}$.



Cartesian equation of the plane passing through the point $A(x, y, z)$ and a, b, c as drs of normal is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Here $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}, \vec{N} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Example

Find the vector and cartesian equation of the plane passing through the point $(1,2,3)$ and perpendicular to the line with direction ratios $2, 3, -4$.

Equation of the Plane Passing Through Three Given Points which are Non-collinear

The **vector equation** of the plane through three non collinear points A, B, C whose position vectors are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively is $(\vec{r} - \vec{a}).\left[\left(\vec{b} - \vec{a}\right) \times (\vec{c} - \vec{a})\right] = 0.$

The cartesian equation of the plane which passes through three points $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

Here $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$, $\vec{b} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}, \vec{c} = x_3\hat{\imath} + y_3\hat{\jmath} + z_3\hat{k}$

Example

Find the equation of the plane whose intercepts on the coordinate axes as a, b, and c respectively.

Angle Between two planes

The angle between two planes is defined as the angle between their normals. Let θ be the angle between two planes $\vec{r}.\vec{N}_1 = d_1$ and $\vec{r}.\vec{N}_2 = d_2$ then θ will be the angle between $\vec{N}_1$ and $\vec{N}_2$

$$\cos\theta = \left|\frac{\vec{N}_1.\vec{N}_2}{|\vec{N}_1||\vec{N}_2|}\right|$$

If θ be the angle between planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$

Here normal has drs a_1, b_1, c_1 & a_2, b_2, c_2 then $\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

Note:-

- Two planes are perpendicular if $\vec{N}_1.\vec{N}_2 = 0$ or $a_1a_2 + b_1b_2 + c_1c_2 = 0$
- Two planes are parallel if $\vec{N}_1 = \lambda\vec{N}_2$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Example

Find the the angle between the two planes $\vec{r}.\left(2\hat{\imath}-3\hat{\jmath}+4\hat{k}\right)=1$ and $\vec{r}.(-\hat{\imath}+\hat{\jmath})=4$

Angle Between a line and a Plane

The angle between a line and a plane is defined as the complement of the angle between the line and the normal to plane. when θ is the angle between the line and a plane, then $\frac{\pi}{2} - \theta$ is the angle between the line and the normal to the plane.

Then the angle θ between the line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ and plane $Ax + By + Cz + D = 0$ is

$$\cos\left(\frac{\pi}{2} - \theta\right) = \left|\frac{aA + bB + cC}{\sqrt{a^2 + b^2 + c^2}\sqrt{A^2 + B^2 + C^2}}\right|$$

$$\Rightarrow \sin\theta = \left|\frac{aA + bB + cC}{\sqrt{a^2 + b^2 + c^2}\sqrt{A^2 + B^2 + C^2}}\right|$$

Example

Find the the angle between the line $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and the plane $\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$.

Example

Find the intercepts cut off by the plane $2x + y - z = 5$.

Example

If the planes $\vec{r}.\left(2\hat{\imath}-\hat{\jmath}+\lambda\hat{k}\right)=15$ and $3x+2y+2z=7$ are perpendicular to each other then find the value of λ.

Assignments

1. Find the angle between two planes $3x - 6y + 2z = 7$ and $2x + 2y - 2z = 5$
2. Show that the planes $2x + 6y + 6z = 7$ and $\vec{r}.\left(3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}\right) = 8$ are perpendicular to each other.
3. If the line $\frac{x-1}{2} = \frac{y+4}{1} = \frac{z-7}{2}$ is parallel to the plane $3x - 2y + cz = 14$ then find the value of c.
4. Find the co-ordinate of the point where the line joining points (1, -2, 3) and (2, -1, 5) cuts the plane $x - 2y + 3z = 19$
3. Question no 5, 6,7,8,12,13 from Exercise 11.3 from NCERT book.

ODM EDUCATIONAL GROUP
Changing your Tomorrow

THANKING YOU

ODM EDUCATIONAL GROUP